Theory and Applications of Transmission Lines

Topics

- Introduction
- Types
- General Transmission-Line Equations
- Wave Characteristics on Finite Transmission Lines
- Waveguides
- Optical Fiber

Transmission Lines

- Used for guiding electromagnetic (EM) waves
- Point-to-point "guided" transmission of power and information from "source" to "receiver", e.g., data signal. (unguided=antenna)
- Transverse EM (TEM) waves applied to most transmission lines except waveguides.
- TEM waves -> uniform plane waves

Types classified by materials

- Metallic Transmission Lines (Conductor)
- Hollow or Dielectric-filled Waveguides (Conductor and dielectric)
- Optical Fiber (dielectric)

Transmission Lines

Two fundamental types

- Low Frequency
  - used for power transmission
- High Frequency
  - used for RF transmission

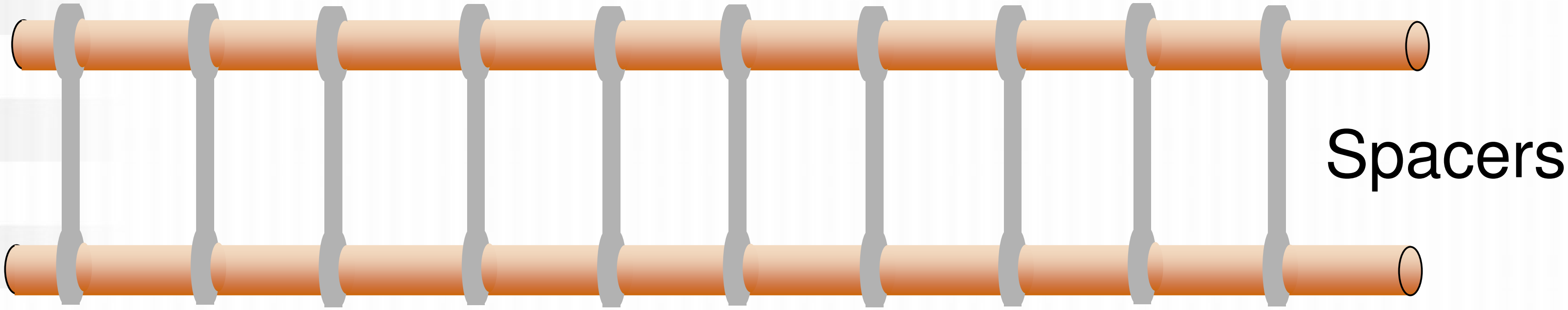
    "wavelengths are shorter than or comparable to the length of cable"

Note - transmission line = conductor - but only use "surface"

Types of Metallic Transmission Lines

- Parallel Line
- Twisted Pair (Shielded & Unshielded)
- Coaxial
- Microstrips
- Strip Line

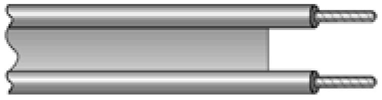
Parallel Pair



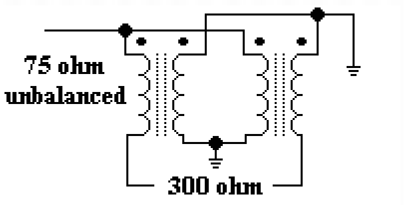
Parallel Line (aka Ribbon Cable)

- Simple Construction
- Used primarily for power lines, rural telephone lines or TV antenna cable
- Freq up to 200MHz over short distances
- High Radiation Loss
  - moving current = Ae
  - need to be aware of other metallic conductors

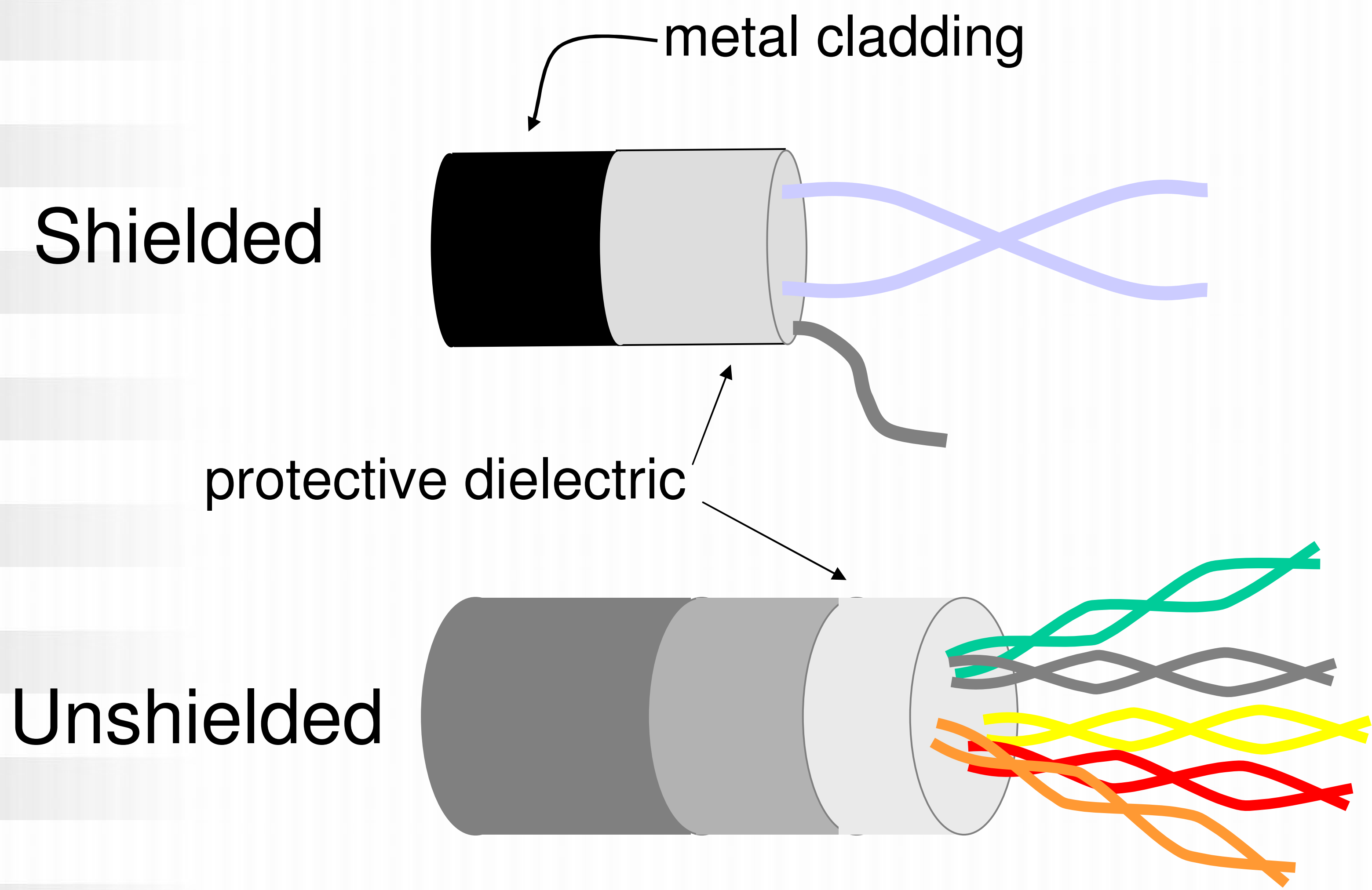
Twin Lead Cable



- Balanced
  - 300 Ω $Z_0 = 276\log(D/r)$
- Balun
  - Balanced to unbalance transformer



Twisted Pair



coating is paper, rubber, PVC…
can also have single pair, each wrapped individually

Twisted Pair

- Twists tend to cancel radiation loss
- Helps reduce crosstalk
- Still fairly inexpensive
- Frequency < 100MHz
- Generally short distances
  - analog ~5-6 km
  - digital ~2-3 km

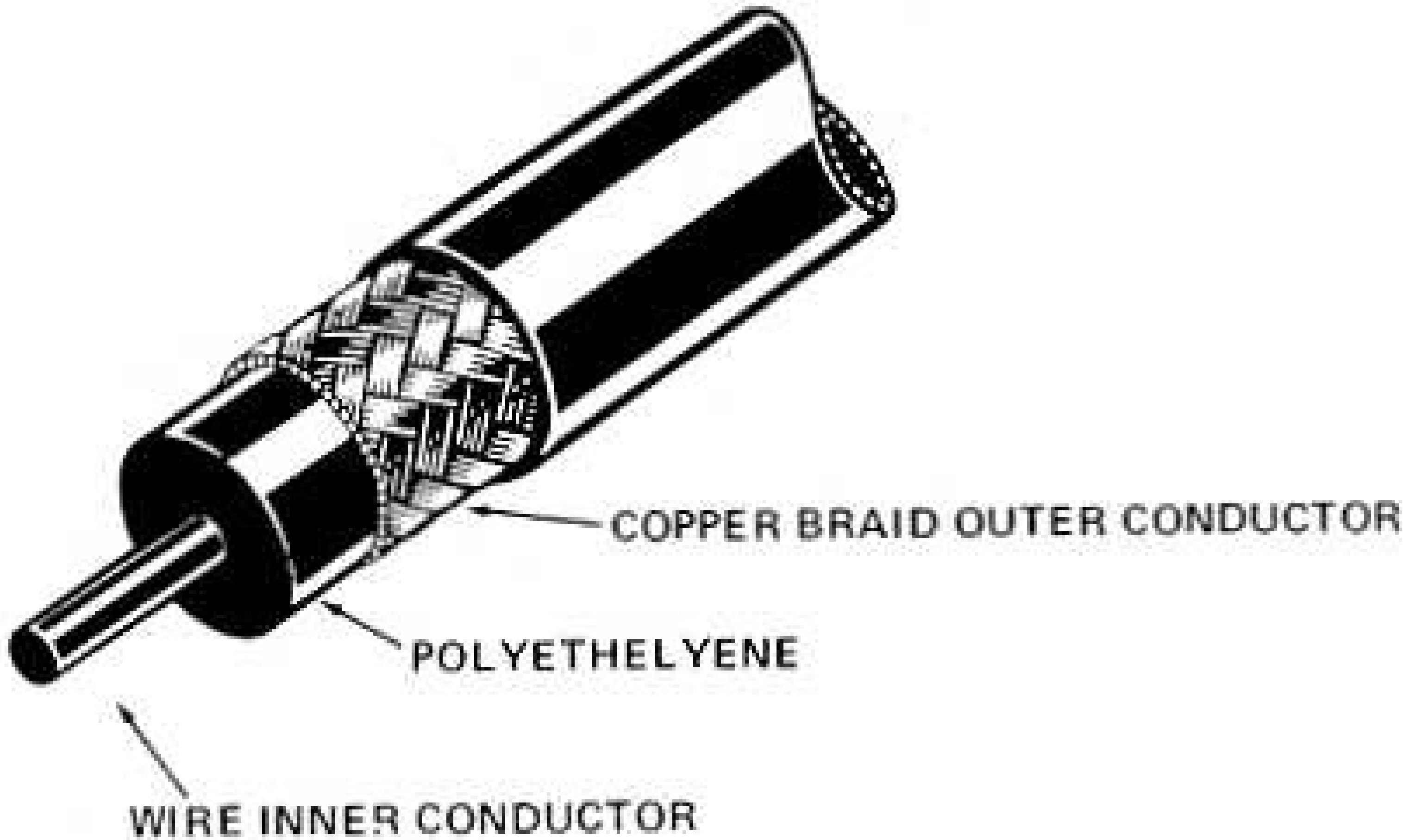
- Note - power line interference

CAT5 Cable

- UTP
- 4 pair
- terminating in RJ45
- 100MHz max frequency
- 1000 Mbps transmit rate

- Aside: Wire Gauge (smaller is bigger)

