the *dielectric constant*. Scientists and engineers usually designate the square root of the relative permittivity as the *index of refraction*. Typical values of dielectric constants at static frequencies of some prominent dielectric materials are listed in Table 2-1.

Thus the dielectric constant of a dielectric material is a parameter that indicates the relative (compared to free-space) charge (energy) storage capabilities of a dielectric material; the larger its value, the greater its ability to store charge (energy). Parallel plate capacitors utilize dielectric material between their plates to increase their charge (energy) storage capacity by forcing extra free charges to be induced on the plates. These free charges neutralize the bound charges on the surface of the dielectric so that the voltage and electric field intensity is maintained constant between the plates.

Example 2-1. The static dielectric constant of water is 81. Assuming the electric field intensity applied to water is 1 V/m, determine the magnitudes of the electric flux density and electric polarization vector within the water.

Solution. Using (2-9), we have

$$D = \varepsilon_s E_a = 81(8.854 \times 10^{-12})(1) = 7.17 \times 10^{-10}\ \text{C/m}^2$$

Using (2-12), we have

$$\chi_e = \varepsilon_{sr} - 1 = 81 - 1 = 80$$

Thus the electric polarization vector is given, using (2-10), by

$$P = \varepsilon_0 \chi_e E_a = 8.854 \times 10^{-12}(80)(1) = 7.08 \times 10^{-12}\ \text{C/m}^2$$

The permittivity of (2-11a), or its relative form of (2-12), represents values at static or quasistatic frequencies. These values vary as a function of the alternating field frequency. The variations of the permittivity as a function of the frequency of the applied fields are examined in Section 2.8.1.

2.3 MAGNETICS, MAGNETIZATION, AND PERMEABILITY

Magnetic materials are those that exhibit magnetic polarization when they are subjected to an applied magnetic field. The magnetization phenomenon is represented by the alignment of the magnetic dipoles of the material with the applied magnetic field, similar to the alignment of the electric dipoles of the dielectric material with the applied electric field.

Accurate results concerning the behavior of magnetic material when they are subjected to applied magnetic fields can only be predicted by the use of quantum theory. This is usually quite complex and unnecessary for most engineering applications. Quite satisfactory quantitative results can be obtained, however, by using simple atomic models to represent the atomic lattice structure of the material. The atomic models used here represent the electrons as negative charges orbiting around the positive charged nucleus, as shown in Figure 2-7*a*. Each orbiting electron can be modeled by an equivalent small electric current loop of area ds whose current flows in the direction opposite to the electron orbit, as shown in Figure 2-7*b*. As long as

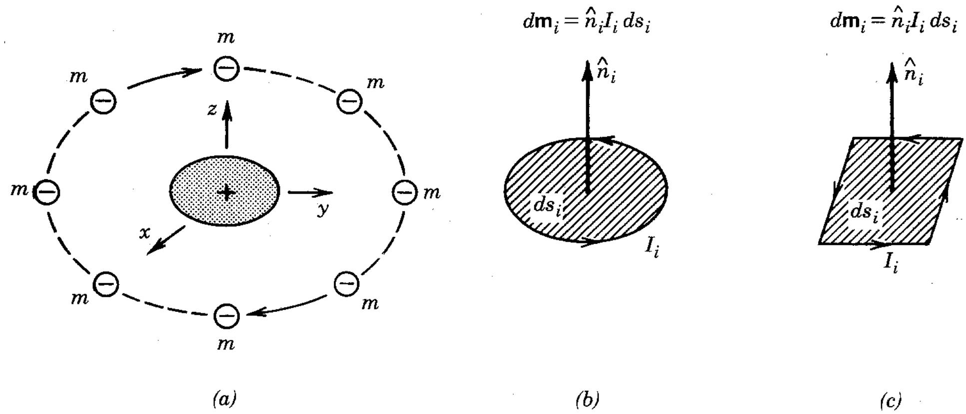

FIGURE 2-7 Atomic models and their equivalents, representing the atomic lattice structure of magnetic material. (*a*) Orbiting electrons. (*b*) equivalent circular electric loop. (*c*) Equivalent square electric loop.

the loop is small, its shape can be circular, square, or any other configuration, as shown in Figure 2-7*c*. The fields produced by a small loop of electric current at large distances are the same as those produced by a linear bar magnet (magnetic dipole) of length d.

