Problem: Derive the simplest sum-of-products expression for the function **Exam**

$$f = x_2\bar{x}_3x_4 + x_1x_3x_4 + x_1\bar{x}_2x_4$$

Solution: Applying the consensus property 17*a* to the first two terms yields

$$\begin{aligned} f &= x_2\bar{x}_3x_4 + x_1x_3x_4 + x_2x_4x_1x_4 + x_1\bar{x}_2x_4 \\ &= x_2\bar{x}_3x_4 + x_1x_3x_4 + x_1x_2x_4 + x_1\bar{x}_2x_4 \end{aligned}$$

Now, using the combining property 14*a* for the last two terms gives

$$f = x_2\bar{x}_3x_4 + x_1x_3x_4 + x_1x_4$$

Finally, using the absorption property 13*a* produces

$$f = x_2\bar{x}_3x_4 + x_1x_4$$

Problem: Derive the simplest product-of-sums expression for the function **Exam**

$$f = (\bar{x}_1 + x_2 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_4)(\bar{x}_1 + x_3 + x_4)$$

Solution: Applying the consensus property 17*b* to the first two terms yields

$$\begin{aligned} f &= (\bar{x}_1 + x_2 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_4)(\bar{x}_1 + x_3 + \bar{x}_1 + \bar{x}_4)(\bar{x}_1 + x_3 + x_4) \\ &= (\bar{x}_1 + x_2 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_4)(\bar{x}_1 + x_3 + \bar{x}_4)(\bar{x}_1 + x_3 + x_4) \end{aligned}$$

Now, using the combining property 14*b* for the last two terms gives

$$f = (\bar{x}_1 + x_2 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_4)(\bar{x}_1 + x_3)$$

Finally, using the absorption property 13*b* on the first and last terms produces

$$f = (\bar{x}_1 + \bar{x}_2 + \bar{x}_4)(\bar{x}_1 + x_3)$$

PROBLEMS

Answers to problems marked by an asterisk are given at the back of the book.

2.1 Use algebraic manipulation to prove that $x + yz = (x + y) \cdot (x + z)$. Note that distributive rule, as stated in identity 12*b* in section 2.5.

2.2 Use algebraic manipulation to prove that $(x + y) \cdot (x + \bar{y}) = x$.

2.3 Use algebraic manipulation to prove that $xy + yz + \bar{x}z = xy + \bar{x}z$. Note that consensus property 17*a* in section 2.5.

2.4 Use the Venn diagram to prove the identity in problem 1.

2.5 Use the Venn diagram to prove DeMorgan's theorem, as given in expressions 15*a* and 15*b* in section 2.5.

2.6 Use the Venn diagram to prove that

$$(x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \overline{x}_3) = x_1 + x_2$$

***2.7** Determine whether or not the following expressions are valid, i.e., whether the left- and right-hand sides represent same function.
(a) $\overline{x}_1 x_3 + x_1 x_2 \overline{x}_3 + \overline{x}_1 x_2 + x_1 \overline{x}_2 = \overline{x}_2 x_3 + x_1 \overline{x}_3 + x_2 \overline{x}_3 + \overline{x}_1 x_2 x_3$
(b) $x_1 \overline{x}_3 + x_2 x_3 + \overline{x}_2 \overline{x}_3 = (x_1 + \overline{x}_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + \overline{x}_3)$
(c) $(x_1 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + x_2) = (x_1 + x_2)(x_2 + x_3)(\overline{x}_1 + \overline{x}_3)$

2.8 Draw a timing diagram for the circuit in Figure 2.19*a*. Show the waveforms that can be observed on all wires in the circuit.

2.9 Repeat problem 2.8 for the circuit in Figure 2.19*b*.

2.10 Use algebraic manipulation to show that for three input variables x_1, x_2, and x_3

$$\sum m(1, 2, 3, 4, 5, 6, 7) = x_1 + x_2 + x_3$$

2.11 Use algebraic manipulation to show that for three input variables x_1, x_2, and x_3

$$\Pi M(0, 1, 2, 3, 4, 5, 6) = x_1 x_2 x_3$$

2.12 Use algebraic manipulation to find the minimum sum-of-products expression for the function $f = x_1 x_3 + x_1 \overline{x}_2 + \overline{x}_1 x_2 x_3 + \overline{x}_1 \overline{x}_2 \overline{x}_3$.

2.13 Use algebraic manipulation to find the minimum sum-of-products expression for the function $f = x_1 \overline{x}_2 \overline{x}_3 + x_1 x_2 x_4 + x_1 \overline{x}_2 x_3 \overline{x}_4$.