Coaxial Cable

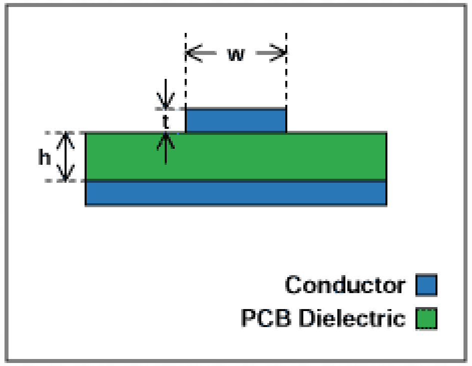


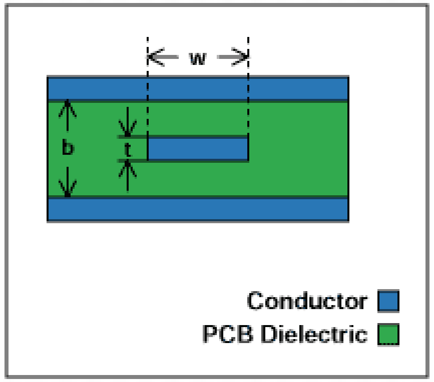
Coaxial Cable

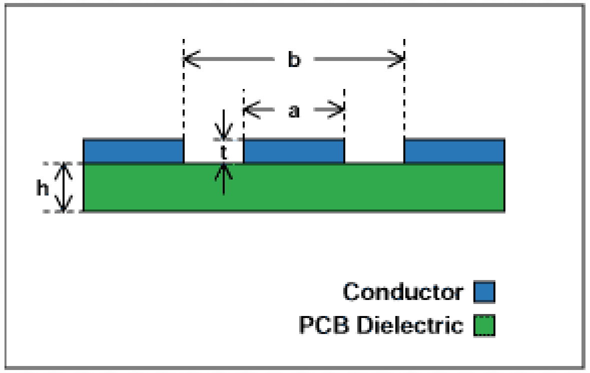
- Geometry creates a “shielded” system
  - no EM energy outside the cable
- Can support frequencies > 100MHz
- Can support data rates > 1GHz
- Low self-inductance allows greater BW
- Used for long-distance telephone trunks, urban networks, TV cables
- Expensive + must keep dielectric dry

Striplines

- Micro Stripline
- Embedded Stripline
- Coplanar Stripline



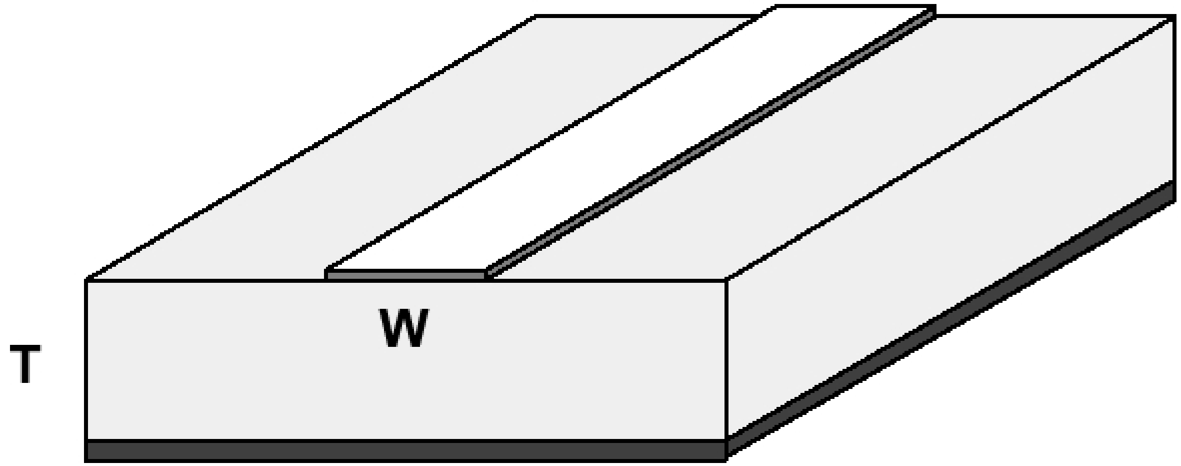




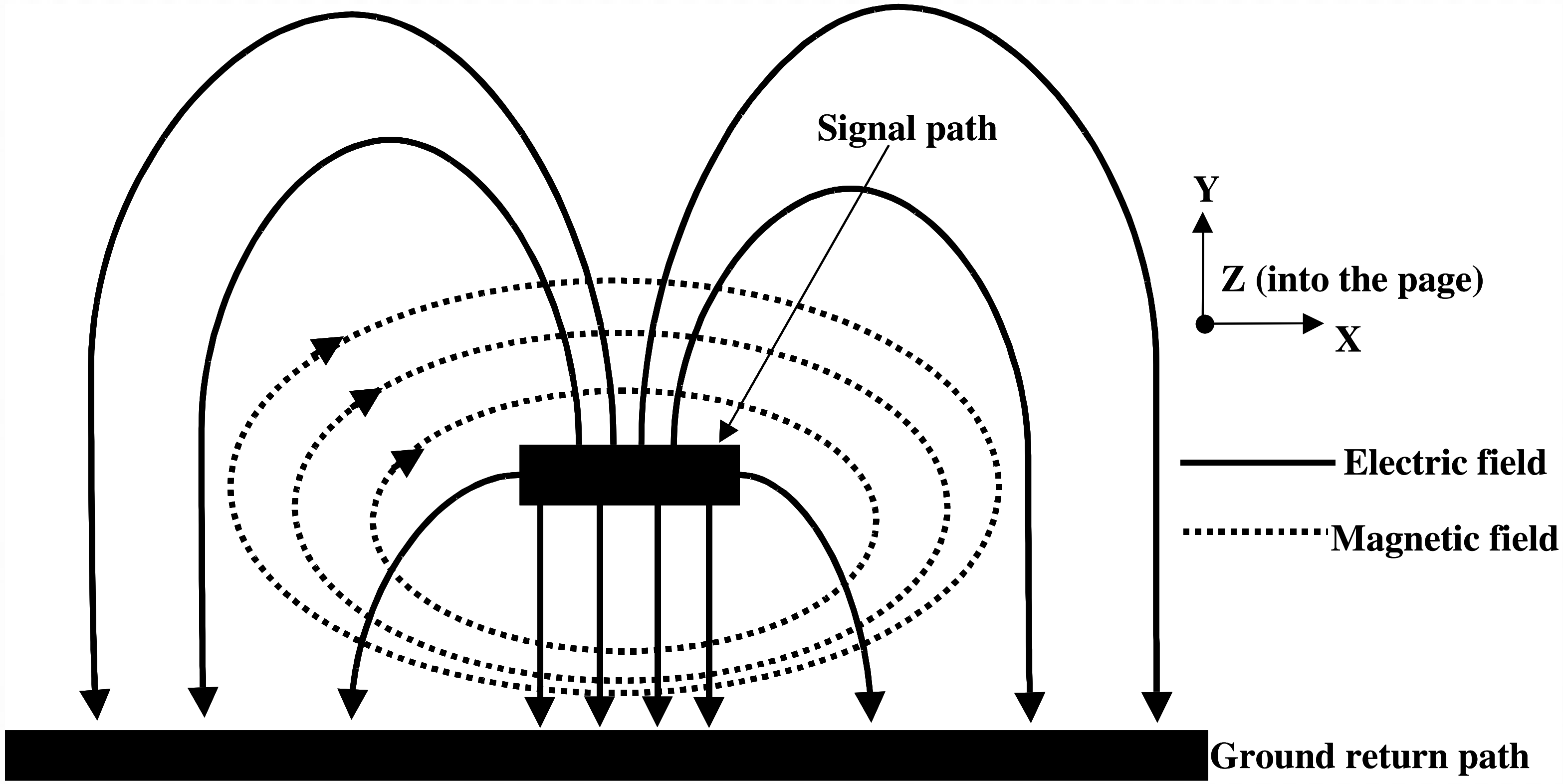
- Loss
  - Metallic
    - Skin depth
    - Localized current flow
  - Dielectric
    - Loss tangent $\varepsilon = \varepsilon' - j\varepsilon'' \Rightarrow Tan\delta = \frac{\varepsilon''}{\varepsilon'}$
  - Surface roughness

Microstrips

- Used for very high frequencies in semi-conductors



E & H Fields – Microstrip Case



The signal is really the wave propagating <u>*between*</u> the conductors

Transmission Theory

- Current and Voltage change with time along the line (the signal)
  - superposition of waves in both directions
  - but over short distances ($<\lambda$) are constant
- Energy is lost (heat - resistance) or stored (magnetic - inductance) / (capacitive - capacitance)

