$x_5x_6x_7$ \ $x_1x_2x_3x_4$	0000	0001	···	1110	1111
000	m_0	m_8		m_{112}	m_{120}
001	m_1	m_9		m_{113}	m_{121}
010	m_2	m_{10}		m_{114}	m_{122}
011	m_3	m_{11}	···	m_{115}	m_{123}
100	m_4	m_{12}		m_{116}	m_{124}
101	m_5	m_{13}		m_{117}	m_{125}
110	m_6	m_{14}		m_{118}	m_{126}
111	m_7	m_{15}		m_{119}	m_{127}

Figure 4.55 A possible format for truth tables of seven-variable functions.

PROBLEMS

Answers to problems marked by an asterisk are given at the back of the book.

***4.1** Find the minimum-cost SOP and POS forms for the function $f(x_1, x_2, x_3) = \sum m(1, 2, 3, 5)$.

***4.2** Repeat problem 4.1 for the function $f(x_1, x_2, x_3) = \sum m(1, 4, 7) + D(2, 5)$.

4.3 Repeat problem 4.1 for the function $f(x_1, \ldots, x_4) = \Pi M(0, 1, 2, 4, 5, 7, 8, 9, 10, 12, 14, 15)$.

4.4 Repeat problem 4.1 for the function $f(x_1, \ldots, x_4) = \sum m(0, 2, 8, 9, 10, 15) + D(1, 3, 6, 7)$.

***4.5** Repeat problem 4.1 for the function $f(x_1, \ldots, x_5) = \Pi M(1, 4, 6, 7, 9, 12, 15, 17, 20, 21, 22, 23, 28, 31)$.

4.6 Repeat problem 4.1 for the function $f(x_1, \ldots, x_5) = \sum m(0, 1, 3, 4, 6, 8, 9, 11, 13, 14, 16, 19, 20, 21, 22, 24, 25) + D(5, 7, 12, 15, 17, 23)$.

4.7 Repeat problem 4.1 for the function $f(x_1, \ldots, x_5) = \sum m(1, 4, 6, 7, 9, 10, 12, 15, 17, 19, 20, 23, 25, 26, 27, 28, 30, 31) + D(8, 16, 21, 22)$.

4.8 Find 5 three-variable functions for which the product-of-sums form has lower cost than the sum-of-products form.

***4.9** A four-variable logic function that is equal to 1 if any three or all four of its variables are equal to 1 is called a *majority* function. Design a minimum-cost SOP circuit that implements this majority function.

4.10 Derive a minimum-cost realization of the four-variable function that is equal to 1 if exactly two or exactly three of its variables are equal to 1; otherwise it is equal to 0.

***4.11** Prove or show a counter-example for the statement: If a function f has a unique minimum-cost SOP expression, then it also has a unique minimum-cost POS expression.

***4.12** A circuit with two outputs has to implement the following functions

$$f(x_1, \ldots, x_4) = \sum m(0, 2, 4, 6, 7, 9) + D(10, 11)$$

$$g(x_1, \ldots, x_4) = \sum m(2, 4, 9, 10, 15) + D(0, 13, 14)$$

Design the minimum-cost circuit and compare its cost with combined costs of two circuits that implement f and g separately. Assume that the input variables are available in both uncomplemented and complemented forms.

4.13 Repeat problem 4.12 for the following functions

$$f(x_1, \ldots, x_5) = \sum m(1, 4, 5, 11, 27, 28) + D(10, 12, 14, 15, 20, 31)$$
$$g(x_1, \ldots, x_5) = \sum m(0, 1, 2, 4, 5, 8, 14, 15, 16, 18, 20, 24, 26, 28, 31) + D(10, 11, 12, 27)$$

***4.14** Implement the logic circuit in Figure 4.23 using NAND gates only.

***4.15** Implement the logic circuit in Figure 4.23 using NOR gates only.

4.16 Implement the logic circuit in Figure 4.25 using NAND gates only.

4.17 Implement the logic circuit in Figure 4.25 using NOR gates only.

***4.18** Consider the function $f = x_3x_5 + \bar{x}_1x_2x_4 + x_1\bar{x}_2\bar{x}_4 + x_1x_3\bar{x}_4 + \bar{x}_1x_3x_4 + \bar{x}_1x_2x_5 + x_1\bar{x}_2x_5$. Derive a minimum-cost circuit that implements this function using NOT, AND, and OR gates.

4.19 Derive a minimum-cost circuit that implements the function $f(x_1, \ldots, x_4) = \sum m(4, 7, 8, 11) + D(12, 15)$.

4.20 Find the simplest realization of the function $f(x_1, \ldots, x_4) = \sum m(0, 3, 4, 7, 9, 10, 13, 14)$, assuming that the logic gates have a maximum fan-in of two.

