**Problem:** Derive the simplest sum-of-products expression for the function

\[ f = x_2 \overline{x}_3 x_4 + x_1 x_3 x_4 + x_1 \overline{x}_2 x_4 \]

**Solution:** Applying the consensus property 17a to the first two terms yields

\[ f = x_2 \overline{x}_3 x_4 + x_1 x_3 x_4 + x_2 x_4 x_1 x_4 + x_1 \overline{x}_2 x_4 \]

\[ = x_2 \overline{x}_3 x_4 + x_1 x_3 x_4 + x_1 x_2 x_4 + x_1 \overline{x}_2 x_4 \]

Now, using the combining property 14a for the last two terms gives

\[ f = x_2 \overline{x}_3 x_4 + x_1 x_3 x_4 + x_1 x_4 \]

Finally, using the absorption property 13a produces

\[ f = x_2 \overline{x}_3 x_4 + x_1 x_4 \]

**Problem:** Derive the simplest product-of-sums expression for the function

\[ f = (\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_4)(\overline{x}_1 + x_3 + x_4) \]

**Solution:** Applying the consensus property 17b to the first two terms yields

\[ f = (\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_4)(\overline{x}_1 + x_3 + \overline{x}_4)(\overline{x}_1 + x_3 + x_4) \]

\[ = (\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_4)(\overline{x}_1 + x_3 + \overline{x}_4)(\overline{x}_1 + x_3 + x_4) \]

Now, using the combining property 14b for the last two terms gives

\[ f = (\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_4)(\overline{x}_1 + x_3) \]

Finally, using the absorption property 13b on the first and last terms produces

\[ f = (\overline{x}_1 + \overline{x}_2 + \overline{x}_4)(\overline{x}_1 + x_3) \]

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**PROBLEMS**

Answers to problems marked by an asterisk are given at the back of the book.

2.1 Use algebraic manipulation to prove that \( x + yz = (x + y) \cdot (x + z) \). Note that distributive rule, as stated in identity 12b in section 2.5.

2.2 Use algebraic manipulation to prove that \( (x + y) \cdot (x + \overline{y}) = x \).

2.3 Use algebraic manipulation to prove that \( xy + yz + \overline{x}z = xy + \overline{x}z \). Note that consensus property 17a in section 2.5.

2.4 Use the Venn diagram to prove the identity in problem 1.
2.5 Use the Venn diagram to prove DeMorgan's theorem, as given in expressions 15a and 15b in section 2.5.

2.6 Use the Venn diagram to prove that

\[(x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \overline{x}_3) = x_1 + x_2\]

2.7 Determine whether or not the following expressions are valid, i.e., whether the left- and right-hand sides represent same function.

(a) \(\overline{x}_1 x_3 + x_1 x_2 \overline{x}_3 + \overline{x}_1 x_2 + x_1 \overline{x}_2 = \overline{x}_2 x_3 + x_1 \overline{x}_3 + x_2 \overline{x}_3 + \overline{x}_1 x_2 x_3\)

(b) \(x_1 \overline{x}_3 + x_2 x_3 + \overline{x}_2 \overline{x}_3 = (x_1 + \overline{x}_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + \overline{x}_3)\)

(c) \((x_1 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + x_2) = (x_1 + x_2)(x_2 + x_3)(\overline{x}_1 + \overline{x}_3)\)

2.8 Draw a timing diagram for the circuit in Figure 2.19a. Show the waveforms that can be observed on all wires in the circuit.

2.9 Repeat problem 2.8 for the circuit in Figure 2.19b.

2.10 Use algebraic manipulation to show that for three input variables \(x_1, x_2,\) and \(x_3\)

\[\sum m(1, 2, 3, 4, 5, 6, 7) = x_1 + x_2 + x_3\]

2.11 Use algebraic manipulation to show that for three input variables \(x_1, x_2,\) and \(x_3\)

\[\Pi M (0, 1, 2, 3, 4, 5, 6) = x_1 x_2 x_3\]

2.12 Use algebraic manipulation to find the minimum sum-of-products expression for the function \(f = x_1 x_3 + x_1 \overline{x}_2 + \overline{x}_1 x_2 x_3 + \overline{x}_1 \overline{x}_2 \overline{x}_3.\)

2.13 Use algebraic manipulation to find the minimum sum-of-products expression for the function \(f = x_1 \overline{x}_2 x_3 + x_1 x_2 x_4 + x_1 \overline{x}_2 x_3 \overline{x}_4.\)