$$v = Ri \qquad v = L\frac{di}{dt} \qquad i = C\frac{dv}{dt}$$

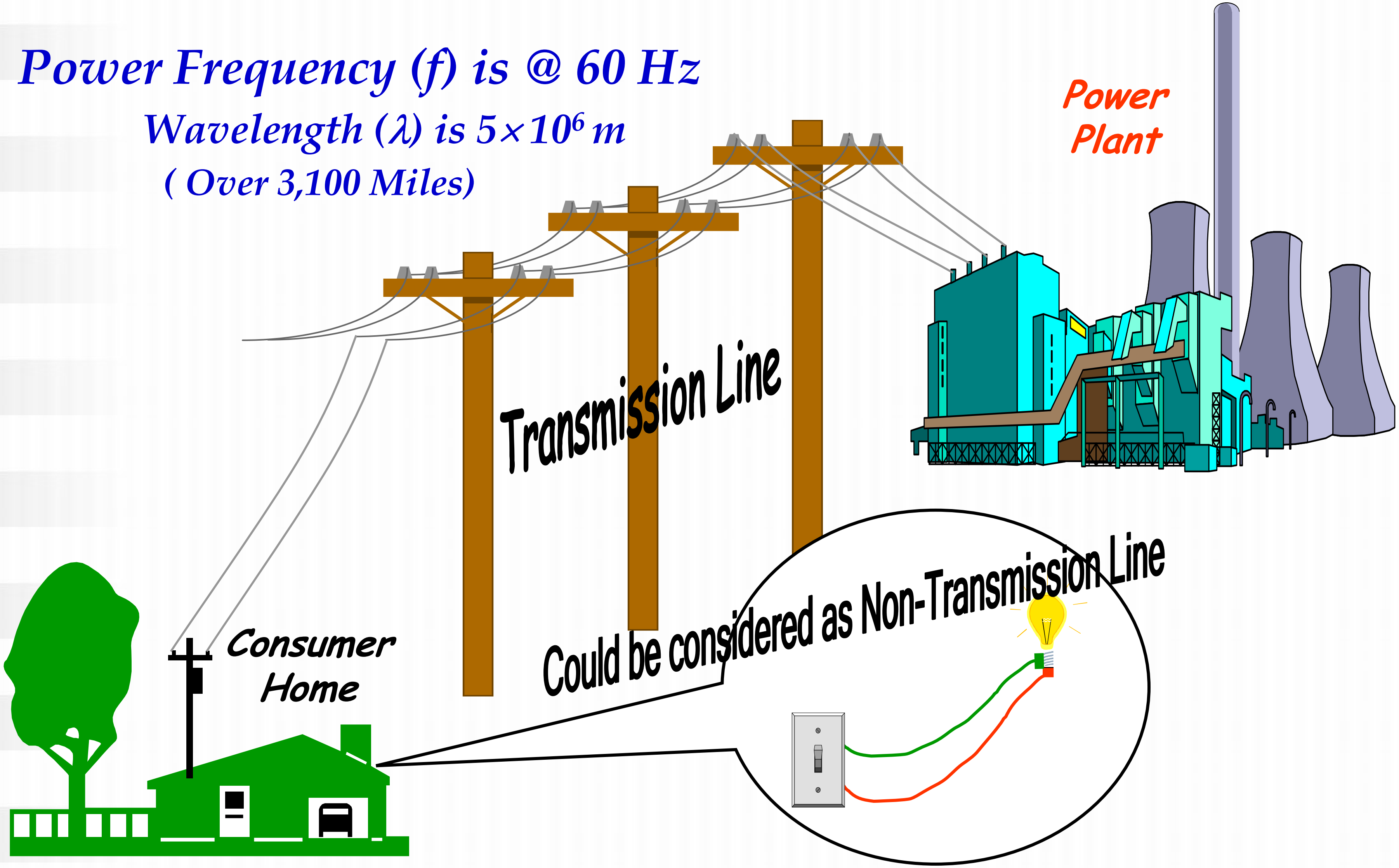
= Attenuation Losses

Transmission Line Concept

Power Frequency (f) is @ 60 Hz

Wavelength (λ) is 5×10^6 m

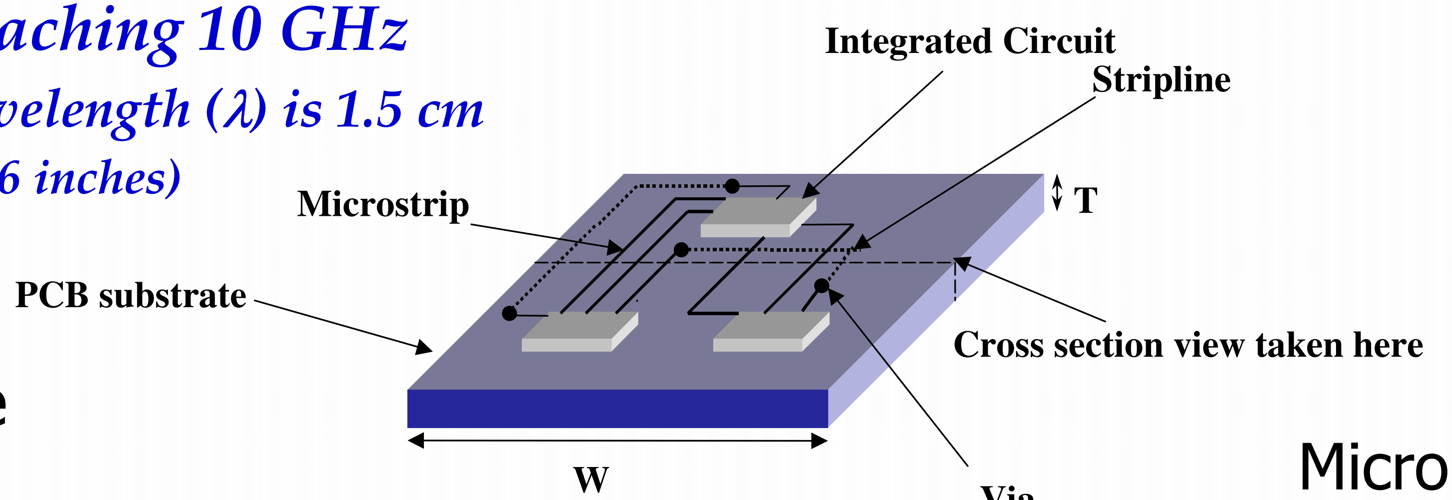
(Over 3,100 Miles)



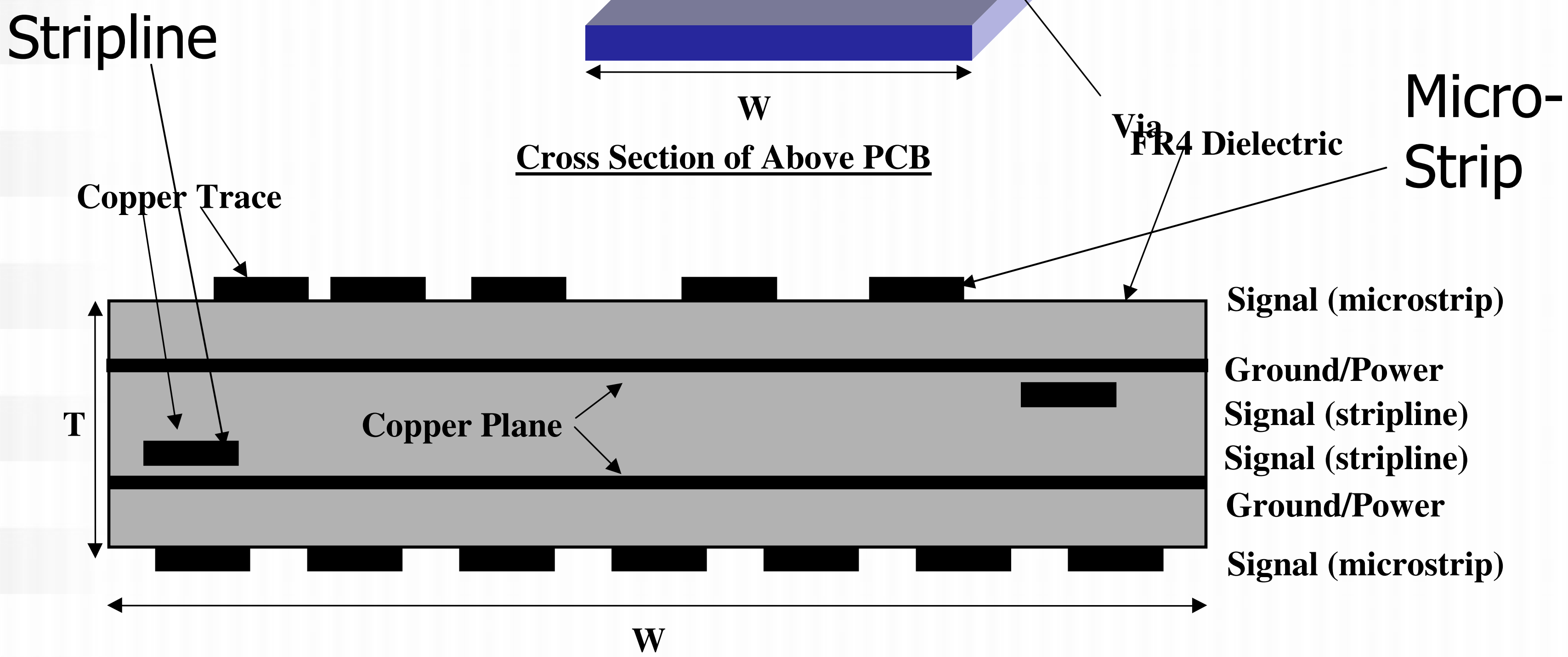
PC Transmission Lines

Signal Frequency (f) is approaching 10 GHz

Wavelength (λ) is 1.5 cm (0.6 inches)



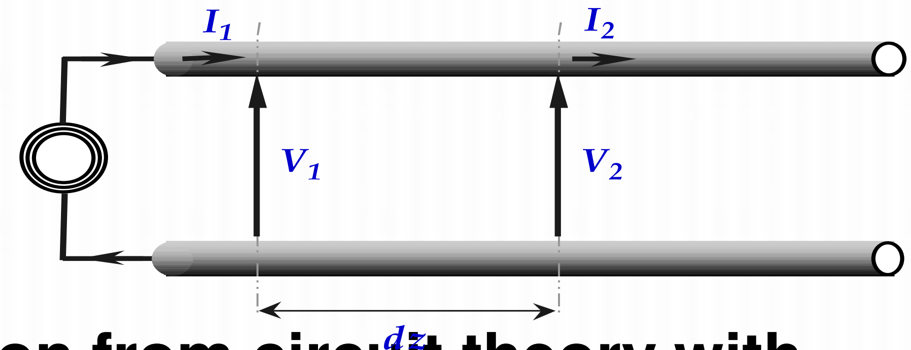
Cross Section of Above PCB



Key point about transmission line operation

Voltage and current on a transmission line is a function of both time and *position*.

$$V = f(z,t)$$
$$I = f(z,t)$$

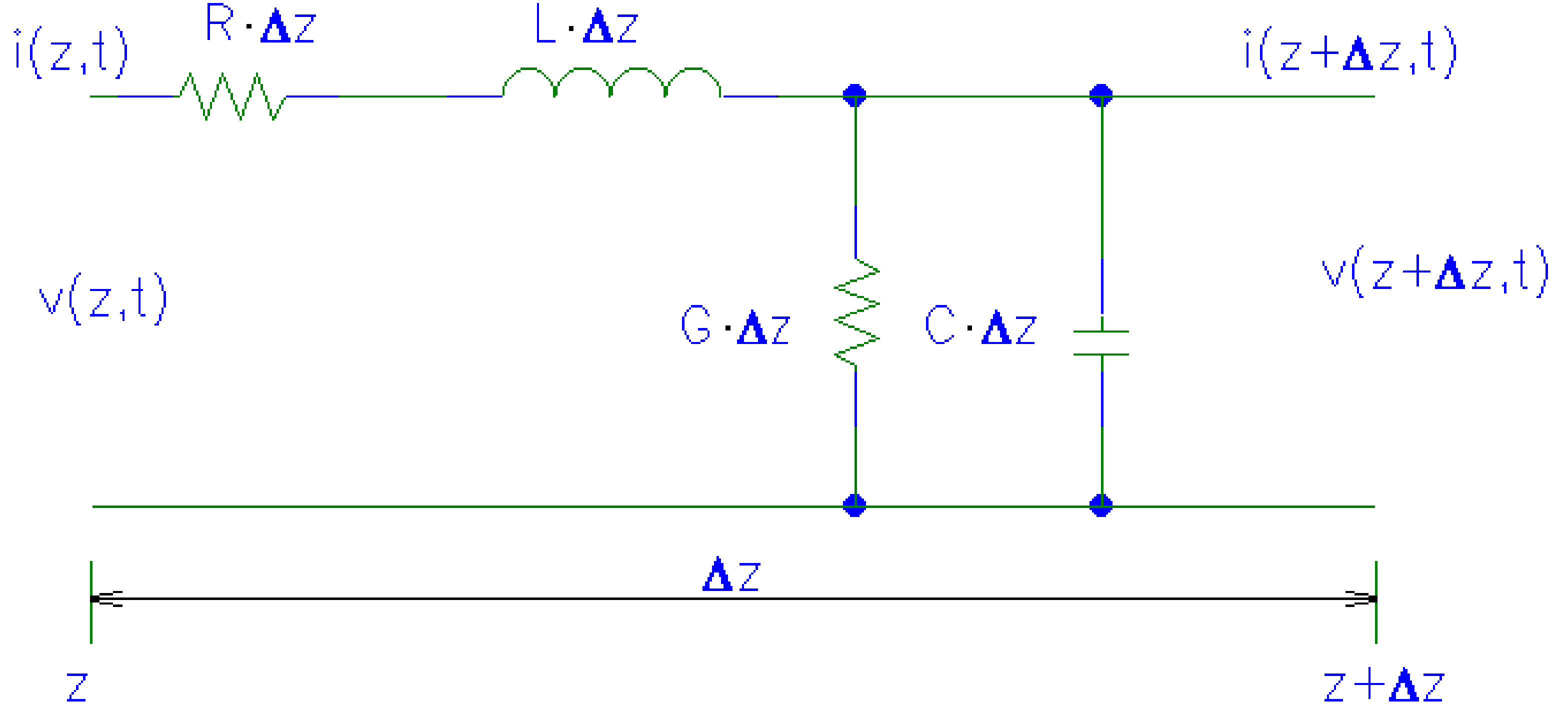


The major deviation from circuit theory with transmission line, distributed networks is this positional dependence of voltage and current!