***4.21** Find the minimum-cost circuit for the function $f(x_1, \ldots, x_4) = \sum m(0, 4, 8, 13, 14, 15)$. Assume that the input variables are available in uncomplemented form only. (Hint: use functional decomposition.)

4.22 Use functional decomposition to find the best implementation of the function $f(x_1, \ldots, x_5) = \sum m(1, 2, 7, 9, 10, 18, 19, 25, 31) + D(0, 15, 20, 26)$. How does your implementation compare with the lowest-cost SOP implementation? Give the costs.

***4.23** Use the tabular method discussed in section 4.9 to find a minimum cost SOP realization for the function

$$f(x_1, \ldots, x_4) = \sum m(0, 2, 4, 5, 7, 8, 9, 15)$$

4.24 Repeat problem 4.23 for the function

$$f(x_1, \ldots, x_4) = \sum m(0, 4, 6, 8, 9, 15) + D(3, 7, 11, 13)$$

4.25 Repeat problem 4.23 for the function

$$f(x_1, \ldots, x_4) = \sum m(0, 3, 4, 5, 7, 9, 11) + D(8, 12, 13, 14)$$

4.26 Show that the following distributive-like rules are valid

$$(A \cdot B)\#C = (A\#C) \cdot (B\#C)$$
$$(A + B)\#C = (A\#C) + (B\#C)$$

4.27 Use the cubical representation and the method discussed in section 4.10 to find a minimum-cost SOP realization of the function $f(x_1, \ldots, x_4) = \sum m(0, 2, 4, 5, 7, 8, 9, 15)$.

4.28 Repeat problem 4.27 for the function $f(x_1, \ldots, x_5) = \overline{x}_1\overline{x}_3\overline{x}_5 + x_1x_2\overline{x}_3 + x_2x_3\overline{x}_4x_5 + x_1\overline{x}_2\overline{x}_3x_4 + x_1x_2x_3x_4\overline{x}_5 + \overline{x}_1x_2x_4\overline{x}_5 + \overline{x}_1\overline{x}_3x_4x_5$.

4.29 Use the cubical representation and the method discussed in section 4.10 to find a minimum-cost SOP realization of the function $f(x_1, \ldots, x_4)$ defined by the ON-set ON = {00x0, 100x, x010, 1111} and the don't-care set DC = {00x1, 011x}.

4.30 In section 4.10.1 we showed how the $*$-product operation can be used to find the prime implicants of a given function f. Another possibility is to find the prime implicants by expanding the implicants in the initial cover of the function. An implicant is *expanded* by removing one literal to create a larger implicant (in terms of the number of vertices covered). A larger implicant is valid only if it does not include any vertices for which $f = 0$. The largest valid implicants obtained in the process of expansion are the prime implicants. Figure P4.1 illustrates the expansion of the implicant $\overline{x}_1x_2x_3$ of the function in Figure 4.9, which is also used in Example 4.16. Note from Figure 4.9 that

$$\overline{f} = x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2x_3 + x_1x_2\overline{x}_3$$

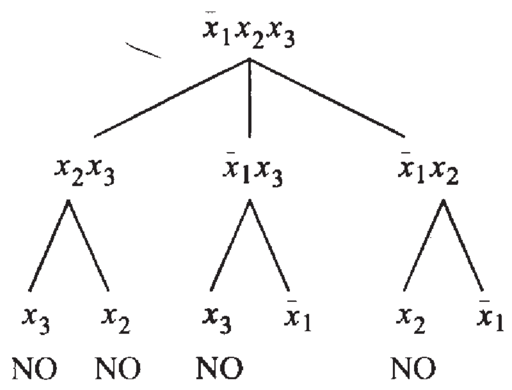

Figure P4.1 Expansion of implicant $\overline{x}_1x_2x_3$.

In Figure P4.1 the word NO is used to indicate that the expanded term is not valid, because it includes one or more vertices from $\overline{f}$. From the graph it is clear that the largest valid implicants that arise from this expansion are x_2x_3 and $\overline{x}_1$; they are prime implicants of f.

Expand the other four implicants given in the initial cover in Example 4.14 to find all prime implicants of f. What is the relative complexity of this procedure compared to the $*$-product technique?