2.14 Use algebraic manipulation to find the minimum product-of-sums expression for the function $f = (x_1 + x_3 + x_4) \cdot (x_1 + \overline{x}_2 + x_3) \cdot (x_1 + \overline{x}_2 + \overline{x}_3 + x_4)$.

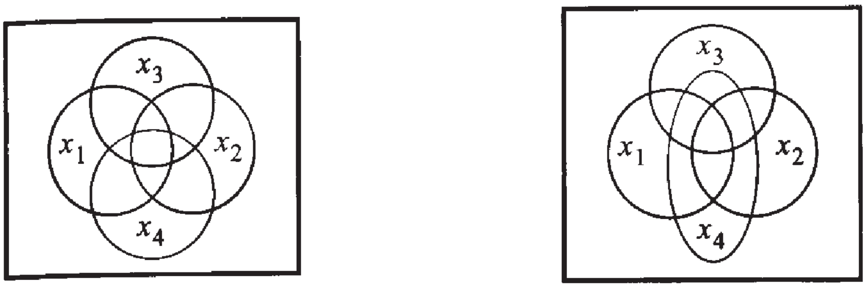
2.15 Use algebraic manipulation to find the minimum product-of-sums expression for the function $f = (x_1 + x_2 + x_3) \cdot (x_1 + \overline{x}_2 + x_3) \cdot (\overline{x}_1 + \overline{x}_2 + x_3) \cdot (x_1 + x_2 + \overline{x}_3)$.

2.16 (a) Show the location of all minterms in a three-variable Venn diagram.
(b) Show a separate Venn diagram for each product term in the function $f = x_1 \overline{x}_2 x_3 + x_1 x_2 + \overline{x}_1 x_3$. Use the Venn diagram to find the minimal sum-of-products form of f.

2.17 Represent the function in Figure 2.18 in the form of a Venn diagram and find its minimal sum-of-products form.

2.18 Figure P2.1 shows two attempts to draw a Venn diagram for four variables. For parts (*a*) and (*b*) of the figure, explain why the Venn diagram is not correct. (Hint: the Venn diagram must be able to represent all 16 minterms of the four variables.)

2.19 Figure P2.2 gives a representation of a four-variable Venn diagram and shows the location of minterms m_0, m_1, and m_2. Show the location of the other minterms in the diagram. Represent the function $f = \overline{x}_1 \overline{x}_2 x_3 \overline{x}_4 + x_1 x_2 x_3 x_4 + \overline{x}_1 x_2$ on this diagram.



(a) (b)

Figure P2.1 Two attempts to draw a four-variable Venn diagram.

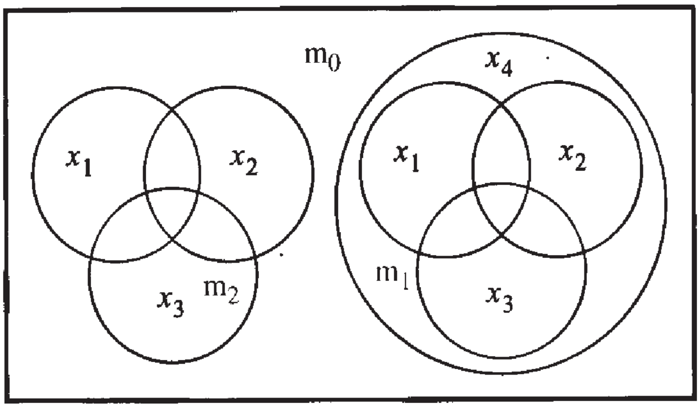


Figure P2.2 A four-variable Venn diagram.

***2.20** Design the simplest sum-of-products circuit that implements the function $f(x_1, x_2, x_3) =$ $\sum m(3, 4, 6, 7)$.

2.21 Design the simplest sum-of-products circuit that implements the function $f(x_1, x_2, x_3) =$ $\sum m(1, 3, 4, 6, 7)$.

2.22 Design the simplest product-of-sums circuit that implements the function $f(x_1, x_2, x_3) =$ $\Pi M(0, 2, 5)$.

***2.23** Design the simplest product-of-sums expression for the function $f(x_1, x_2, x_3) = \Pi M(0, 1$ $5, 7)$.

2.24 Derive the simplest sum-of-products expression for the function $f(x_1, x_2, x_3, x_4) =$ $x_1\overline{x}_3\overline{x}_4 + x_2\overline{x}_3x_4 + x_1\overline{x}_2\overline{x}_3$.

2.25 Derive the simplest sum-of-products expression for the function $f(x_1, x_2, x_3, x_4, x_5) =$ $\overline{x}_1\overline{x}_3\overline{x}_5 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1x_4x_5 + x_1\overline{x}_2\overline{x}_3x_5$. (Hint: Use the consensus property 17*a*.)

