LE 200 Complex Variables Assignment

1. Regarding the following functions:

(A) z^{-1} (B) $(z+1)^{-1}$ (C) $(z-1)(z+1)^{-1}$ (D) $(z+1)(z-1)^{-1}$
(E) $(2z-1)(z+1)^{-1}$ (F) $(z+j)^{-1}$ (G) $(j-z)(j+z)^{-1}$ (H) $(j+z)(j-z)^{-1}$ (I) $(z+1)z^{-1}$ (J) $(4z+1)^{-1}$
(K) $z(z+j)^{-1}$ (L) $z(z+1)^{-1}$ (M) $(z-j)(z+1)^{-1}$ (N) $(z+1)(z-j)^{-1}$ (O) $(z+j2)(z-j)^{-1}$

(i) Sketch the images of the following <u>lines</u> in <u>*w*-plane</u>.

(a) $x = 1$ (b) $x = 1/2$ (c) $x = 3$ (d) $y = 1$ (e) $y = 1/2$ (f) $y = 3$

(ii) Sketch the images of the following <u>lines</u> in <u>*z*-plane</u>.

(a) $u = -1$ (b) $u = 1/2$ (c) $u = 3$ (d) $v = -1$ (e) $v = 1/2$ (f) $v = 3$

2. Show that

(a) $\tanh^{-1} z = \frac{1}{2}\ln\left(\frac{1+z}{1-z}\right)$ (b) $\sinh^{-1} z = \ln\left(z+\sqrt{z^2+1}\right)$

(c) $\tan^{-1} z = \frac{1}{j2}\ln\left(\frac{1+jz}{1-jz}\right)$ (d) $\cos^{-1} z = -j\ln\left(z+\sqrt{z^2-1}\right)$

3. Regarding the following functions $f(z)$:

(a) $\sinh z$ (b) $\cosh z$ (c) $1/z$ (d) e^z (e) e^{z^2} (f) $(z-1)(z+1)^{-1}$ (g) $(z+1)(z-1)^{-1}$ (h) $(z+1)^{-1}$
(i) $\cos z$ (j) z^{-2} (k) $z + z^{-1}$ (l) $\sin z \cos z$ (m) $\cosh^2 z$ (n) $\sin^2 z$ (o) $\cos^2 z$ (p) $\sinh^2 z$

(i) Find the derivatives from <u>the definition</u>.

(ii) Use Cauchy-Riemann equations to show their analyticity and verify that their derivatives are given by $f'(z) = u_x + jv_x$.

4. Evaluate the following integrals along the specified paths:

(i) $\int_{z=0}^{z=1+j} \mathrm{Re}\, z\, dz$

(a) $x = y$ (b) C_1: (0,0)->(1,0) and C_2: (1,0)->(1,*j*) (c) C_3: (0,0)->(0,*j*) and C_4: (0,*j*)->(1,*j*)

(ii) $\int_{z=0}^{z=1+j} |z|^2\, dz$

(a) $x = y$ (b) C_1: (0,0)->(1,0) and C_2: (1,0)->(1,*j*) (c) C_3: (0,0)->(0,*j*) and C_4: (0,*j*)->(1,*j*)

(iii) $\int_{-j}^{j} \sinh z\, dz$

(a) C_1: (0,-*j*)->(0,*j*) (b) C_2: (0,-*j*)->(1,-*j*), C_3: (1,-*j*)->(1,*j*) and C_4: (1,*j*)->(0,*j*)

5. Find the Taylor's series of the following functions $f(z)$ about the specified points.

(a) $z \sinh z$, $z=0$ (b) $z \cosh z$, $z=0$ (c) $z \sin z$, $z=0$ (d) $z \cos z$, $z=0$ (e) ze^z, $z=0$ (f) ze^{-z}, $z=0$
(g) $\cos z^2$, $z=0$ (h) e^{z^2}, $z=0$ (i) e^{-z^2}, $z=0$ (j) ze^{jz}, $z=0$ (k) z^2e^{-jz}, $z=0$ (l) $z \sinh z$, $z=1$
(m) $z \cosh z$, $z=1$ (n) $z \sin z$, $z=1$ (o) $\sin z^2$, $z=0$ (p) $\cosh z^2$, $z=0$