4.31 Repeat problem 4.30 for the function in Example 4.17. Expand the implicants given in the initial cover C^0.

***4.32** Consider the logic expressions

$$f = x_1\bar{x}_2\bar{x}_5 + \bar{x}_1\bar{x}_2\bar{x}_4\bar{x}_5 + x_1x_2x_4x_5 + \bar{x}_1\bar{x}_2x_3\bar{x}_4 + x_1\bar{x}_2x_3x_5 + \bar{x}_2\bar{x}_3x_4\bar{x}_5 + x_1x_2x_3x_4\bar{x}_5$$
$$g = \bar{x}_2x_3\bar{x}_4 + \bar{x}_2\bar{x}_3\bar{x}_4\bar{x}_5 + x_1x_3x_4\bar{x}_5 + x_1\bar{x}_2x_4\bar{x}_5 + x_1x_3x_4x_5 + \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_5 + x_1x_2\bar{x}_3x_4x_5$$

Prove or disprove that $f = g$.

4.33 Consider the circuit in Figure P4.2, which implements functions f and g. What is the cost of this circuit, assuming that the input variables are available in both true and complemented Redesign the circuit to implement the same functions, but at as low a cost as possible. What is the cost of your circuit?

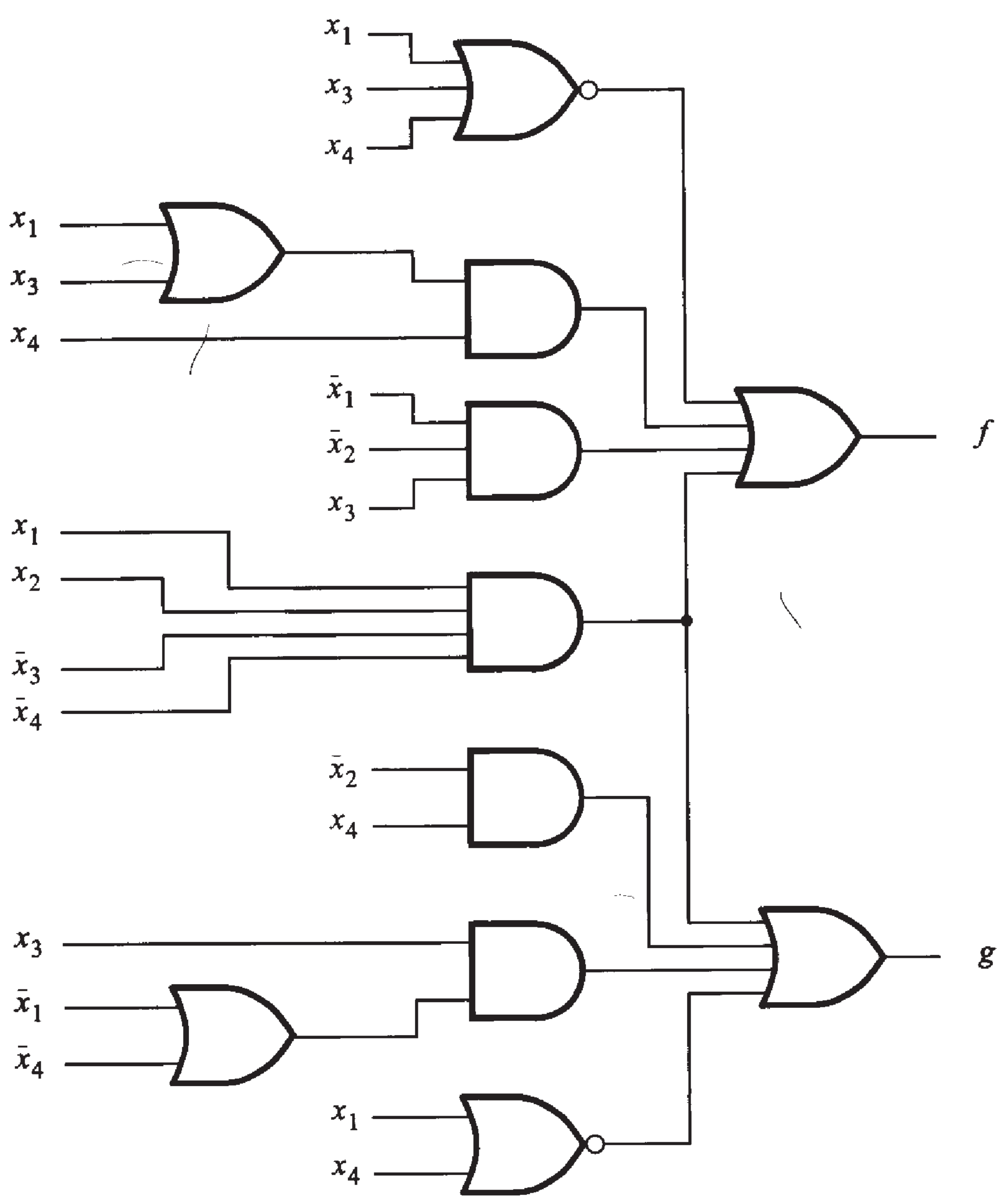

Figure P4.2 Circuit for problem 4.33.

