2.11 EXERCISES

1. Show a truth table for a 1-bit full subtractor that has a borrow input b_{in} and inputs x and y, and produces a difference, d, and a borrow output, b_{out}.

$$\begin{array}{r} b^{in} \\ x \\ \underline{-y} \\ b_{out}\ d \end{array}$$

2. Show truth tables for each of the following.

*a. There are four inputs and three outputs. The inputs, w, x, y, z, are codes for the grade that may be received:

0000 A	0100 B−	1000 D+	1100 Incomplete
0001 A−	0101 C+	1001 D	1101 Satisfactory
0010 B+	0110 C	1010 D−	1110 Unsatisfactory
0011 B	0111 C−	1011 F	1111 Pass

The outputs are

1: a 1 if and only if the grade is C or better (only letter grades; C− is not C or better)

2: a 1 if and only if the university will count it toward the 120 credits required for a degree (passing grade only)

3: a 1 if and only if it will be counted in computing a grade point average (letter grades only).

b. This system has four inputs and three outputs. The first two inputs, a and b, represent a 2-bit binary number (range of 0 to 3). A second binary number (same range) is represented by the other two inputs, c and d. The output f is to be 1 if and only if the two numbers differ by exactly 2. Output g is to be 1 if and only if the numbers are equal. Output h is to be 1 if and only if the second number is larger than the first.

c. The system has four inputs. The first two, a and b, represent a number in the range 1 to 3 (0 is not used). The other two, c and d, represent a second number in the same range. The output, y, is to be 1 if and only if the first number is greater than the second or the second is 2 greater than the first.

*d. A system has one output, F, and four inputs, where the first two inputs (A, B) represent one 2-bit binary number (in the range 0 to 3) and the second two inputs (C, D) represent another binary number (same range). F is to be 1 if and only if the two numbers are equal or if they differ by exactly 1.

e. A system has one output, F, and four inputs, where the first two inputs (A, B) represent one 2-bit binary number (in the range 0 to 3) and the second two inputs (C, D) represent another binary number (same range). F is to be 1 if and only if the sum of the two numbers is odd.

f. The system has four inputs. The first two, a and b, represent a number in the range 0 to 2 (3 is not used). The other two, c and d, represent a second number in the same range. The output, y, is to be 1 if and only if the two numbers do not differ by more than 1.

*g. The months of the year are coded in four variables, $abcd$, such that January is 0000, February is 0001, . . . , and December is 1011. The remaining 4 combinations are never used. (Remember: 30 days has September, April, June, and November. All the rest have 31, except February. . . .) Show a truth table for a function, g, that is 1 if the month has 31 days and 0 if it does not.

h. The months of the year are coded as in 2g, except that February of a leap year is coded as 1100. Show a truth table with five outputs, v, w, x, y, z that indicates the number of days in the selected month.

i. Repeat 2h, except that the outputs are to be in BCD (8421 code). There are now six outputs, u, v, w, x, y, z (where the first decimal digit is coded 0, 0, u, v and the second digit is coded w, x, y, z).

j. The system has four inputs, a, b, c, and d, and one output, f. The last three inputs (b, c, d) represent a binary number, n, in the range 0 to 7; however, the input 0 never occurs. The first input (a) specifies which of two computations is made.

$a = 0$: f is 1 iff n is a multiple of 2

$a = 1$: f is 1 iff n is a multiple of 3

k. The system has four inputs, a, b, c, and d, and one output, f. The first two inputs (a, b) represent one binary number (in the range 0 to 3) and the last two (c, d) represent another number in the range 1 to 3 (0 never occurs). The output, f, is to be 1 iff the second number is at least two larger than the first.

l. Show the truth table for a system with four inputs, a, b, c, and d, and two outputs, f and g. The inputs represent a BCD digit between 1 and 9 (8421 code). All other inputs never happen. The output f is 1 if and only if the input represents an odd number larger than 6 or an even number less than 7. The output g is 1 iff the input represents a perfect square. (A perfect square is a number whose square root is an integer.)