6. Find the Laurent's series of the following functions $f(z)$ about the specified poles.

(a) $\frac{1}{z^2+4z+3}$, $z = -1$ (b) $\frac{z}{z^2+4z+3}$, $z = -3$ (c) $\frac{z^2}{z^2+4z+3}$, $z = -1$ (d) $\frac{z^2+1}{z^2+4z+3}$, $z = -3$

(e) $\frac{1}{z^3+2z^2+z}$, $z = 0$ (f) $\frac{z^2+1}{z^3+2z^2+z}$, $z = -1$ (g) $\frac{z+3}{z^3+2z^2+z}$, $z = 0$ (h) $\frac{3z+2}{z^3+2z^2+z}$, $z = -1$

(i) $\frac{1}{z^2+5z+4}$, $z = -1$ (j) $\frac{z^2}{z^2+5z+4}$, $z = -4$ (k) $\frac{z}{z^2+5z+4}$, $z = -1$ (l) $\frac{z+2}{z^2+5z+4}$, $z = -4$

(m) $\frac{(z+1)^2}{z^2+5z}$, $z = 0$ (n) $\frac{(z+2)^2}{z^2+5z}$, $z = -5$ (o) $\frac{z+1}{z^2+5z}$, $z = 0$ (p) $\frac{z+3}{z^2+5z}$, $z = -5$

7. Find the Residues at all poles of functions given in 6.

8. Consider $f(z)$ in 6, evaluate $\oint_C f(z)dz$ for the specified contours.

(a) C:|z+1|=1 (b) C:|z+4|=2 (c) C:|z|=1 (d) C:|z+5|=3 (e) C:|z+1|=2 (f) C:|2z+1|=1
(g) C:|3z|=1 (h) C:|2z+1|=2 (i) C:|z+1|=1 (j) C:|z+4|=2 (k) C:|z+1|=1 (l) C:|z+4|=2
(m) C:|z+1|=1 (n) C:|z+4|=2 (o) C:|z+1|=2 (p) C:|z+4|=3

9. Evaluate the following improper integrals:

(a) $\int_0^\infty \frac{dx}{1+x^4}$ (b) $\int_0^\infty \frac{x^2dx}{2+3x^2+x^4}$ (c) $\int_0^\infty \frac{x^2dx}{5+6x^2+x^4}$ (d) $\int_0^\infty \frac{dx}{6+5x^2+x^4}$ (e) $\int_0^\infty \frac{x^2dx}{x^4+16}$

(f) $\int_0^\infty \frac{x^2dx}{1+x^6}$ (g) $\int_0^\infty \frac{dx}{(1+x^2)^2}$ (h) $\int_0^\infty \frac{dx}{2+3x^2+x^4}$ (i) $\int_0^\infty \frac{dx}{1+x^6}$ (j) $\int_0^\infty \frac{dx}{1+x^8}$

(k) $\int_0^\infty \frac{x^4dx}{(1+x^2)^2}$ (l) $\int_0^\infty \frac{x^2dx}{1+x^8}$ (m) $\int_0^\infty \frac{x^4dx}{8+x^6}$ (n) $\int_0^\infty \frac{(x^2+4)dx}{4+3x^2+x^4}$ (o) $\int_0^\infty \frac{x^4dx}{(1+x^4)^2}$

10. Find the inverse Laplace transform required in the Laplace transform practice by evaluating Bromwich integral.

11. Find the partial fraction of

(a) $\frac{15x+9}{x^3-9x}$ (b) $\frac{5x^2+2x+4}{x^3-6x^2+11x-6}$ (c) $\frac{5x^2+3x+9}{x^3+4x^2+4x}$ (d) $\frac{4x^2+2x+7}{x^3+4x^2+4x}$

(e) $\frac{x^2+2x+3}{x^3+x^2-x-1}$ (f) $\frac{2x^2+3x+2}{x^3+x^2-x-1}$ (g) $\frac{x^2+2x+3}{x^3-5x^2+8x-4}$ (h) $\frac{x^2+4x+5}{x^3-5x^2+8x-4}$

(i) $\frac{3x^2+4x+2}{x^3-6x^2+11x-6}$ (j) $\frac{8x+15}{x^3-9x}$ (k) $\frac{5x^2+4x+2}{x^3+6x^2+11x+6}$ (l) $\frac{x^2+4x+1}{x^3+4x^2+5x+2}$

(m) $\frac{x^2+2x+4}{x^3-3x-2}$ (n) $\frac{x^2-2x+4}{x^3-x^2-10x-8}$ (o) $\frac{2x^2+3}{x^3-2x^2-x+2}$