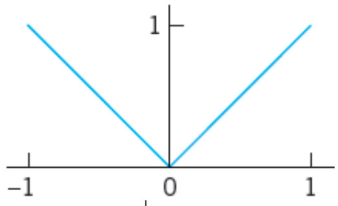
Fourier Transform Practice Problems

<table>
<tr><th colspan="2">$f(x)$ or $f(t)$</th><th>ODE</th></tr>
<tr>
<td>
1


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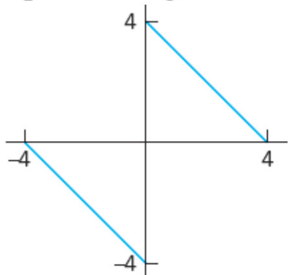

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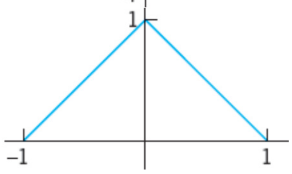

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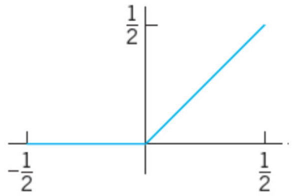

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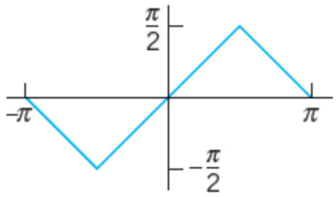

6 $f(x) = x^2$ $(-1<x<1)$, $p = 2$

7 $f(x) = x|x|$ $(-1<x<1)$, $p = 2$
</td>
<td>
a. $y''-y'-6y = f(t)$
b. $y''-6y'+5y = f(t)$
c. $y''-4y'+4y = f(t)$
d. $y''+5y'+6y = f(t)$
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Question

1. Find the Fourier series of $f(x)$, then find the "steady-state" solution for the ODE.
2. Find the Fourier transform of $f(x)$ assuming that $p \to \infty$.
3. Prove $f(x) \leftrightarrow \hat{f}(w), g(x) \leftrightarrow \hat{g}(w) \to fg \leftrightarrow \dfrac{1}{2\pi}\hat{f} * \hat{g}$.
4. Find the Fourier cosine transforms of:

(a) $-x + a$, $0 < x < a$ (b) e^{-ax^2} $(a > 0)$

5. Find the Fourier sine transforms of:

(a) $-x + a$, $0 < x < a$ (b) $x\,e^{-ax^2}$ $(a > 0)$

6. Show that

(a)
$$\int_0^\infty \frac{\cos xw + w \sin xw}{1 + w^2}\, dx = \begin{cases} 0 & \text{if} \quad x < 0 \\ \pi/2 & \text{if} \quad x = 0 \\ \pi e^{-x} & \text{if} \quad x > 0 \end{cases}$$

(b)
$$\int_0^\infty \frac{\sin \pi w \sin xw}{1 - w^2}\, dw = \begin{cases} \frac{\pi}{2} \sin x & \text{if} \quad 0 \leqq x \leqq \pi \\ 0 & \text{if} \quad x > \pi \end{cases}$$

(c)
$$\int_0^\infty \frac{\cos \frac{1}{2}\pi w}{1 - w^2} \cos xw\, dw = \begin{cases} \frac{1}{2}\pi \cos x & \text{if} \quad 0 < |x| < \frac{1}{2}\pi \\ 0 & \text{if} \quad |x| \geqq \frac{1}{2}\pi \end{cases}$$

(d)
$$\int_0^\infty \frac{w^3 \sin xw}{w^4 + 4}\, dw = \frac{1}{2}\pi e^{-x} \cos x \quad \text{if} \quad x > 0$$

7. Solve the following integral equations; (i.e., find $f(x)$)

(a) $\int_0^\infty f(x) \cos wx dx = e^{-w^2/4a}$ (b) $\int_0^\infty f(x) \sin wx dx = we^{-w^2/2}$