3. Show a block diagram of a circuit using AND and OR gates for each side of each of the following equalities:

*a. P2a: $a + (b + c) = (a + b) + c$

b. P8a: $a(b + c) = ab + ac$

4. Show a truth table for the following functions:

*a. $F = X'Y + Y'Z + XYZ$

b. $G = X'Y + (X' + Z')(Y + Z)$

c. $H = W'X + XY' + WX'Z + XYZ' + W'XY'$

5. Determine, using truth tables, which expressions in each of the groups are equal:

a. $f = ac' + a'c + bc$
$g = (a + c)(a' + b + c')$

*b. $f = a'c' + bc + ab'$
$g = b'c' + a'c' + ac$
$h = b'c' + ac + a'b$

c. $f = ab + ac + a'bd$
$g = bd + ab'c + abd'$

6. For each of the following expressions, indicate which (if any) of the following apply (more than one may apply):

i. Product term
ii. SOP expression
iii. Sum term
iv. POS expression

a. $abc'd + b'cd + ad'$
*b. $a' + b + cd$
c. $b'c'd'$
*d. $(a + b)c'$
e. $a' + b$
*f. a'
*g. $a(b + c) + a'(b' + d)$
h. $(a + b' + d)(a' + b + c)$

***7.** For the expressions of problem 4, how many literals are in each?

8. Using properties 1 to 10, reduce the following expressions to a minimum SOP form. Show each step (number of terms and number of literals in minimum shown in parentheses).

*a. $x'z + xy'z + xyz$ (1 term, 1 literal)
b. $x'y'z' + x'yz + xyz$ (2 terms, 5 literals)
c. $x'y'z' + x'y'z + xy'z + xyz'$ (3 terms, 7 literals)
*d. $a'b'c' + a'b'c + abc + ab'c$ (2 terms, 4 literals)
e. $x'y'z' + x'yz' + x'yz + xyz$ (2 terms, 4 literals)
*f. $x'y'z' + x'y'z + x'yz + xyz + xyz'$
(2 solutions, each with 3 terms, 6 literals)

g. $x'y'z' + x'y'z + x'yz + xy'z + xyz + xyz'$

(3 terms, 5 literals)

h. $a'b'c' + a'bc' + a'bc + ab'c + abc' + abc$

(3 terms, 5 literals)

9. Using Properties 1 to 10, reduce the following expressions to a minimum POS form. The number of terms and number of literals are shown in parentheses.

a. $(a + b + c)(a + b' + c)(a + b' + c')(a' + b' + c')$

(2 terms, 4 literals)

b. $(x + y + z)(x + y + z')(x + y' + z)(x + y' + z')$

(1 term, 1 literal)

*c. $(a + b + c')(a + b' + c')(a' + b' + c')(a' + b' + c)$
$(a' + b + c)$ (2 solutions, each with 3 terms, 6 literals)

10. Show a block diagram of a system using AND, OR, and NOT gates to implement the following functions. Assume that variables are available only uncomplemented. Do not manipulate the algebra.

a. $PQ + PR + QR$

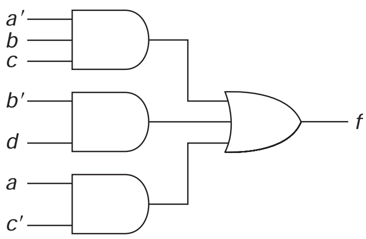
b. $ab + c(a + b)$

*c. $wx'(v + y'z) + (w'y + v')(x + yz)'$

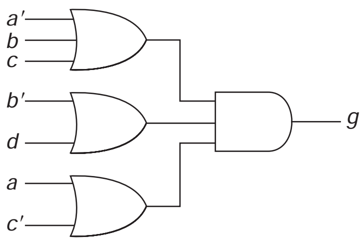
11. For each of the following circuits,

i. find an algebraic expression.

ii. put it in sum of product form.



a.



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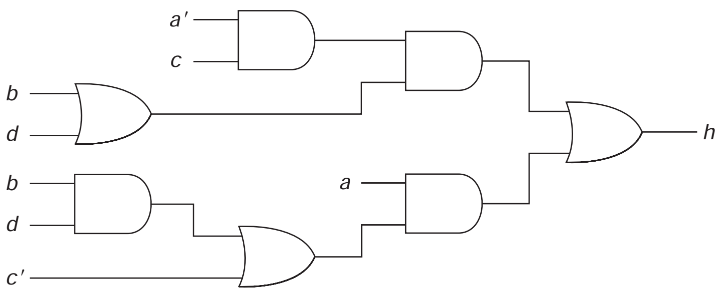


c.