4.34 Repeat problem 4.33 for the circuit in Figure P4.3. Use only NAND gates in your circuit.

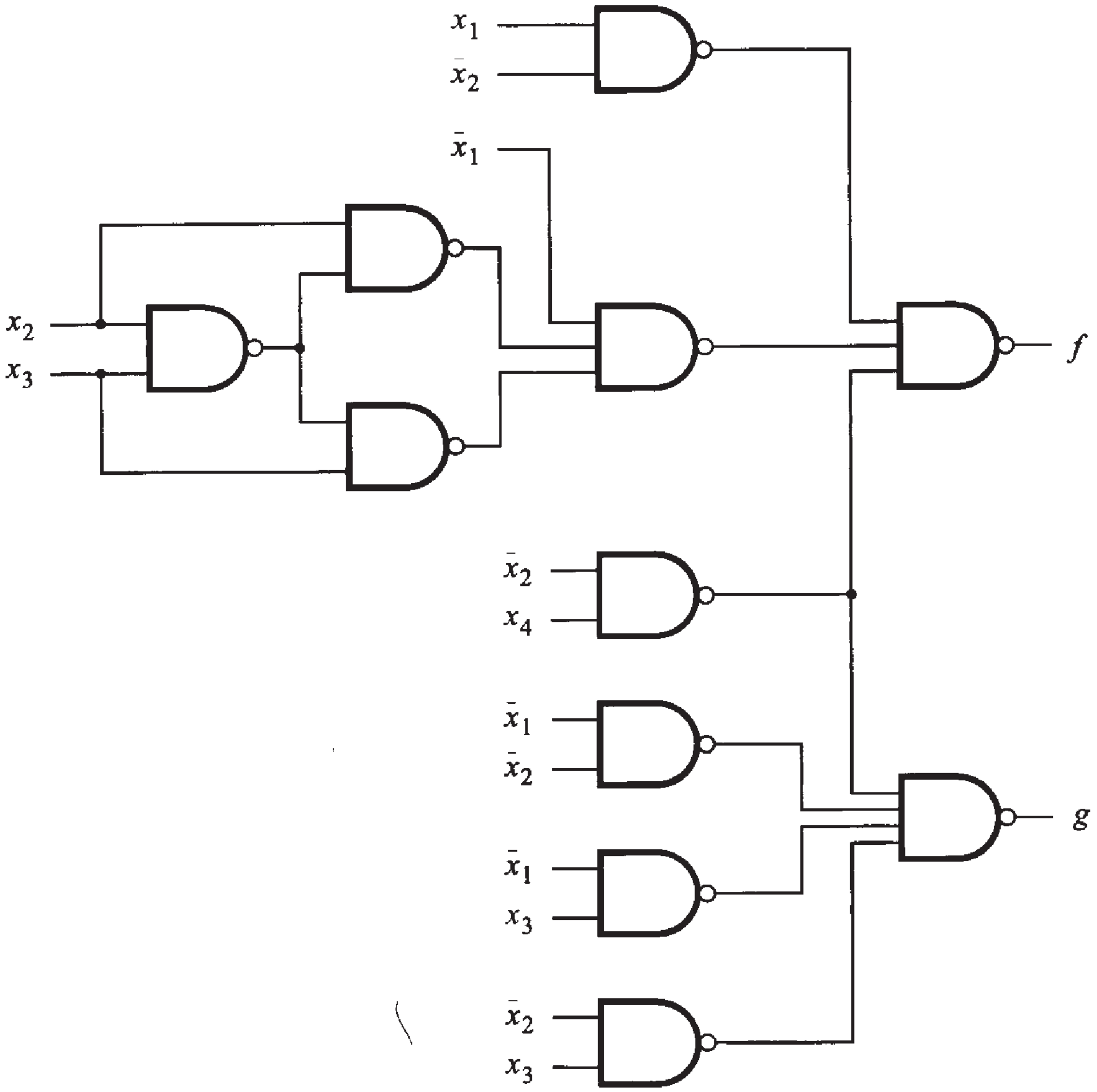

Figure P4.3 Circuit for problem 4.34.

4.35 Write VHDL code to implement the circuit in Figure 4.25*b*.

4.36 Write VHDL code to implement the circuit in Figure 4.27*c*.

4.37 Write VHDL code to implement the circuit in Figure 4.28*b*.

4.38 Write VHDL code to implement the function $f(x_1, \ldots, x_4) = \sum m(0, 1, 2, 4, 5, 7, 8, 9, 11, 12, 14, 15)$.

4.39 Write VHDL code to implement the function $f(x_1, \ldots, x_4) = \sum m(1, 4, 7, 14, 15) + D(0, 5, 9)$.