2.26 Derive the simplest product-of-sums expression for the function $f(x_1, x_2, x_3, x_4) =$ $(\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3)$. (Hint: Use the consensus property 17*b*.)

2.27 Derive the simplest product-of-sums expression for the function $f(x_1, x_2, x_3, x_4, x_5) = (\bar{x}_2 + x_3 + x_5)(x_1 + \bar{x}_3 + x_5)(x_1 + x_2 + x_5)(x_1 + \bar{x}_4 + \bar{x}_5)$. (Hint: Use the consensus property 17*b*.)

***2.28** Design the simplest circuit that has three inputs, x_1, x_2, and x_3, which produces an output value of 1 whenever two or more of the input variables have the value 1; otherwise, the output has to be 0.

2.29 Design the simplest circuit that has three inputs, x_1, x_2, and x_3, which produces an output value of 1 whenever exactly one or two of the input variables have the value 1; otherwise, the output has to be 0.

2.30 Design the simplest circuit that has four inputs, x_1, x_2, x_3, and x_4, which produces an output value of 1 whenever three or more of the input variables have the value 1; otherwise, the output has to be 0.

2.31 For the timing diagram in Figure P2.3, synthesize the function $f(x_1, x_2, x_3)$ in the simplest sum-of-products form.

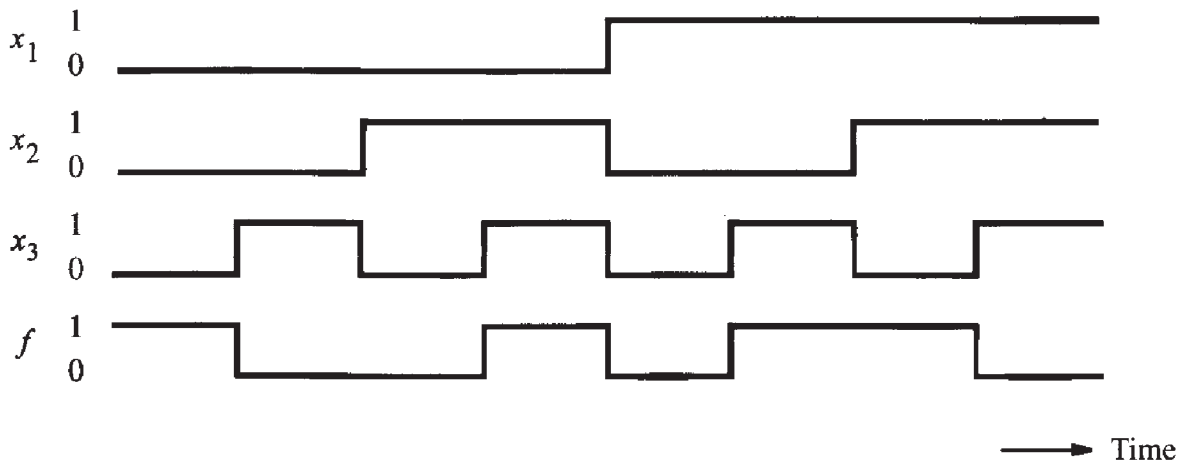


Figure P2.3 A timing diagram representing a logic function.

***2.32** For the timing diagram in Figure P2.3, synthesize the function $f(x_1, x_2, x_3)$ in the simplest product-of-sums form.

***2.33** For the timing diagram in Figure P2.4, synthesize the function $f(x_1, x_2, x_3)$ in the simplest sum-of-products form.

2.34 For the timing diagram in Figure P2.4, synthesize the function $f(x_1, x_2, x_3)$ in the simplest product-of-sums form.

2.35 Design a circuit with output f and inputs x_1, x_0, y_1, and y_0. Let $X = x_1x_0$ be a number, where the four possible values of X, namely, 00, 01, 10, and 11, represent the four numbers 0, 1, 2, and 3, respectively. (We discuss representation of numbers in Chapter 5.) Similarly, let $Y = y_1y_0$ represent another number with the same four possible values. The output f should be 1 if the numbers represented by X and Y are equal. Otherwise, f should be 0.

(a) Show the truth table for f.

(b) Synthesize the simplest possible product-of-sums expression for f.