- **Must think in terms of position and time to understand transmission line behavior**
- **This positional dependence is added when the assumption of the size of the circuit being small compared to the signaling wavelength**

Transmission Line Model

- Distributed circuit concept



R is the resistance in both conductors per unit length in W /m
L is the inductance in both conductors per unit length in H/m
G is the conductance of the dielectric media per unit length in S/m
C is the capacitance between the conductors per unit length in F/m

Transmission Line Model (cont'd)

Using Kirchhoff´s voltage law on the circuit in the figure

$$v(z,t) - R \cdot \Delta z \cdot i(z,t) - L \cdot \Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

$$-\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = R \cdot i(z,t) + L\frac{\partial i(z,t)}{\partial t}$$

letting $\Delta z \rightarrow 0$ we get

$$-\frac{\partial v(z,t)}{\partial z} = R \cdot i(z,t) + L\frac{\partial i(z,t)}{\partial t} \qquad (1)$$

Transmission Line Model (cont'd)

To get another equation relating G and C we apply Kirchhoff´s current law on the circuit and get:

$$i(z,t) - G \cdot \Delta z \cdot v(z+\Delta z,t) - C \cdot \Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

letting $\Delta z \to 0$ in this equation also we get:

$$-\frac{\partial i(z,t)}{\partial z} = Gv(z,t) + C\frac{\partial v(z,t)}{\partial t} \qquad (2)$$

(1),(2) : General Transmission-line Equations

Transmission Line Model (cont'd)

These equations can be simplified if the voltage *v(z,t)* and the current *i(z,t)* are time-harmonic cosine functions

$$v(t,z) = \mathrm{Re}(V(z)e^{j\omega t}) \qquad i(z,t) = \mathrm{Re}(I(z)e^{j\omega t})$$

the general transmission line equations become:

$$-\frac{dV(z)}{dz} = (R + j\omega L)I(z) \qquad (3)$$

$$-\frac{dI(z)}{dz} = (G + j\omega C)V(z) \qquad (4)$$

Wave equations & solutions

By combining (3) and (4):

$$\frac{d^2V(z)}{dz^2} = \gamma^2 V(z) \quad (5) \qquad \frac{d^2I(z)}{dz^2} = \gamma^2 I(z) \quad (6)$$

where γ is the propagation constant:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

The general solution of (5), (6)

$$V(z) = V^+(z) + V^-(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (7)$$

$$I(z) = I^+(z) + I^-(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad (8)$$

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{R + j\omega L}{\gamma} = \frac{\gamma}{G + j\omega C} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Characteristic Impedance

Special Cases

Lossless Line (R=0,G=0)

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC};$$

$$u_p = \omega / \beta = 1/\sqrt{LC}; Z_0 = \sqrt{L/C}$$

Distortionless Line (R/L,G/C)

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(RC/L + j\omega C)}$$

$$= \sqrt{C/L}(R + j\omega L);$$

$$\alpha = R\sqrt{C/L}; \beta = \omega\sqrt{LC}$$

$$u_p = \omega / \beta = 1/\sqrt{LC}; Z_0 = \sqrt{L/C}$$

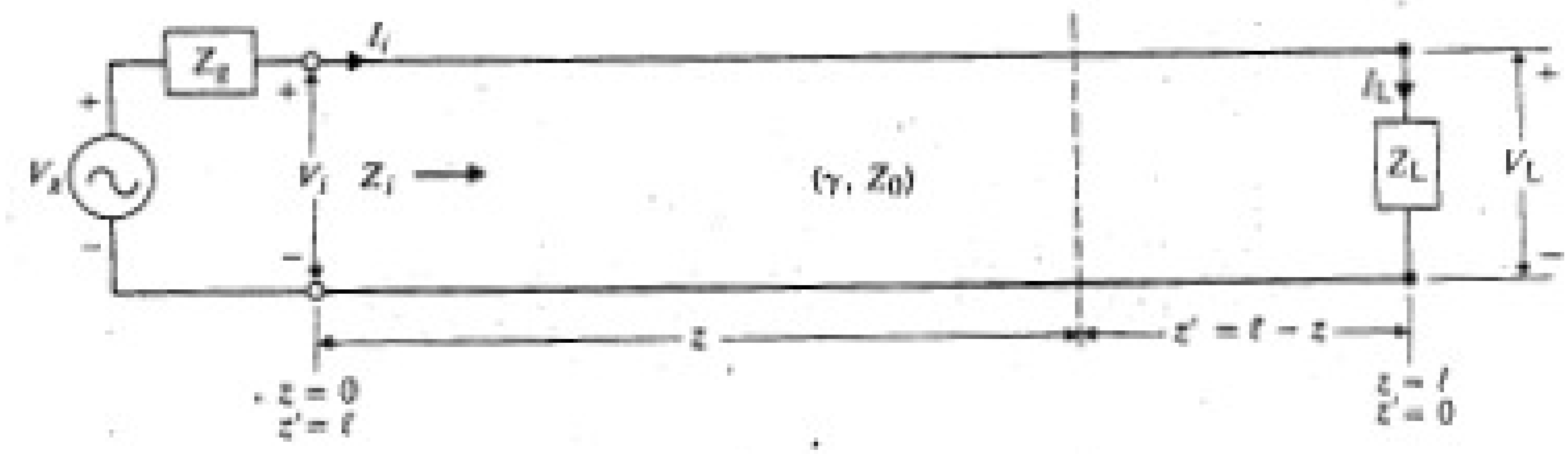
Finite Transmission Lines

In an infinitely long line there are only forward travelling waves and no reflected waves. The second term in (7) and (8) will be zero. This is however also true for a line terminated with its characteristic impedance. A line is called a matched line when the load impedance is equal to the characteristic impedance. If we consider a line with the characteristic impedance Z_0, a propagation constant γ and with the length l terminated with a load impedance Z_L connected to a sinusoidal voltage source, and then the voltage and current distribution on the line can be calculated as:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}; I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

Finite Transmission Lines (2)



$$Z_L = \left(\frac{V}{I}\right)_{z=l} = \frac{V_L}{I_L} = \frac{V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l}}{I_0^+ e^{-\gamma l} + I_0^- e^{\gamma l}}$$

$$= \frac{V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l} / Z_0 - V_0^- e^{\gamma l} / Z_0}$$

Finite Transmission Lines (3)

$$V_0^+ = \frac{1}{2}\left(V_L + I_L Z_0\right)e^{\gamma l} = \frac{I_L}{2}\left(Z_L + Z_0\right)e^{\gamma l};$$

$$V_0^- = \frac{1}{2}\left(V_L - I_L Z_0\right)e^{-\gamma l} = \frac{I_L}{2}\left(Z_L - Z_0\right)e^{-\gamma l}$$

$$V(z) = \frac{I_L}{2}\left[\left(Z_L + Z_0\right)e^{\gamma(l-z)} + \left(Z_L - Z_0\right)e^{-\gamma(l-z)}\right];$$

$$I(z) = \frac{I_L}{2Z_0}\left[\left(Z_L + Z_0\right)e^{\gamma(l-z)} - \left(Z_L - Z_0\right)e^{-\gamma(l-z)}\right]$$

Finite Transmission Lines (4)

$$V(z') = \frac{I_L}{2}\left[(Z_L + Z_0)e^{\gamma z'} + (Z_L - Z_0)e^{-\gamma z'}\right];$$

$$I(z') = \frac{I_L}{2Z_0}\left[(Z_L + Z_0)e^{\gamma z'} - (Z_L - Z_0)e^{-\gamma z'}\right];\ z' = l - z$$

$$V(z') = I_L(Z_L \cosh \gamma z' + Z_0 \sinh \gamma z')$$

$$I(z') = \frac{I_L}{Z_0}(Z_L \sinh \gamma z' + Z_0 \cosh \gamma z')$$

Finite Transmission Lines (5)

$$Z(z') = Z_0 \frac{Z_L + Z_0 \tanh \gamma z'}{Z_0 + Z_L \tanh \gamma z'}$$

Input Impedance:

$$Z_i = Z(z' = l) = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

Matched load if $Z_L = Z_0$

Reflection Coefficient

$$V(z') = \frac{I_L}{2}\left[(Z_L + Z_0)e^{\gamma z'} + (Z_L - Z_0)e^{-\gamma z'}\right]$$

$$= \frac{I_L}{2}(Z_L + Z_0)e^{\gamma z'}\left[1 + \frac{Z_L - Z_0}{Z_L + Z_0}e^{-2\gamma z'}\right]$$

$$= \frac{I_L}{2}(Z_L + Z_0)e^{\gamma z'}\left[1 + \Gamma e^{-2\gamma z'}\right]$$

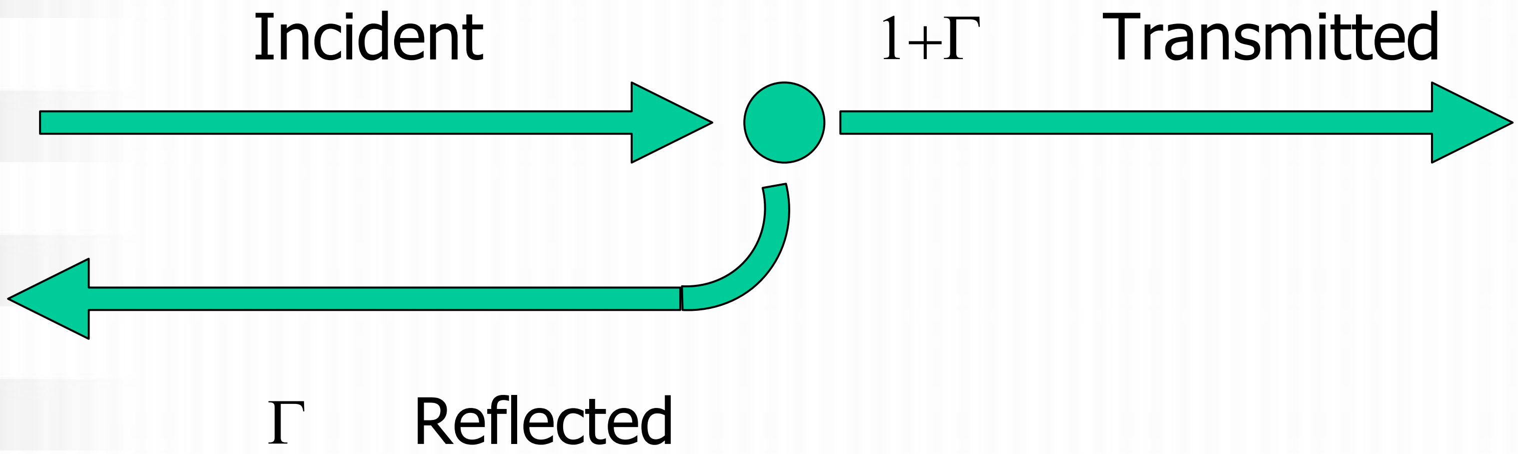
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}$$

Reflection Coefficient

$$S = \frac{|V_{max}|}{|V_{min}|} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Standing Wave Ratio (SWR)