By referring to the equivalent loop models of Figure 2-7, the angular momentum associated with an orbiting electron can be represented by a magnetic dipole moment $d\mathbf{m}_i$ of

$$d\mathbf{m}_i = I_i\, d\mathbf{s}_i = I_i \hat{n}_i\, ds_i = \hat{n}_i I_i\, ds_i \text{ (A-m}^2\text{)} \tag{2-13}$$

For atoms that possess many orbiting electrons, the total magnetic dipole moment $\mathbf{m}_t$ is equal to the vector sum of all the individual magnetic dipole moments each represented by (2-13). Thus we can write that

$$\mathbf{m}_t = \sum_{i=1}^{N_m \Delta v} d\mathbf{m}_i = \sum_{i=1}^{N_m \Delta v} \hat{n}_i I_i\, ds_i \tag{2-14}$$

where N_m is equal to the number of orbiting electrons (equivalent loops) per unit volume. A magnetic polarization (magnetization) vector $\mathbf{M}$ is then defined as

$$\mathbf{M} = \lim_{\Delta v \to 0}\left[\frac{1}{\Delta v}\mathbf{m}_t\right] = \lim_{\Delta v \to 0}\left[\frac{1}{\Delta v}\sum_{i=1}^{N_m \Delta v} d\mathbf{m}_i\right] = \lim_{\Delta v \to 0}\left[\frac{1}{\Delta v}\sum_{i=1}^{N_m \Delta v} \hat{n}_i I_i\, ds_i\right] \text{ (A/m)} \tag{2-15}$$

Assuming for each of the loops an average magnetic moment of

$$d\mathbf{m}_i = d\mathbf{m}_{\text{av}} = \hat{n}(I\, ds)_{\text{av}} \tag{2-16}$$

the magnetic polarization vector $\mathbf{M}$ of (2-15) can be written (assuming all the loops are aligned in the parallel planes) as

$$\mathbf{M} = \lim_{\Delta v \to 0}\left[\frac{1}{\Delta v}\sum_{i=1}^{N_m \Delta v} d\mathbf{m}_i\right] = N_m\, d\mathbf{m}_{\text{av}} = \hat{n} N_m (I\, ds)_{\text{av}} \tag{2-17}$$

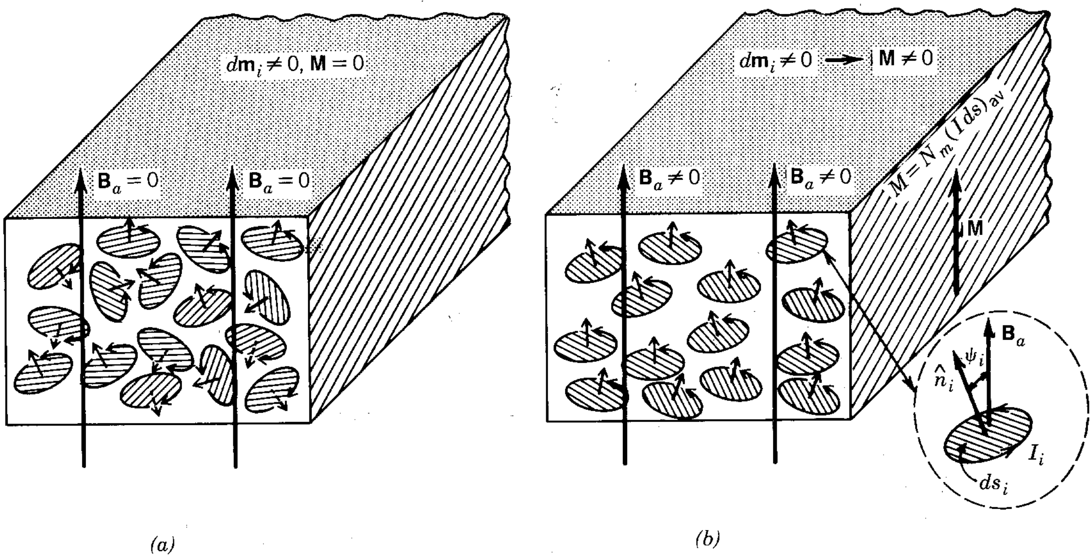

FIGURE 2-8 Random orientation of magnetic dipoles and their alignment (a) in the absence of and (b) under an applied field.