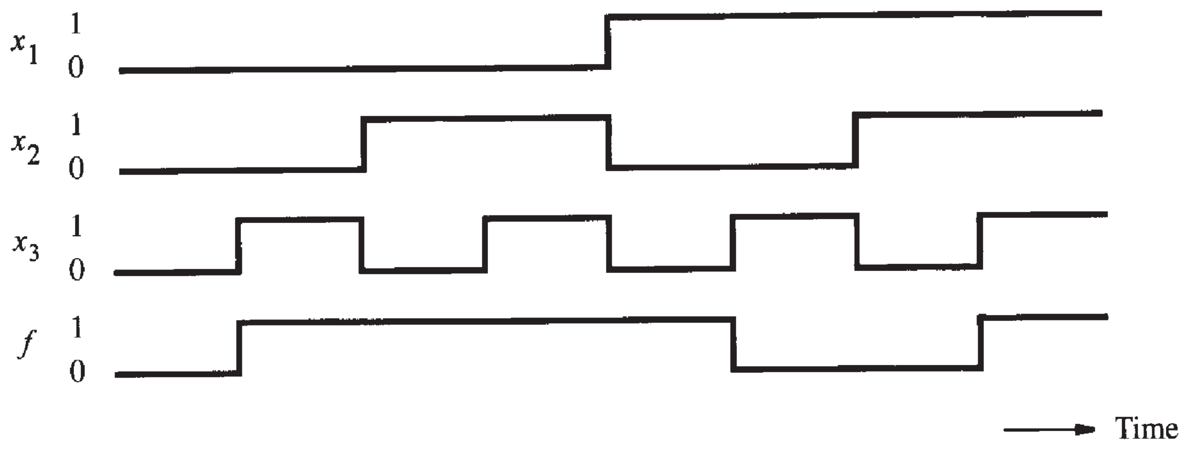


Figure P2.4 A timing diagram representing a logic function.

2.36 Repeat problem 2.35 for the case where f should be 1 only if $X \geq Y$.
(a) Show the truth table for f.
(b) Show the canonical sum-of-products expression for f.
(c) Show the simplest possible sum-of-products expression for f.

2.37 Implement the function in Figure 2.26 using only NAND gates.

2.38 Implement the function in Figure 2.26 using only NOR gates.

2.39 Implement the circuit in Figure 2.35 using NAND and NOR gates.

***2.40** Design the simplest circuit that implements the function $f(x_1, x_2, x_3) = \sum m(3, 4, 6,$ using NAND gates.

2.41 Design the simplest circuit that implements the function $f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6,$ using NAND gates.

***2.42** Repeat problem 2.40 using NOR gates.

2.43 Repeat problem 2.41 using NOR gates.

2.44 (a) Use a schematic capture tool to draw schematics for the following functions

$$f_1 = x_2\overline{x}_3\overline{x}_4 + \overline{x}_1x_2x_4 + \overline{x}_1x_2x_3 + x_1x_2x_3$$
$$f_2 = x_2\overline{x}_4 + \overline{x}_1x_2 + x_2x_3$$

(b) Use functional simulation to prove that $f_1 = f_2$.

2.45 (a) Use a schematic capture tool to draw schematics for the following functions

$$f_1 = (x_1 + x_2 + \overline{x}_4) \cdot (\overline{x}_2 + x_3 + \overline{x}_4) \cdot (\overline{x}_1 + x_3 + \overline{x}_4) \cdot (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)$$
$$f_2 = (x_2 + \overline{x}_4) \cdot (x_3 + \overline{x}_4) \cdot (\overline{x}_1 + \overline{x}_4)$$

(b) Use functional simulation to prove that $f_1 = f_2$.

2.46 Write VHDL code to implement the function $f(x_1, x_2, x_3) = \sum m(0, 1, 3, 4, 5, 6)$.

2.47 (a) Write VHDL code to describe the following functions

$$f_1 = x_1\bar{x}_3 + x_2\bar{x}_3 + \bar{x}_3\bar{x}_4 + x_1x_2 + x_1\bar{x}_4$$
$$f_2 = (x_1 + \bar{x}_3) \cdot (x_1 + x_2 + \bar{x}_4) \cdot (x_2 + \bar{x}_3 + \bar{x}_4)$$

(b) Use functional simulation to prove that $f_1 = f_2$.

2.48 Consider the following VHDL assignment statements

```
f1 <= ((x1 AND x3) OR (NOT x1 AND NOT x3)) OR ((x2 AND x4) OR
      (NOT x2 AND NOT x4)) ;
f2 <= (x1 AND x2 AND NOT x3 AND NOT x4) OR (NOT x1 AND NOT x2 AND x3 AND x4)
      OR (x1 AND NOT x2 AND NOT x3 AND x4) OR
      (NOT x1 AND x2 AND x3 AND NOT x4) ;
```

(a) Write complete VHDL code to implement f1 and f2.
(b) Use functional simulation to prove that $f1 = \overline{f2}$.

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