Reflection and Transmission



Special Cases to Remember

A: Terminated in Zo

$$\rho = \frac{Zo - Zo}{Zo + Zo} = 0$$

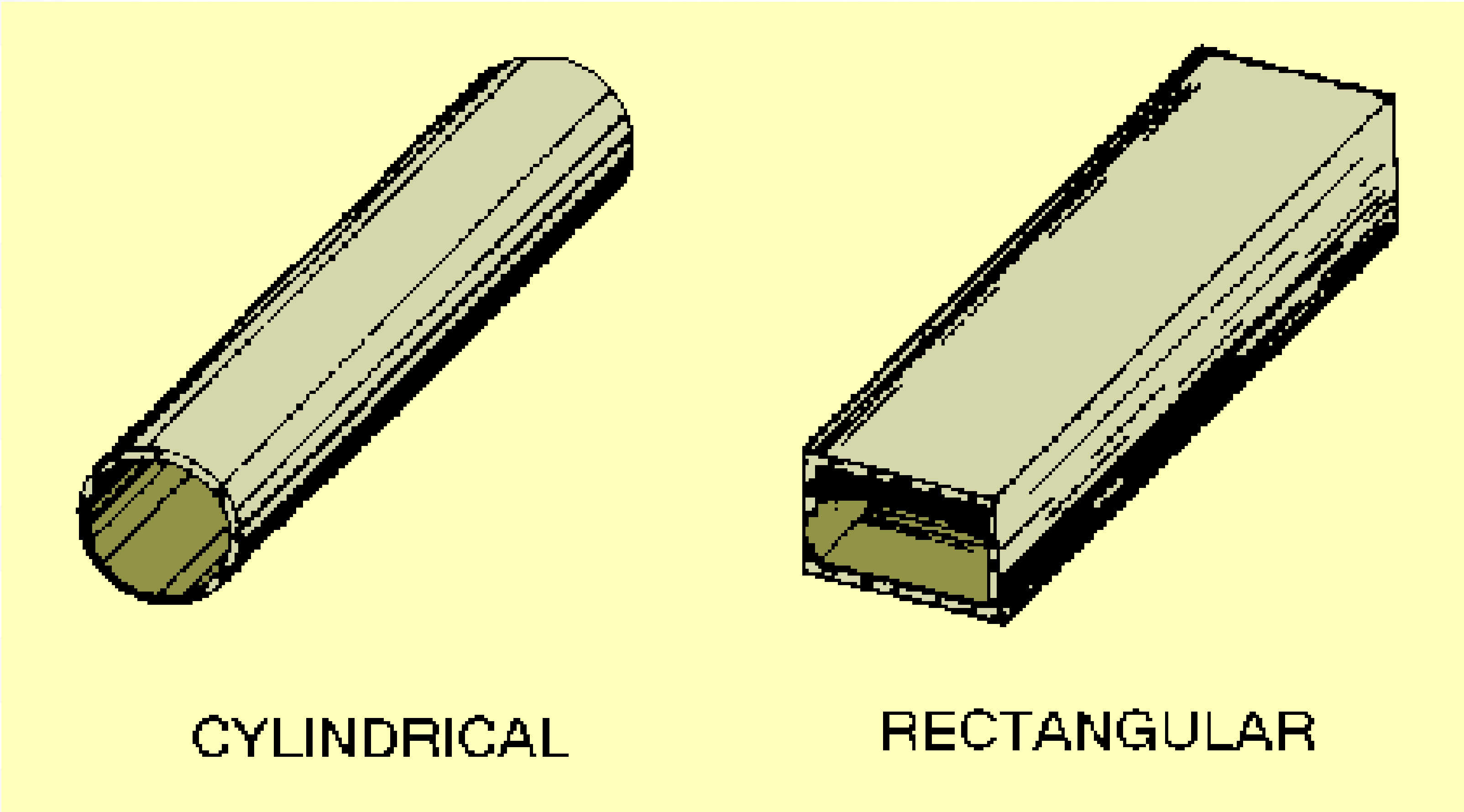
B: Short Circuit

$$\rho = \frac{0 - Zo}{0 + Zo} = -1$$

C: Open Circuit

$$\rho = \frac{\infty - Zo}{\infty + Zo} = 1$$

Waveguides aka plumbing

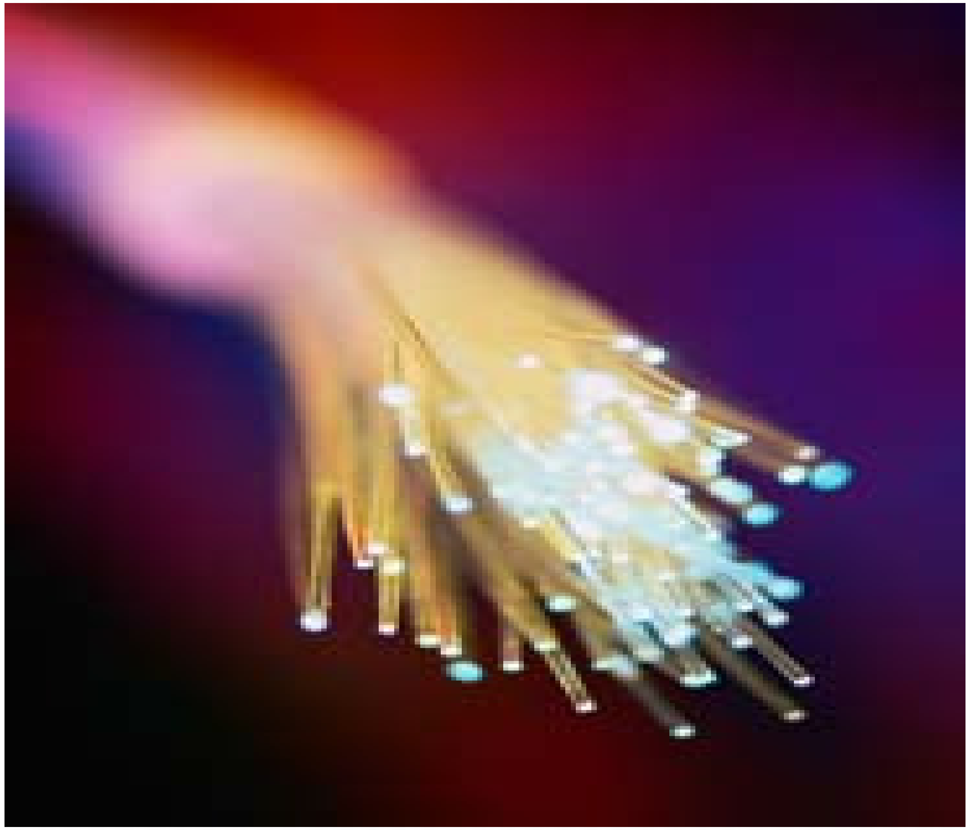


- width is ~ wavelength

Waveguides

- Uses a different transmission method
- “Ducting” not “conducting”
- >1GHz
- Expensive
- May need to be filled
- Cannot turn sharp corners
- Any defects will cause significant attenuation (sparking)

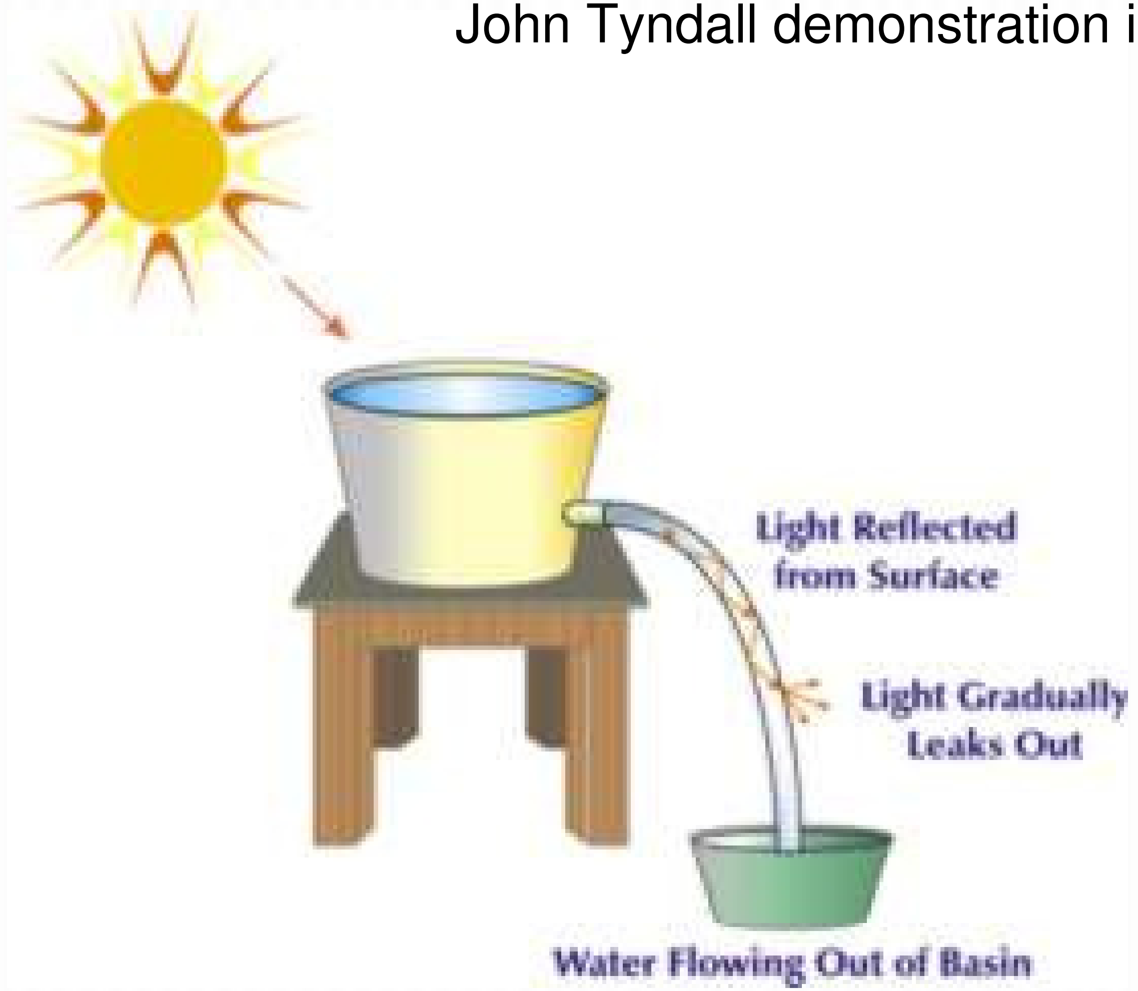
Optical Fiber



Can be considered “circular waveguides”

History of Fiber Optics

John Tyndall demonstration in 1870



Total Internal reflection is the basic idea of fiber optic

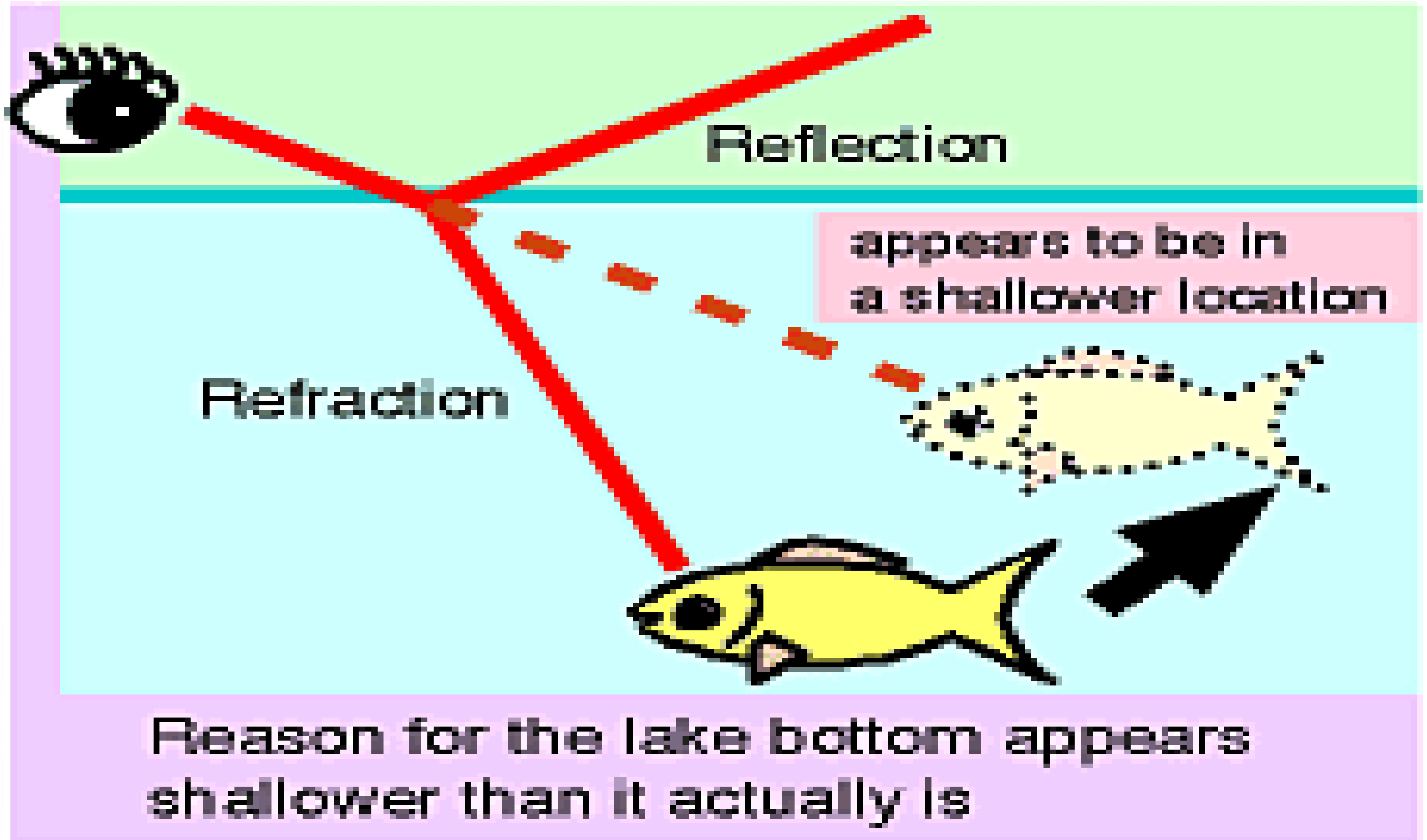
History of Fiber optics

- During 1930, other ideas were developed with this fiber optic such as transmitting images through a fiber.
- During the 1960s, Lasers were introduced as efficient light sources
- In 1970s All glass fibers experienced excessive optical loss, the loss of the light signal as it traveled the fiber limiting transmission distance.