A magnetic material is represented by a number of magnetic dipoles and thus by many magnetic moments. In the absence of an applied magnetic field the magnetic dipoles and their corresponding electric loops are oriented in a random fashion so that on a macroscopic scale the vector sum of the magnetic moments of (2-14) and the magnetic polarization of (2-15) are equal to zero. The random orientation of the magnetic dipoles and loops is illustrated in Fig. 2-8a. When the magnetic material is subjected to an applied magnetic field, represented by the magnetic flux density $\mathbf{B}_a$ in Figure 2-8b, the magnetic dipoles of most material will tend to align in the direction of the $\mathbf{B}_a$ since a torque given by

$$|\Delta\mathbf{T}| = |d\mathbf{m}_i \times \mathbf{B}_a| = |d\mathbf{m}_i|\,|\mathbf{B}_a|\sin(\psi_i) = |(\hat{n}_i I_i\, ds_i) \times \mathbf{B}_a| = |I_i\, ds_i\, B_a \sin(\psi_i)| \tag{2-18}$$

will be exerted in each of the magnetic dipole moments. This is shown in the insert to Figure 2-8b. Ideally, if there were no other magnetic moments to consider, torque would be exerted. The torque would exist until each of the orbiting electrons shifted in such a way that the magnetic field produced by each of its equivalent electric loops (or magnetic moments) was aligned with the applied field and its value represented by (2-18) vanished. Thus the resultant magnetic field at every point in the material would be greater than its corresponding value at the same point when the material is absent.

The magnetization vector $\mathbf{M}$ resulting from the realignment of the magnetic dipoles is better illustrated by considering a slab of magnetic material across which a magnetic field $\mathbf{B}_a$ is applied, as shown in Figure 2-9. Ideally, on a microscopic scale, for most magnetic material all the magnetic dipoles will align themselves so that their individual magnetic moments are pointed in the direction of the applied field, as shown in Figure 2-9. In the limit, as the number of magnetic dipoles and their corresponding equivalent electric loops become very large, the currents of the loops found in the interior parts of the slab are canceled by those of the neighboring

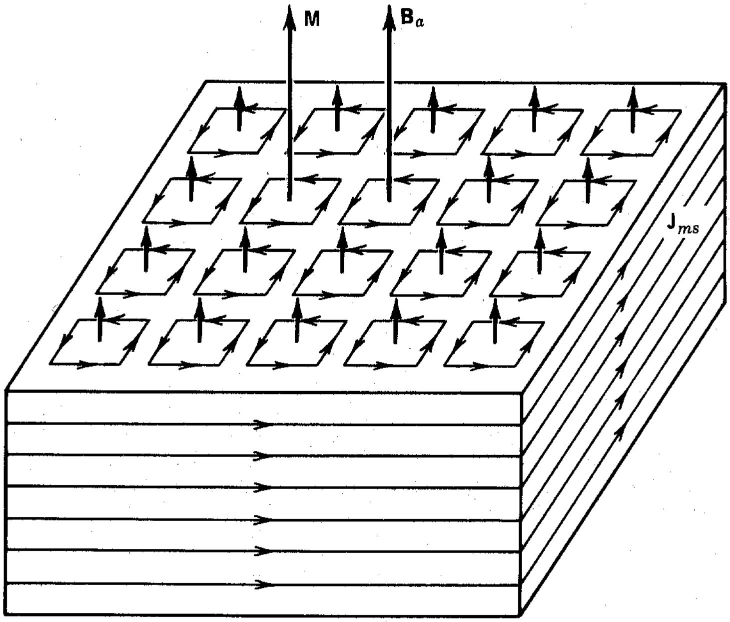

FIGURE 2-9 Magnetic slab subjected to an applied magnetic field and the formation of the magnetization current density $\mathbf{J}_{ms}$.