2.14 Use algebraic manipulation to find the minimum product-of-sums expression for the function \(f = (x_1 + x_3 + x_4) \cdot (x_1 + \overline{x}_2 + x_3) \cdot (x_1 + \overline{x}_2 + \overline{x}_3 + x_4).\)

2.15 Use algebraic manipulation to find the minimum product-of-sums expression for the function \(f = (x_1 + x_2 + x_3) \cdot (x_1 + \overline{x}_2 + x_3) \cdot (\overline{x}_1 + \overline{x}_2 + x_3) \cdot (x_1 + x_2 + \overline{x}_3).\)

2.16 (a) Show the location of all minterms in a three-variable Venn diagram.

(b) Show a separate Venn diagram for each product term in the function \(f = x_1 \overline{x}_2 x_3 + x_1 x_2 + \overline{x}_1 x_3.\) Use the Venn diagram to find the minimal sum-of-products form of \(f.\)

2.17 Represent the function in Figure 2.18 in the form of a Venn diagram and find its minimal sum-of-products form.

2.18 Figure P2.1 shows two attempts to draw a Venn diagram for four variables. For parts (a) and (b) of the figure, explain why the Venn diagram is not correct. (Hint: the Venn diagram must be able to represent all 16 minterms of the four variables.)

2.19 Figure P2.2 gives a representation of a four-variable Venn diagram and shows the location of minterms \(m_0, m_1,\) and \(m_2.\) Show the location of the other minterms in the diagram. Represent the function \(f = \overline{x}_1 \overline{x}_2 x_3 x_4 + x_1 x_2 x_3 x_4 + \overline{x}_1 x_2\) on this diagram.
Design the simplest sum-of-products circuit that implements the function \( f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7) \).

Design the simplest sum-of-products circuit that implements the function \( f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6, 7) \).

Design the simplest product-of-sums circuit that implements the function \( f(x_1, x_2, x_3) = \Pi M (0, 2, 5) \).

Design the simplest product-of-sums expression for the function \( f(x_1, x_2, x_3) = \Pi M (0, 1, 5, 7) \).

Derive the simplest sum-of-products expression for the function \( f(x_1, x_2, x_3, x_4) = x_1\overline{x}_3\overline{x}_4 + x_2\overline{x}_3 x_4 + x_1\overline{x}_2\overline{x}_3 \).

Derive the simplest sum-of-products expression for the function \( f(x_1, x_2, x_3, x_4, x_5) = \overline{x}_1\overline{x}_3\overline{x}_5 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1 x_4 x_5 + x_1\overline{x}_2\overline{x}_3 x_5 \). (Hint: Use the consensus property 17a.)

Derive the simplest product-of-sums expression for the function \( f(x_1, x_2, x_3, x_4) = (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3) \). (Hint: Use the consensus property 17b.)
2.27 Derive the simplest product-of-sums expression for the function \( f(x_1, x_2, x_3, x_4, x_5) = (\bar{x}_2 + x_3 + x_5)(\bar{x}_1 + \bar{x}_3 + x_5)(x_1 + x_2 + x_5)(x_1 + \bar{x}_4 + \bar{x}_5) \). (Hint: Use the consensus property 17b.)

*2.28 Design the simplest circuit that has three inputs, \( x_1, x_2, \) and \( x_3 \), which produces an output value of 1 whenever two or more of the input variables have the value 1; otherwise, the output has to be 0.

2.29 Design the simplest circuit that has three inputs, \( x_1, x_2, \) and \( x_3 \), which produces an output value of 1 whenever exactly one or two of the input variables have the value 1; otherwise, the output has to be 0.

2.30 Design the simplest circuit that has four inputs, \( x_1, x_2, x_3, \) and \( x_4 \), which produces an output value of 1 whenever three or more of the input variables have the value 1; otherwise, the output has to be 0.

2.31 For the timing diagram in Figure P2.3, synthesize the function \( f(x_1, x_2, x_3) \) in the simplest sum-of-products form.

\[
\begin{array}{c|c|c|c|c}
& x_1 & x_2 & x_3 & f \\
\hline
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

**Figure P2.3** A timing diagram representing a logic function.

*2.32 For the timing diagram in Figure P2.3, synthesize the function \( f(x_1, x_2, x_3) \) in the simplest product-of-sums form.

*2.33 For the timing diagram in Figure P2.4, synthesize the function \( f(x_1, x_2, x_3) \) in the simplest sum-of-products form.