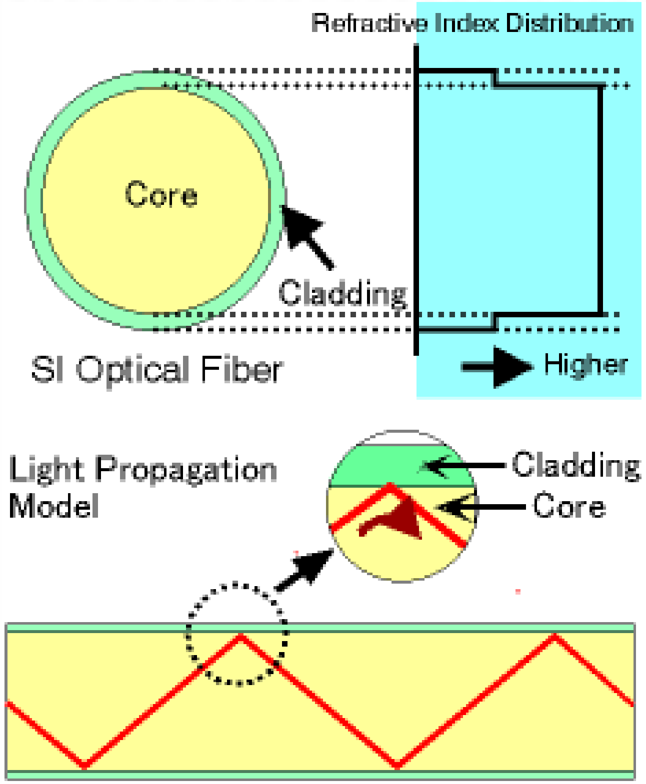
- This motivated the scientists to develop glass fibers that include a separating glass coating. The innermost region was used to transmit the light, while the glass coating prevented the light from leaking out of the core by reflecting the light within the boundaries of the core.

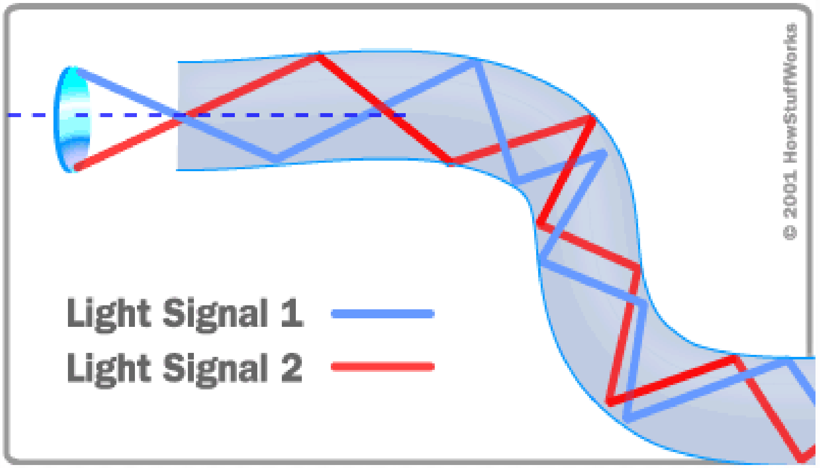
- Today, you can find fiber optics used in variety of applications such as medical environment to the broadcasting industry. It is used to transmit voice, television, images and data signals through small flexible threads of glass or plastic.

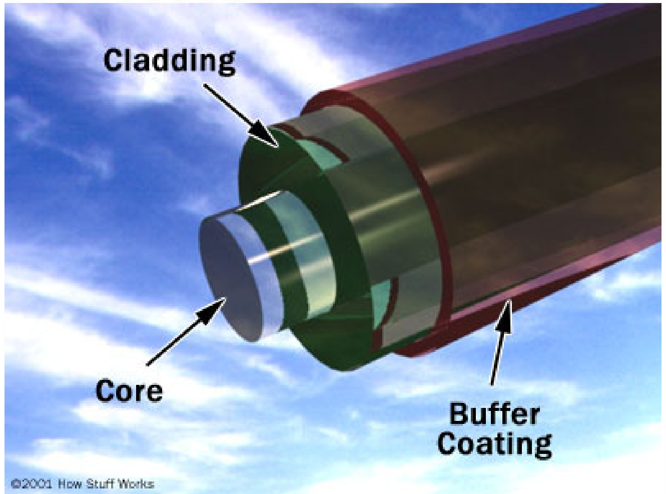
Optical fiber transmits light. But, what prevents the light from escaping from the fiber?



How Does fiber optic transmit light?





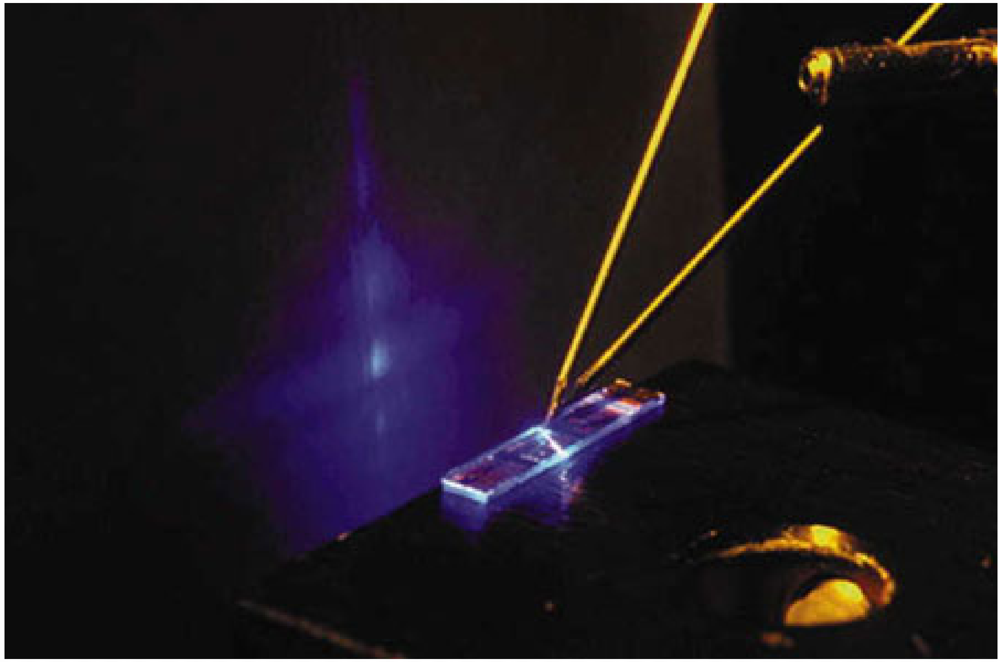


Source and transmitters

- A basic fiber optic communications system consists of three basic elements:
  - Fiber media
  - Light sources
  - Light detector

A Light Sources

LED (Light emitting diode)



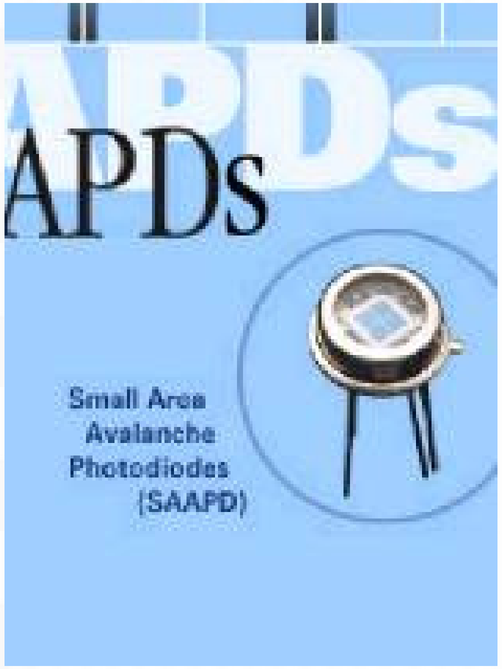
ILD (injection laser diode)

Detectors

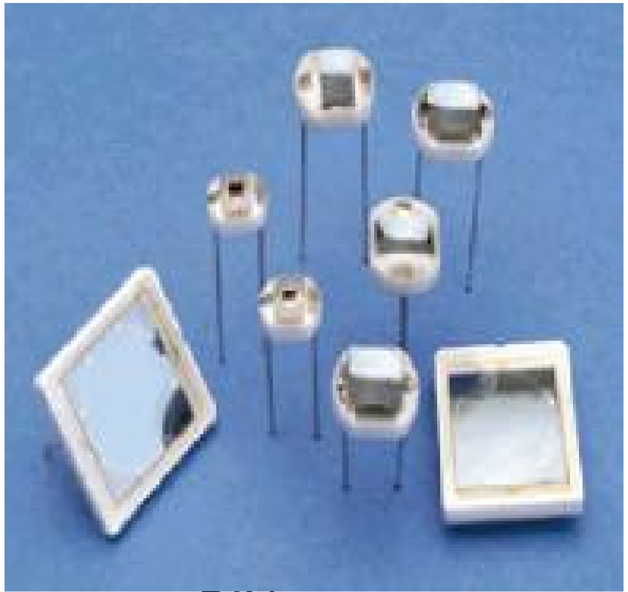
•Detector is the receiving end of a fiber optic link.

There are two kinds of Detectors

1. PIN (Positive Intrinsic Negative)
2. APD (Avalanche photo diodes)



APD



PIN

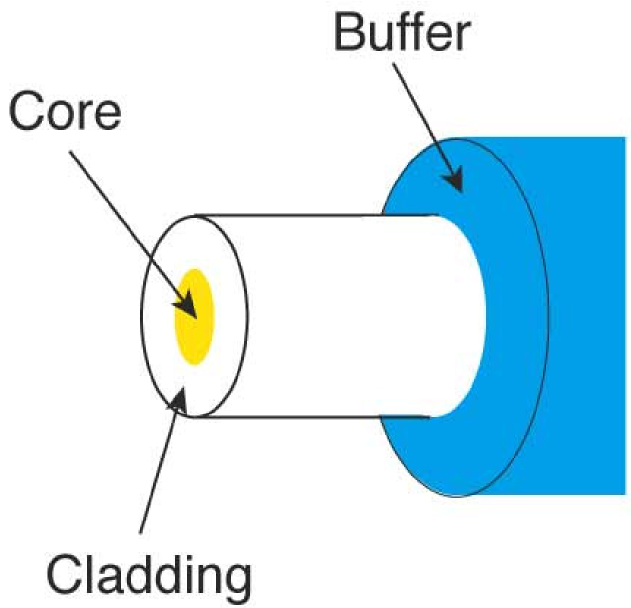
The advantages of fiber optic over wire cable

- Thinner
- Higher carrying capacity
- Less signal degradation
- Light signal
- Low power
- Flexible
- Non-flammable
- Lightweight

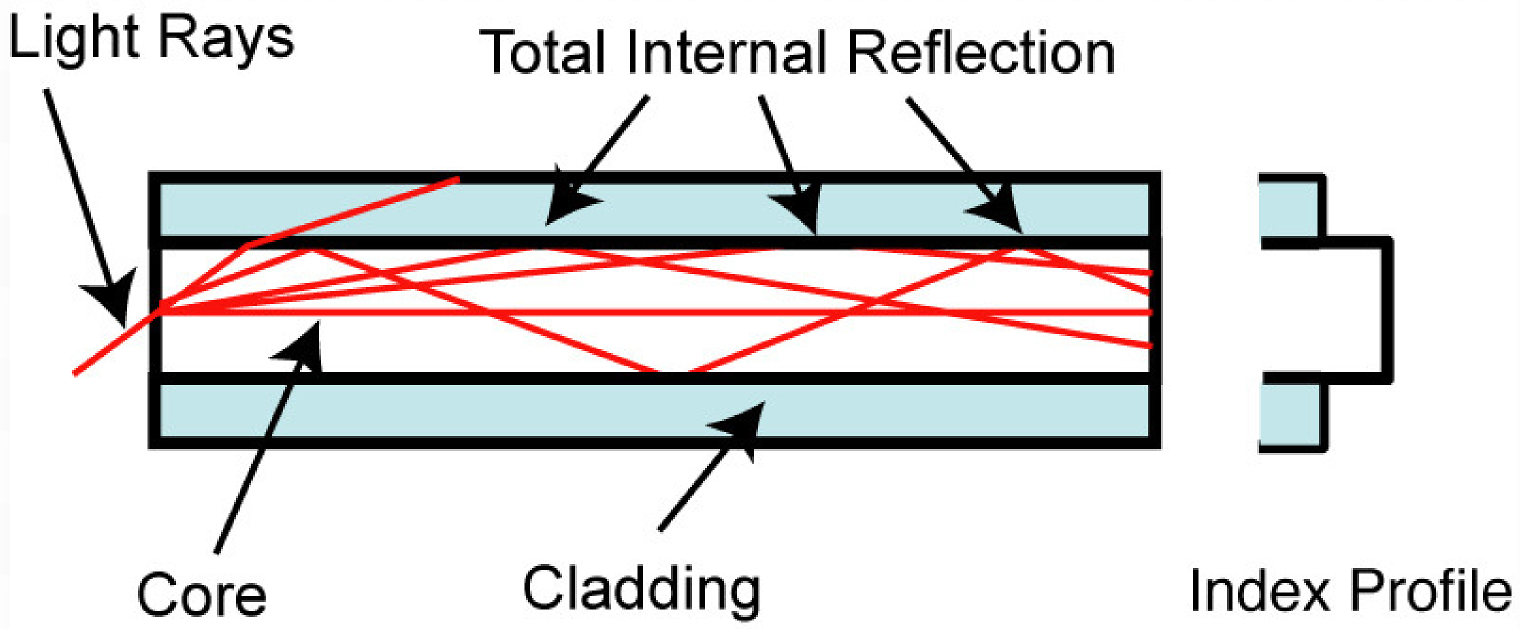
Disadvantage of fiber optic over copper wire cable

- Optical fiber is more expensive per meter than copper
- Optical fiber can not be join together as easily as copper cable. It requires training and expensive splicing and measurement equipment.