loops. On a macroscopic scale a net nonzero equivalent magnetic current, resulting in an equivalent magnetic current surface density (A/m), is found on the exterior surface of the slab. This equivalent magnetic current density $\mathbf{J}_{ms}$ is responsible for the introduction of the magnetization vector $\mathbf{M}$ in the direction of $\mathbf{B}_a$.

The magnetic flux density across the slab is increased by the presence of $\mathbf{M}$ so that the net magnetic flux density at any interior point of the slab is given by

$$\mathbf{B} = \mu_0(\mathbf{H}_a + \mathbf{M}) \tag{2-19}$$

It should be pointed out that $\mathbf{M}$, as given by (2-15), has the units of amperes per meter and correspond to those of the magnetic field intensity. In general, we can relate the magnetic flux density to the magnetic field intensity by a parameter that is designated as μ_s (henries/meter). Thus we can write that

$$\mathbf{B} = \mu_s \mathbf{H}_a \tag{2-20}$$

Comparing (2-19) and (2-20) indicates that $\mathbf{M}$ is also related to $\mathbf{H}_a$ by

$$\mathbf{M} = \chi_m \mathbf{H}_a \tag{2-21}$$

where χ_m is called the *magnetic susceptibility* (dimensionless quantity).

Substituting (2-21) into (2-19) and equating the result to (2-20) leads to

$$\mathbf{B} = \mu_0(\mathbf{H}_a + \chi_m \mathbf{H}_a) = \mu_0(1 + \chi_m)\mathbf{H}_a = \mu_s \mathbf{H}_a \tag{2-22}$$

Therefore we can define

$$\mu_s = \mu_0(1 + \chi_m) \tag{2-22a}$$

TABLE 2-2
Approximate static relative permeabilities of magnetic materials

Material	Class	Relative permeability (μ_{sr})
Bismuth	Diamagnetic	0.999834
Silver	Diamagnetic	0.99998
Lead	Diamagnetic	0.999983
Copper	Diamagnetic	0.999991
Water	Diamagnetic	0.999991
Vacuum	Nonmagnetic	1.0
Air	Paramagnetic	1.0000004
Aluminum	Paramagnetic	1.00002
Nickel chloride	Paramagnetic	1.00004
Palladium	Paramagnetic	1.0008
Cobalt	Ferromagnetic	250
Nickel	Ferromagnetic	600
Mild steel	Ferromagnetic	2,000
Iron	Ferromagnetic	5,000
Silicon iron	Ferromagnetic	7,000
Mumetal	Ferromagnetic	100,000
Purified iron	Ferromagnetic	200,000
Supermalloy	Ferromagnetic	1000,000

In (2-22a) μ_s is the *static permeability* of the medium whose relative value μ_{sr} (compared to that of free space μ_0) is given by

$$\mu_{sr} = \frac{\mu_s}{\mu_0} = 1 + \chi_m \tag{2-23}$$

Static values of μ_{sr} for some representative material are listed in Table 2-2.

Within the material, a *bound* magnetic current density $\mathbf{J}_m$ is induced that is related to the magnetic polarization vector $\mathbf{M}$ by

$$\mathbf{J}_m = \nabla \times \mathbf{M}\ (\text{A/m}^2) \tag{2-24}$$

To account for this current density, we modify the Maxwell–Ampere equation 1-71b and write it as

$$\nabla \times \mathbf{H} = \mathbf{J}_i + \mathbf{J}_c + \mathbf{J}_m + \mathbf{J}_d = \mathbf{J}_i + \sigma\mathbf{E} + \nabla \times \mathbf{M} + j\omega\varepsilon\mathbf{E} \tag{2-24a}$$