2.34 For the timing diagram in Figure P2.4, synthesize the function \( f(x_1, x_2, x_3) \) in the simplest product-of-sums form.

2.35 Design a circuit with output \( f \) and inputs \( x_1, x_0, y_1 \), and \( y_0 \). Let \( X = x_1x_0 \) be a number, where the four possible values of \( X \), namely, 00, 01, 10, and 11, represent the four numbers 0, 1, 2, and 3, respectively. (We discuss representation of numbers in Chapter 5.) Similarly, let \( Y = y_1y_0 \) represent another number with the same four possible values. The output \( f \) should be 1 if the numbers represented by \( X \) and \( Y \) are equal. Otherwise, \( f \) should be 0.

(a) Show the truth table for \( f \).

(b) Synthesize the simplest possible product-of-sums expression for \( f \).
2.36 Repeat problem 2.35 for the case where $f$ should be 1 only if $X \geq Y$.
   (a) Show the truth table for $f$.
   (b) Show the canonical sum-of-products expression for $f$.
   (c) Show the simplest possible sum-of-products expression for $f$.

2.37 Implement the function in Figure 2.26 using only NAND gates.

2.38 Implement the function in Figure 2.26 using only NOR gates.

2.39 Implement the circuit in Figure 2.35 using NAND and NOR gates.

2.40 Design the simplest circuit that implements the function $f(x_1, x_2, x_3) = \sum m(3, 4, 6)$, using NAND gates.

2.41 Design the simplest circuit that implements the function $f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6)$, using NAND gates.

2.42 Repeat problem 2.40 using NOR gates.

2.43 Repeat problem 2.41 using NOR gates.

2.44 (a) Use a schematic capture tool to draw schematics for the following functions

   $f_1 = x_2 \overline{x}_3 \overline{x}_4 + \overline{x}_1 x_2 x_4 + \overline{x}_1 x_2 x_3 + x_1 x_2 x_3$

   $f_2 = x_2 \overline{x}_4 + \overline{x}_1 x_2 + x_2 x_3$

   (b) Use functional simulation to prove that $f_1 = f_2$.

2.45 (a) Use a schematic capture tool to draw schematics for the following functions

   $f_1 = (x_1 + x_2 + \overline{x}_4) \cdot (\overline{x}_2 + x_3 + \overline{x}_4) \cdot (\overline{x}_1 + x_3 + \overline{x}_4) \cdot (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)$

   $f_2 = (x_2 + \overline{x}_4) \cdot (x_3 + \overline{x}_4) \cdot (\overline{x}_1 + \overline{x}_4)$

   (b) Use functional simulation to prove that $f_1 = f_2$.

2.46 Write VHDL code to implement the function $f(x_1, x_2, x_3) = \sum m(0, 1, 3, 4, 5, 6)$.  

2.47 (a) Write VHDL code to describe the following functions
\[ f_1 = x_1 \overline{x}_3 + x_2 \overline{x}_3 + \overline{x}_3 x_4 + x_1 x_2 + x_1 \overline{x}_4 \]
\[ f_2 = (x_1 + \overline{x}_3) \cdot (x_1 + x_2 + \overline{x}_4) \cdot (x_2 + \overline{x}_3 + \overline{x}_4) \]
(b) Use functional simulation to prove that \( f_1 = f_2 \).

2.48 Consider the following VHDL assignment statements
\[
\begin{align*}
f_1 & \leq ((x_1 \text{ AND} x_3) \text{ OR} (\text{NOT} x_1 \text{ AND NOT} x_3)) \text{ OR} ((x_2 \text{ AND} x_4) \text{ OR} (\text{NOT} x_2 \text{ AND NOT} x_4)) \\
f_2 & \leq (x_1 \text{ AND} x_2 \text{ AND NOT} x_3 \text{ AND NOT} x_4) \text{ OR} (\text{NOT} x_1 \text{ AND NOT} x_2 \text{ AND} x_3 \text{ AND} x_4) \text{ OR} (x_1 \text{ AND NOT} x_2 \text{ AND NOT} x_3 \text{ AND} x_4) \text{ OR} (\text{NOT} x_1 \text{ AND} x_2 \text{ AND} x_3 \text{ AND NOT} x_4) \\
\end{align*}
\]
(a) Write complete VHDL code to implement \( f_1 \) and \( f_2 \).
(b) Use functional simulation to prove that \( f_1 = f_2 \).

REFERENCES