LE 230 Homework : Partial Differential Equations

Please show all details of your solutions.

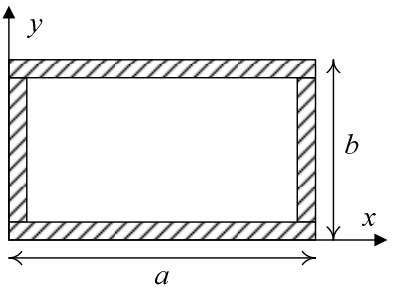
8-1 Solve the following partial differential equation

$$\frac{\partial u}{\partial t}=\frac{1}{16}\frac{\partial^2 u}{\partial x^2};0\le x\le 1,t>0;u(0,t)=u(1,t)=0,t>0;u(x,0)=2\sin 2\pi x,0\le x\le 1$$

and compare your results to the actual solution

$$u(x,t)=2e^{-\pi^2 t/4}\sin 2\pi x$$

8-2 Consider the static electric potential V in the enclosed region shown in the right figure with all planes assumed to be infinite in extent in the z-direction. Assume that V satisfy the Laplace equation ($\nabla^2 V=0$), determine the potential distribution within this region when the boundary conditions are given by:



(a) $V(0,y)=V_0$; $V(a,y)=V(x,0)=V(x,b)=0$.

(b) $V(a,y)=V_0$; $V(0,y)=V(x,0)=V(x,b)=0$.

(c) $V(x,0)=V_0$; $V(0,y)=V(a,y)=V(x,b)=0$.

(d) $V(x,b)=V_0$; $V(0,y)=V(a,y)=V(x,0)=0$.

8-3 Solve the following partial differential equation

$$\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2};0\le x\le 1,t>0;u(0,t)=u(1,t)=0,t>0;$$

$$u(x,0)=2\sin 3\pi x,u_t(x,0)=-12\sin 2\pi x,0\le x\le 1$$

and compare your results to the actual solution

$$u(x,t)=2\cos 6\pi t\sin 3\pi x-\frac{3}{\pi}\cos 4\pi t\sin 2\pi x$$