On the surface of the material, the *bound* magnetization surface current density $\mathbf{J}_{ms}$ is related to the magnetic polarization vector $\mathbf{M}$ at the surface by

$$\mathbf{J}_{ms} = \mathbf{M} \times \hat{n}|_{\text{surface}}\ (\text{A/m}) \tag{2-25}$$

where $\hat{n}$ is a unit vector normal to the surface of the material. The *bound* magnetization current I_m flowing through a cross section S_0 of the material can be obtained by using

$$I_m = \iint_S \mathbf{J}_m \cdot d\mathbf{s} = \iint_{S_0} (\nabla \times \mathbf{M}) \cdot d\mathbf{s}\ (\text{A}) \tag{2-26}$$

In addition to orbiting, the electrons surrounding the nucleus of an atom also spin about their own axis. Therefore magnetic moments of the order of $\pm 9 \times 10^{-24}$ A-m^2 are also associated with the spinning of the electrons that aid or oppose the applied magnetic field (the + sign is used for addition and the − for subtraction). For atoms that have many electrons in their shells, only the spins associated with the electrons found in shells that are not completely filled will contribute to the magnetic moment of the atoms. A third contributor to the total magnetic moment of an atom is that associated with the spinning of the nucleus, which is referred to as *nuclear spin*. However, this nuclear spin magnetic moment is usually much smaller (typically by a factor of about 10^{-3}) than those attributed to the orbiting and the spinning electrons.

Example 2-2. A bar of magnetic material of finite length, which is placed along the z axis as shown in Figure 2-10, has a cross section of 0.3 m in the x direction ($0 \le x \le 0.3$) and 0.2 m in the y direction ($0 \le y \le 0.2$). The bar is subjected to a magnetic field so that the magnetization vector inside the bar is given by

$$\mathbf{M} = \hat{a}_z(4y)$$

Determine the volumetric current density $\mathbf{J}_m$ at any point inside the bar, the surface current density $\mathbf{J}_{ms}$ on the surface of each of the four faces, and the total current I_m per unit length flowing through the bar face that is parallel to the y axis at $x = 0.3$ m.

Solution. Using (2-24), we have

$$\mathbf{J}_m = \nabla \times \mathbf{M} = \hat{a}_x \frac{\partial M_z}{\partial y} = \hat{a}_x 4$$

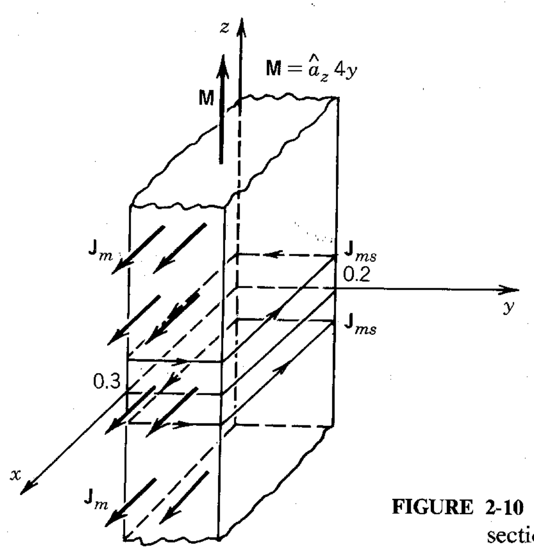

FIGURE 2-10 Magnetic bar of rectangular cross section subjected to a magnetic field.

Using (2-25), we have

$$\mathbf{J}_{ms} = \mathbf{M} \times \hat{n}|_{\text{surface}}$$

Therefore at

$x=0$:

$$\mathbf{J}_{ms} = (\hat{a}_z 4y) \times (-\hat{a}_x)|_{x=0} = -\hat{a}_y(4y) \qquad \text{for } 0 \le y \le 0.2$$

$y=0$:

$$\mathbf{J}_{ms} = (\hat{a}_z 4y) \times (-\hat{a}_y)|_{y=0} = \hat{a}_x(4y) = 0 \qquad \text{for } 0 \le x \le 0.3$$

