Homework #9 Solution

1. First find the magnetic flux using the magnetic circuit.

$$NI = \mathcal{R}\Phi; \mathcal{R} = \frac{2\ell_g}{\mu_0 S} + \frac{\ell_i}{\mu_0 \mu_r S} \rightarrow \Phi = \frac{NI}{\frac{2\ell_g}{\mu_0 S} + \frac{\ell_i}{\mu_0 \mu_r S}}.$$

Then, the force from the electromagnet is given by

$$F = \frac{\Phi^2}{\mu_0 S} = \frac{(NI)^2}{\mu_0 S\left(\frac{2\ell_g}{\mu_0 S} + \frac{\ell_i}{\mu_0 \mu_r S}\right)^2} = \frac{(NI)^2 \mu_0 S}{(2\ell_g + \frac{\ell_i}{\mu_r})^2}.$$

2. $\mathbf{B} = \hat{\mathbf{z}} 3\cos(5\pi 10^7 t - \frac{2}{3}\pi x)$ (μT)

$$\int_S \mathbf{B}\cdot d\mathbf{s} = \int_0^{0.6} \hat{\mathbf{z}} 3\cos(5\pi 10^7 t - \frac{2}{3}\pi x) 10^{-6} \cdot (\hat{\mathbf{z}} 0.2 dx) = -\frac{0.18}{2\pi}\left[\sin(5\pi 10^7 t - 0.4\pi) - \sin 5\pi 10^7 t\right] 10^{-6} \text{ (Wb)}$$

$$\mathcal{V} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s} = 45\left[\cos(5\pi 10^7 t - 0.4\pi) - \cos 5\pi 10^7 t\right] \text{(V)}$$

$$i = \frac{\mathcal{V}}{2R} = 1.5\left[\cos(5\pi 10^7 t - 0.4\pi) - \cos 5\pi 10^7 t\right] = 1.76\sin(5\pi 10^7 t - 0.2\pi) \text{(A)}$$

3. There are two ways to solve this problem.

Method 1: direct method

$$\mathcal{V} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s} = -\int_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s} + \oint_C (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$= \int_{x=0}^{0.06}\int_0^y 4(10^6)\sin(10^6 t - y')dy'dx + \int_{0.06}^0 [\hat{\mathbf{y}} 20 \times \hat{\mathbf{z}} 4\cos(10^6 t - y)]\cdot \hat{\mathbf{x}} dx$$

$$= 24(10^4)\cos(10^6 t - y')\Big|_0^y - 80(0.06)\cos(10^6 t - y)$$

$$= 240000[\cos(10^6 t - y) - \cos 10^6 t] - 4.8\cos(10^6 t - y) \qquad \text{(V)}$$

Method 2: First determine the magnetic flux

$$\Phi = \int_S \mathbf{B}\cdot d\mathbf{s} = \int_0^y \int_{x=0}^{0.06} \hat{\mathbf{z}} 4\cos(10^6 t - y')\cdot \hat{\mathbf{z}} dx dy' = -4(0.06)\sin(10^6 t - y')\Big|_0^y$$

$$= -0.24[\sin(10^6 t - y) - \sin 10^6 t] \qquad \text{(Wb)}$$

Note that $u = dy/dt$ -> $y = ut$. Thus,

$$\mathcal{V} = -\frac{d\Phi}{dt} =$$

$$= 240000[\cos(10^6 t - y) - \cos 10^6 t] - 4.8\cos(10^6 t - y) \qquad \text{(V)}$$