Fiber Technology

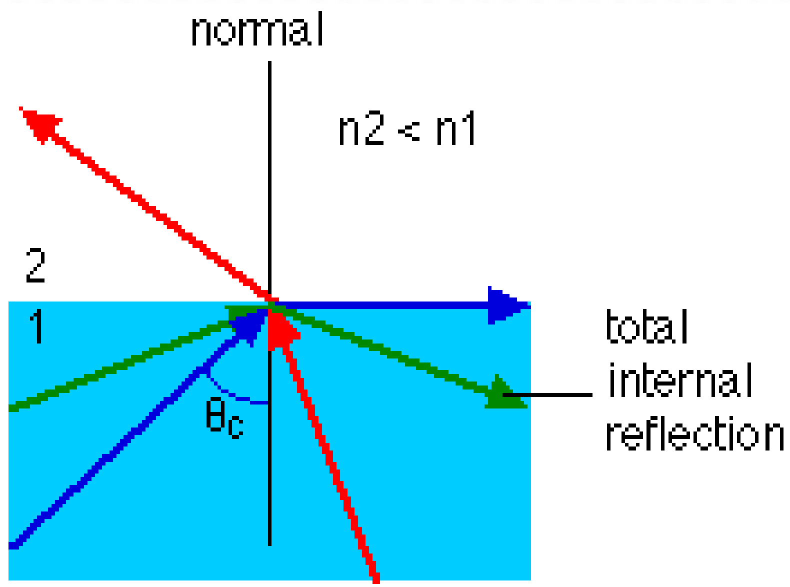


Fiber Technology



Total internal Reflection

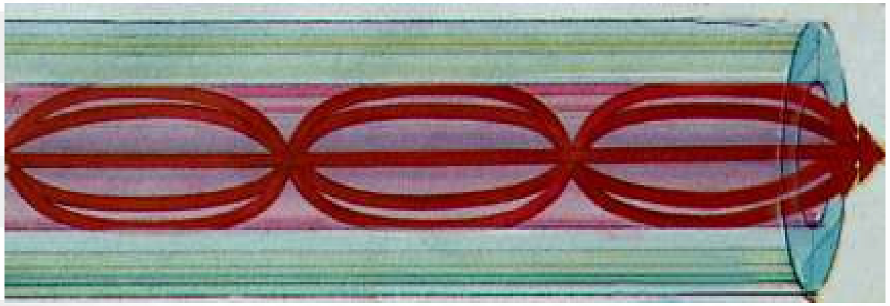
critical angle : $\sin \theta_c = n_2 / n_1$ $(n_1 > n_2)$



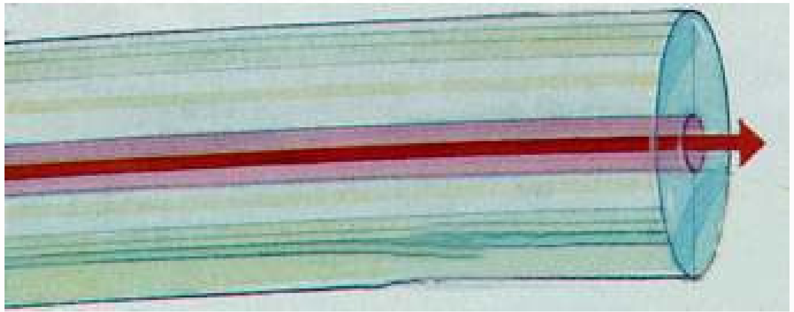
Fiber media

Optical fibers are the actual media that guides the light

There are three types of fiber optic cable commonly used



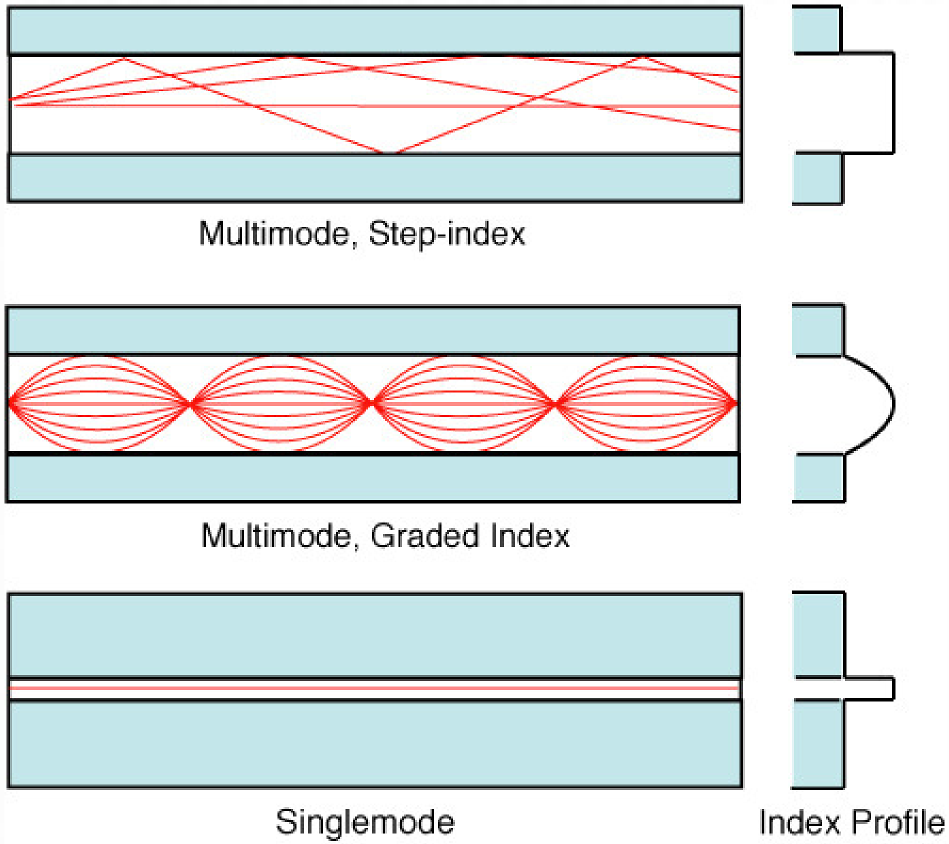
Step-index Multimode fiber



Single Mode

Plastic optic fiber

Fiber Types

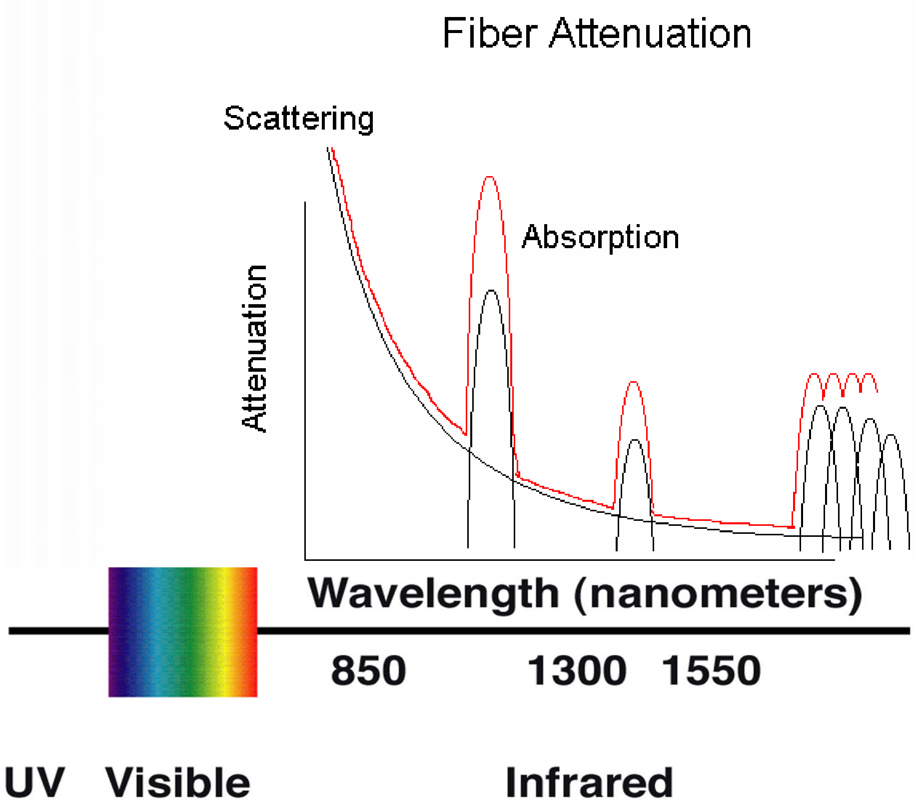

Multimode, Step-index
Multimode, Graded Index
Singlemode
Index Profile

Fiber Types

The loss of fiber optic

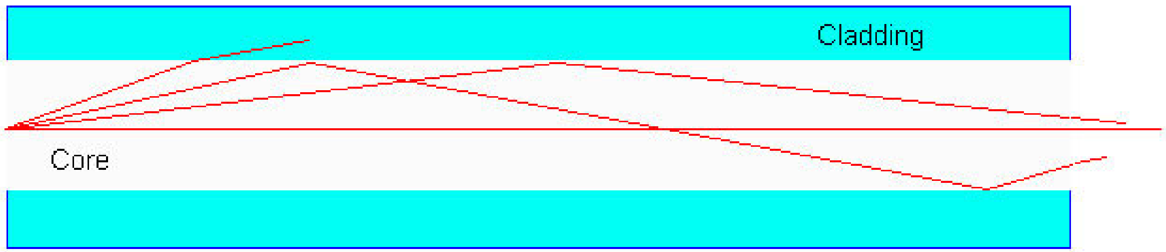
- Material absorption
- Material Scattering
- Waveguide scattering
- Fiber bending
- Fiber coupling loss