$x=0.3$:

$$\mathbf{J}_{ms} = (\hat{a}_z 4y) \times (\hat{a}_x)|_{x=0.3} = \hat{a}_y(4y) \qquad \text{for } 0 \le y \le 0.2$$

$y=0.2$

$$\mathbf{J}_{ms} = (\hat{a}_z 4y) \times (\hat{a}_y)|_{y=0.2} = -\hat{a}_x(4y) = -\hat{a}_x 0.8 \quad \text{for } 0 \le x \le 0.3$$

According to (2-26) the current (per unit length) flowing through the bar face at $x = 0.3$ is given by

$x = 0.3$:

$$I_m = \iint_S \mathbf{J}_m \cdot d\mathbf{s} = \int_0^1 \int_0^{0.2} (\hat{a}_x 4) \cdot (\hat{a}_x \, dy \, dz) = 4(1)(0.2) = 0.8$$

Consistent with the relative permittivity (dielectric constant), the values of μ, and thus μ_r, vary as a function of frequency. These variations will be discussed in Section 2.8.2. The values of μ_r listed in Table 2-2 are representative of frequencies related to static or quasistatic fields. Excluding ferromagnetic material, it is apparent that most relative permeabilities are very near unity, so that for engineering problems a value of unity is almost always used.

According to the direction in which the net magnetization vector **M** is pointing (either aiding or opposing the applied magnetic field), material are classified into two groups, Group A and Group B as shown:

Group A	**Group B**
Diamagnetic	Paramagnetic
	Ferromagnetic
	Antiferromagnetic
	Ferrimagnetic

In general, for material in Group A the net magnetization vector (although small in magnitude) opposes the applied magnetic field, resulting in a relative permeability slightly smaller than unity. *Diamagnetic* materials fall into that group. For material in Group B the net magnetization vector is aiding the applied magnetic field, resulting in relative permeabilities greater than unity. Some of them (*paramagnetic* and *antiferromagnetic*) have only slightly greater than unity relative permeabilities whereas others (*ferromagnetic* and *ferrimagnetic*) have relative permeabilities much greater than unity.

In the absence of an applied magnetic field, the moments of the electron spins of *diamagnetic* material are opposite to each other as well as to the moments associated with the orbiting electrons so that a zero net magnetic moment $\mathbf{m}_t$ is produced on a macroscopic scale. In the presence of an external applied magnetic field, each atom has a net nonzero magnetic moment, and on a macroscopic scale there is a net total magnetic moment for all the atoms that results in a magnetization vector **M**. For diamagnetic material, this vector **M** is very small, opposes the applied magnetic field, leads to a negative magnetic susceptibility χ_m, and results in values of relative permeability that are slightly less than unity. For example, copper is a diamagnetic material with a magnetic susceptibility $\chi_m = -9 \times 10^{-6}$ and a relative permeability $\mu_r = 0.999991$.

In *paramagnetic* material, the magnetic moments associated with the orbiting and spinning electrons of an atom do not quite cancel each other in the absence of an applied magnetic field. Therefore each atom possesses a small magnetic moment. However, because the orientation of the magnetic moment of each atom is random, the net magnetic moment of a large sample (macroscopic scale) of dipoles, and the magnetization vector **M**, are zero when there is no applied field. When the paramagnetic material is subjected to an applied magnetic field, the magnetic dipoles align slightly with the applied field to produce a small nonzero **M** in its direction and a small increase in the magnetic flux density within the material. Thus the magnetic susceptibilities have small positive values and the relative permeabilities are slightly greater than unity. For example, aluminum possesses a susceptibility of $\chi_m = 2 \times 10^{-5}$ and a relative permeability of $\mu_r = 1.00002$.

