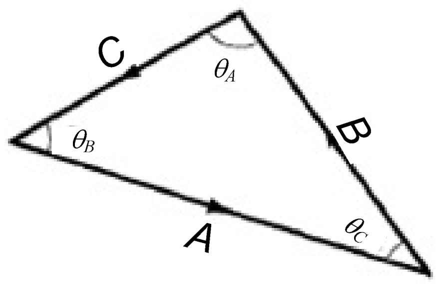
Homework #1 Solution

1-2. Consider the triangle in the figure on the right and let S denote its area, then



$$S = \frac{1}{2}|\mathbf{B}\times\mathbf{C}| = \frac{1}{2}|\mathbf{A}\times\mathbf{C}| = \frac{1}{2}|\mathbf{A}\times\mathbf{B}|.$$

Thus, $BC\sin\theta_A = AC\sin\theta_B = AB\sin\theta_C$.

Dividing every term by ABC yields

$$\frac{\sin\theta_A}{A} = \frac{\sin\theta_B}{B} = \frac{\sin\theta_C}{C}.$$

1-3. $\overline{PQ} = \overline{PB} + \overline{BQ} = \frac{1}{2}\overline{AB} + \frac{1}{2}\overline{BC};\ \overline{RS} = \overline{RD} + \overline{DS} = \frac{1}{2}\overline{CD} + \frac{1}{2}\overline{DA}$

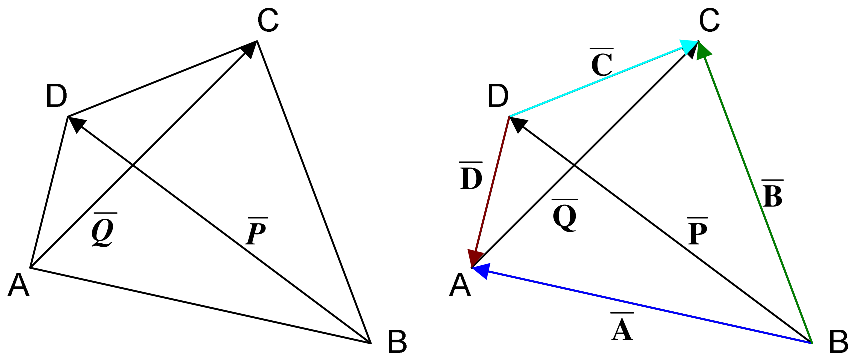
But $\overline{AB} + \overline{BC} + \overline{CD} + \overline{DA} = \overline{0}$, thus

$$\overline{RS} = \frac{1}{2}\overline{CD} + \frac{1}{2}\overline{DA} = -\frac{1}{2}\left(\overline{AB} + \overline{BC}\right) = -\overline{PQ}$$

Likewise, $\overline{QR} = -\overline{SP}$. Hence, $\overline{PQ} + \overline{QR} + \overline{RS} + \overline{SP} = \overline{0}$ and PQRS is a parallelogram.

1-4. Show that the area of the quadrilateral shown in figure 2 is given by:

$$Area = \frac{1}{2}\left|\overline{\boldsymbol{P}}\times\overline{\boldsymbol{Q}}\right|$$



Define vectors **A, B, C, D** as shown above.

$$\overline{\mathbf{P}}\times\overline{\mathbf{Q}} = (\overline{\mathbf{A}} - \overline{\mathbf{B}})\times(\overline{\mathbf{A}} - \overline{\mathbf{D}}) = -\overline{\mathbf{B}}\times\overline{\mathbf{A}} + \overline{\mathbf{B}}\times\overline{\mathbf{D}} - \overline{\mathbf{A}}\times\overline{\mathbf{D}}$$

Since vectors **A, B, C, D** form a rectangle, $\overline{\mathbf{A}} - \overline{\mathbf{D}} + \overline{\mathbf{C}} - \overline{\mathbf{B}} = \overline{\mathbf{0}}$ or $\overline{\mathbf{B}} - \overline{\mathbf{A}} = -\overline{\mathbf{D}} + \overline{\mathbf{C}}$

$$\therefore \overline{\mathbf{P}}\times\overline{\mathbf{Q}} = -\overline{\mathbf{B}}\times\overline{\mathbf{A}} + (\overline{\mathbf{B}} - \overline{\mathbf{A}})\times\overline{\mathbf{D}} = \overline{\mathbf{A}}\times\overline{\mathbf{B}} + (-\overline{\mathbf{D}} + \overline{\mathbf{C}})\times\overline{\mathbf{D}} = \overline{\mathbf{A}}\times\overline{\mathbf{B}} + \overline{\mathbf{C}}\times\overline{\mathbf{D}}$$

It follows that

$$\frac{1}{2}\left|\overline{\mathbf{P}}\times\overline{\mathbf{Q}}\right| = \frac{1}{2}\left|\overline{\mathbf{A}}\times\overline{\mathbf{B}} + \overline{\mathbf{C}}\times\overline{\mathbf{D}}\right| = \frac{1}{2}\left|\overline{\mathbf{A}}\times\overline{\mathbf{B}}\right| + \frac{1}{2}\left|\overline{\mathbf{C}}\times\overline{\mathbf{D}}\right| = \text{Area}$$