Fiber Attenuation

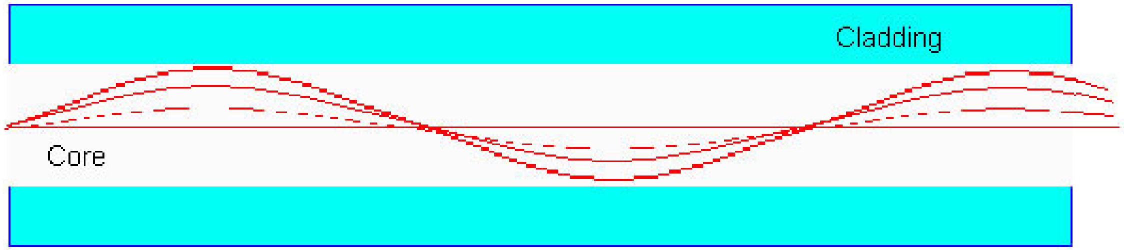


Fiber Bandwidth

Modal Dispersion

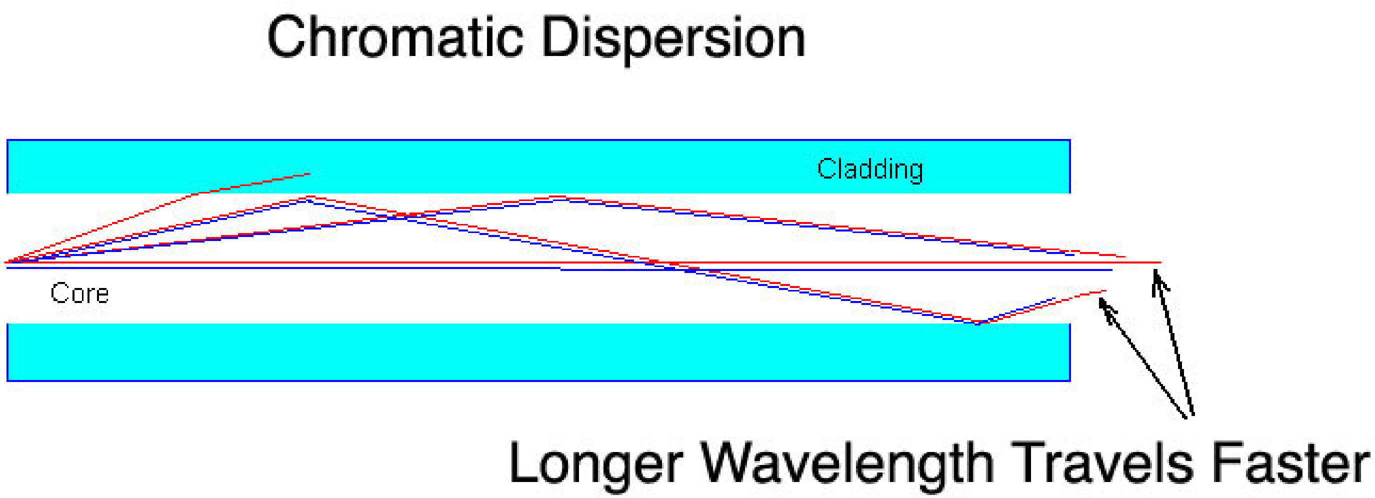


Step Index Fiber

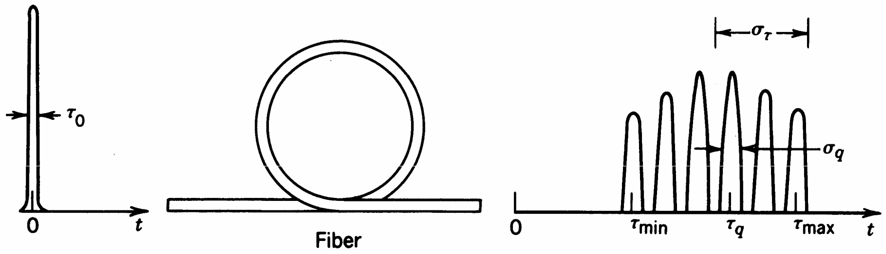


Graded Index Fiber

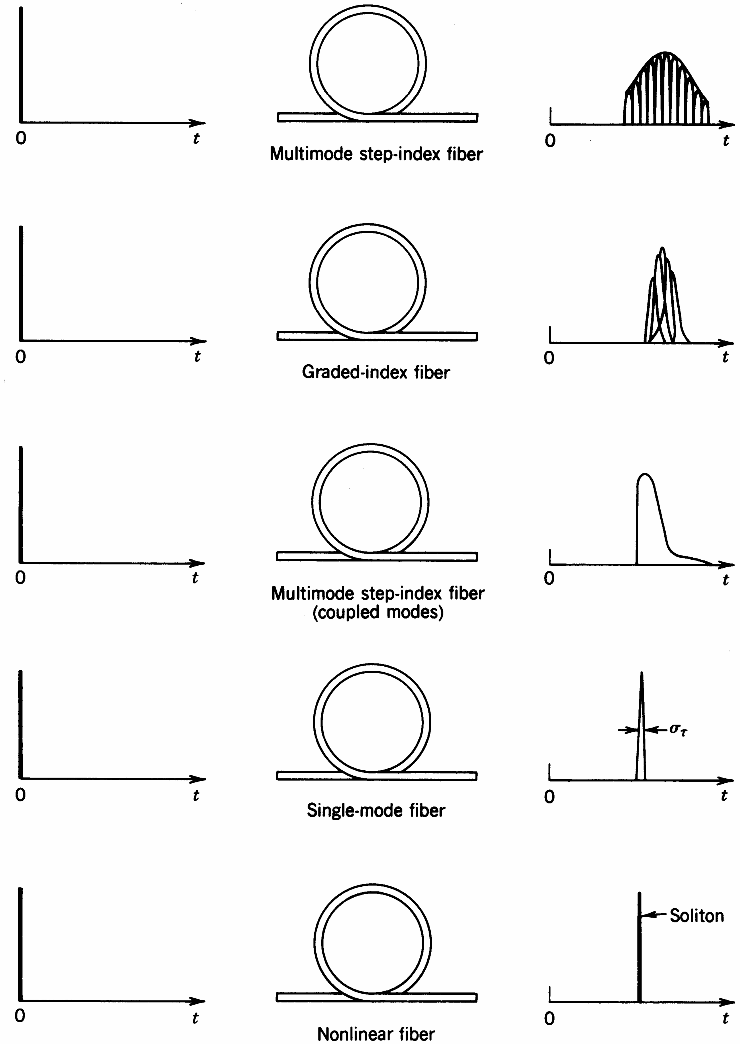
Fiber Bandwidth



Pulse Propagation through Fibers



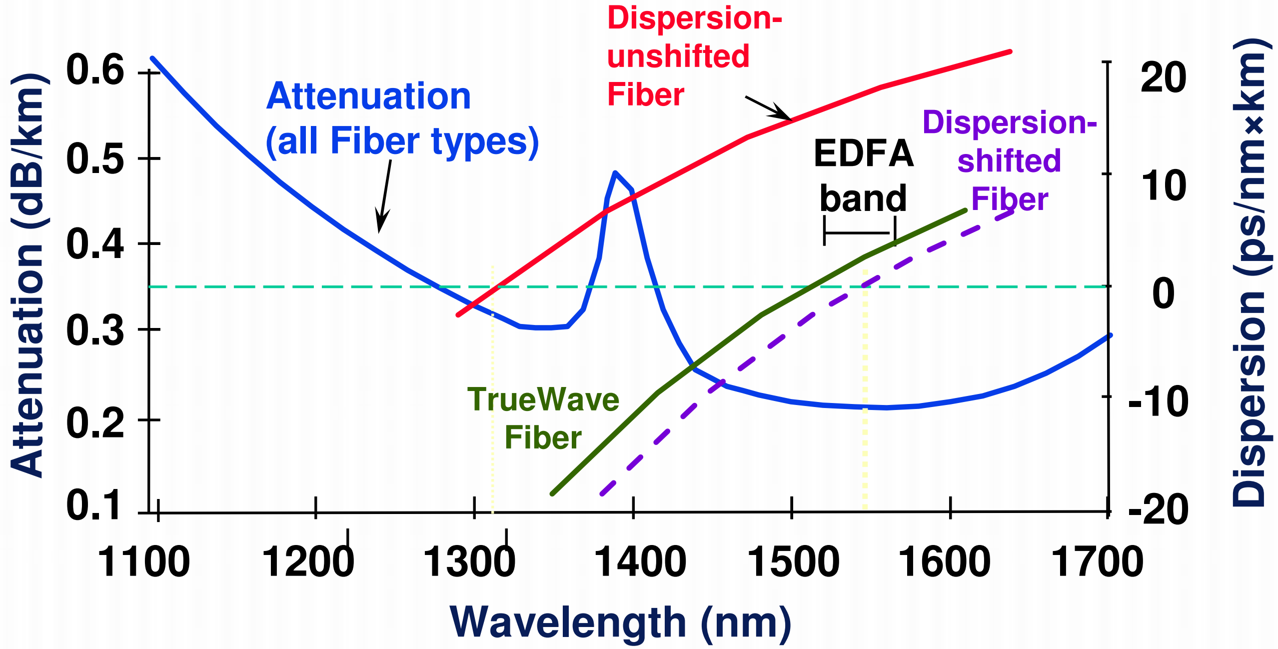
Response of a multi-mode fiber to a single short pulse

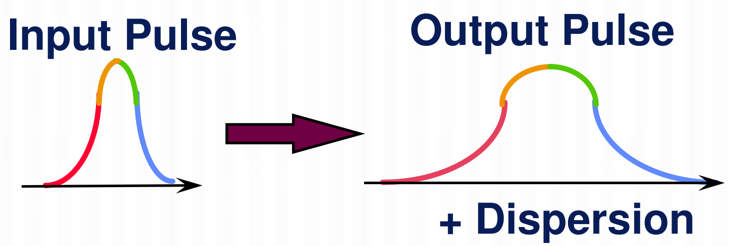


Broadening of a short pulse after transmission through different types of fibers

Fundamentals of Photonics - Saleh and Teich

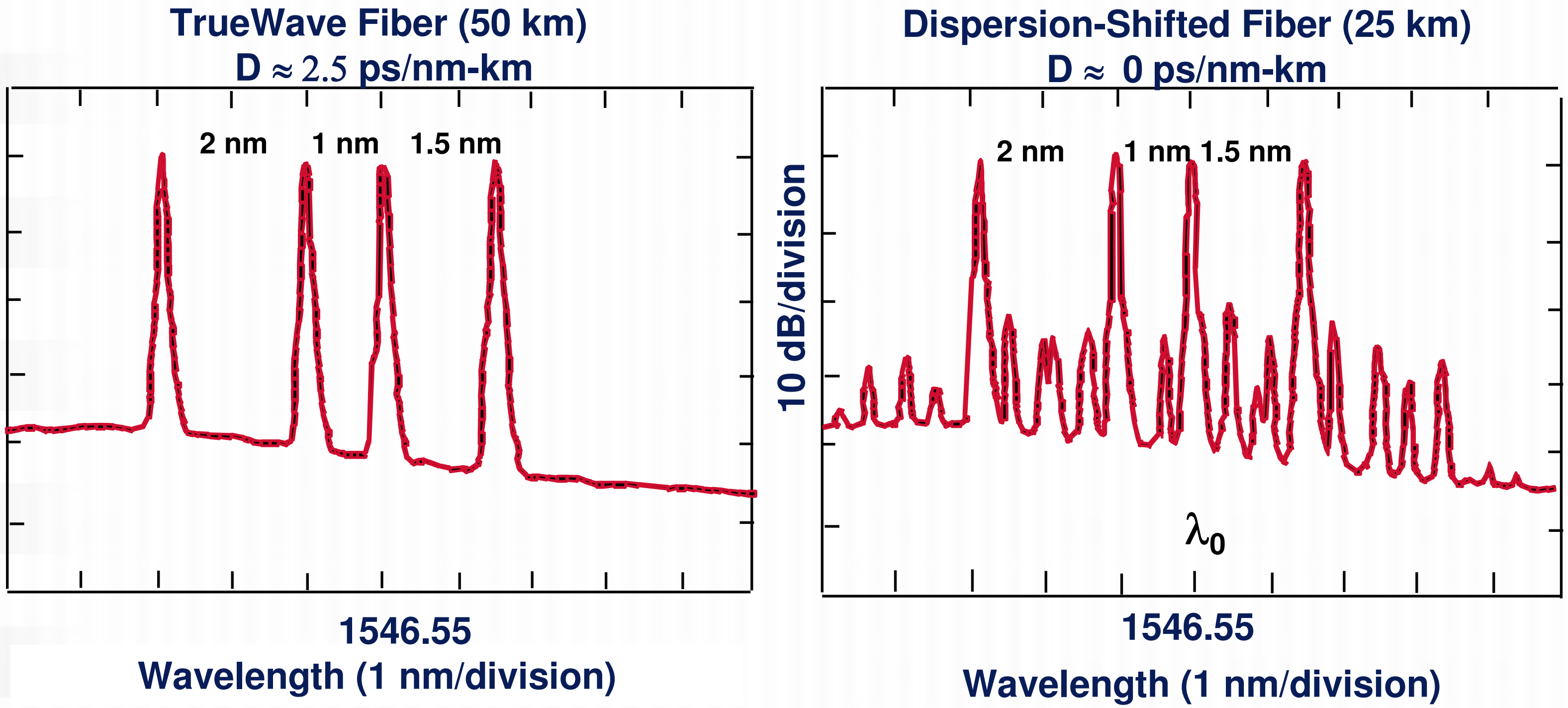
Fiber Attenuation and Chromatic Dispersion





Slide Courtesy of Stan Lumish

Four Wave Mixing (FWM)



Optical Launch Power = 3 dBm/channel

Slide Courtesy of Stan Lumish