The individual atoms of *ferromagnetic* material possess, in the absence of an applied magnetic field, very strong magnetic moments caused primarily by uncompensated electron spin moments. The magnetic moments of many atoms (usually as many as five to six) reinforce one another and form regions called *domains*, which have various sizes and shapes. The dimensions of the domains depend on the material's past magnetic state and history, and range from 1 μm to a few millimeters. On a macroscopic scale, however, the net magnetization vector **M** in the absence of an applied field is zero because the domains are randomly oriented and the magnetic moments of the various atoms cancel one another. When a ferromagnetic material is subjected to an applied field, there are not only large magnetic moments associated with the individual atoms, but the vector sum of all the magnetic moments and the associated vector magnetization **M** are very large, leading to extreme values of magnetic susceptibility χ_m and relative permeability. Typical values of μ_r for some representative ferromagnetic material are found in Table 2-2. When the applied field is removed, the magnetic moments of the various atoms do not attain a random orientation and a net nonzero residual magnetic moment remains. Since the magnetic moment of a ferromagnetic material on a macroscopic scale is different after the applied field is removed, its magnetic state depends on the material's past history. Therefore a plot of the magnetic flux density $\mathscr{B}$ versus $\mathscr{H}$ leads to a double-valued curve known as the *hysteresis loop*. Material with such properties are very desirable in the design of transformers, induction cores, and coatings for magnetic recording tapes.

Materials that possess strong magnetic moments, but whose adjacent atoms are about equal in magnitude and opposite in direction, with zero net total magnetic moment in the absence of an applied magnetic field, are called *antiferromagnetic*. The presence of an applied magnetic field has a minor effect on the material and leads to relative permeabilities slightly greater than unity.

If the adjacent opposing magnetic moments of a material are very large in magnitude but greatly unequal in the absence of an applied magnetic field, the material is known as *ferrimagnetic*. The presence of an applied magnetic field has a large effect on the material and leads to large permeabilities (but not as large as those of ferromagnetic material). *Ferrites* make up a group of ferrimagnetic materials that have low conductivities (several orders smaller than those of semiconductors). Because of their large resistances, smaller currents are induced in them that result in lower ohmic losses when they are subjected to alternating fields. They find wide applications in the design of nonreciprocal microwave components (isolators, hybrids, gyrators, phase shifters, etc.) and they will be discussed briefly in Section 2.8.3.

CURRENT, CONDUCTORS, AND CONDUCTIVITY

The prominent characteristic of dielectric materials is the electric polarization introduced through the formation of electric dipoles between opposite charges of atoms. Magnetic dipoles, modeled by equivalent small electric loops, were introduced to account for the orbiting of electrons in atoms of magnetic material. This phenomenon was designated as magnetic polarization. Conductors are material whose prominent characteristic is the motion of electric charges and the creation of a current flow.

Current

Let us assume that an electric volume charge density, represented here by q_v, is distributed uniformly in an infinitesimal circular cylinder of cross-sectional area Δs and volume ΔV, as shown in Figure 2-11. The total electric charge ΔQ within the volume ΔV is moving in the z direction with a uniform velocity v_z. Thus we can write that

$$\frac{\Delta Q_e}{\Delta t} = q_v \frac{\Delta V}{\Delta t} = q_v \frac{\Delta s \, \Delta z}{\Delta t} = q_v \, \Delta s \, \frac{\Delta z}{\Delta t} \tag{2-27}$$

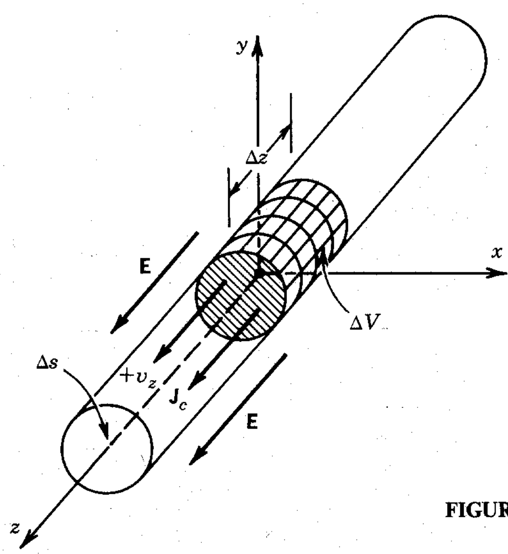

FIGURE 2-11 Charge uniformly distributed in an infinitesimal circular cylinder.