12. Find the complement of the following expressions. Only single variables may be complemented in the answer.

*a. $f = abd' + b'c' + a'cd + a'bc'd$

b. $g = (a + b' + c)(a' + b + c)(a + b' + c')$

c. $h = (a + b)(b' + c) + d'(a'b + c)$

13. For each of the following functions:

$f(x, y, z) = \Sigma m(1, 3, 6)$
$g(x, y, z) = \Sigma m(0, 2, 4, 6)$

a. Show the truth table.

b. Show an algebraic expression in sum of minterms form.

c. Show a minimum SOP expression (a: 2 terms, 5 literals; b: 1 term, 1 literal).

d. Show the minterms of f' (complement of f) in numeric form.

e. Show an algebraic expression in product of maxterms form.

f. Show a minimum POS expression (f: 2 solutions, 3 terms, 6 literals; g: 1 term, 1 literal)

***14.** For each of the following functions,

a b c	*f*	*g*
0 0 0	0	1
0 0 1	1	1
0 1 0	0	0
0 1 1	0	0
1 0 0	0	1
1 0 1	1	1
1 1 0	1	1
1 1 1	1	0

a. Show the minterms in numerical form.

b. Show an algebraic expression in sum of minterms form.

c. Show a minimum SOP expression (f: 2 terms, 4 literals; g: 2 terms, 3 literals).

d. Show the minterms of f' (complement of f) in numeric form.

e. Show an algebraic expression in product of maxterms form.

f. Show a minimum POS expression (f: 2 terms, 4 literals; g: 2 terms, 4 literals)

15. For each of the following functions:

$F = AB' + BC + AC$

$G = (A + B)(A + C') + AB'$

a. Show the truth table.

b. Show an algebraic expression in sum of minterms form.

c. Show a minimum sum of products expression (F: 2 terms, 4 literals; G: 2 terms, 3 literals).

d. Show the minterms of the complement of each function in numeric form.

e. Show an algebraic expression in product of maxterms form.

f. Show a minimum POS expression (F: 2 terms, 4 literals; G: 2 terms, 4 literals)

***16.** Consider the following function with don't cares:

$$G(X, Y, Z) = \Sigma m(5, 6) + \Sigma d(1, 2, 4)$$

For each of the following expressions, indicate whether it could be used as a solution for G. (Note: It may not be a minimum solution.)

a. $XYZ + XY'Z$ d. $Y'Z + XZ + X'Z$

b. $Z + XY'Z$ e. $XZ + X'Z$

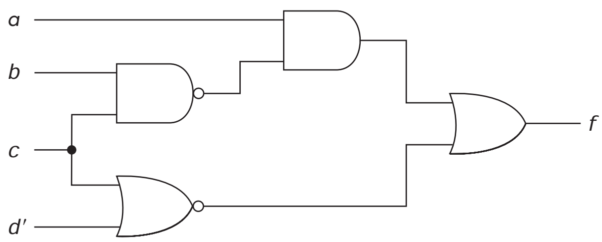
c. $X(Y' + Z)$ f. $YZ + Y'Z$

17. Show that the NOR is functionally complete by implementing a NOT, a two-input AND, and a two-input OR using only two-input NORs.

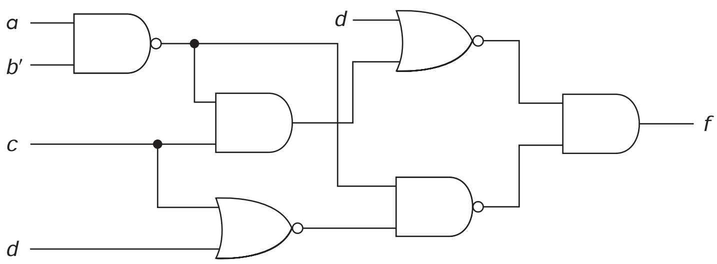
18. For each of the following circuits,

i. find an algebraic expression.

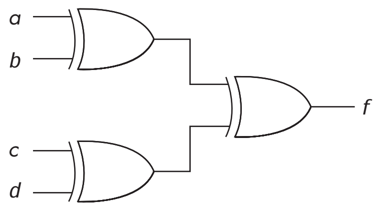
ii. put it in SOP form.



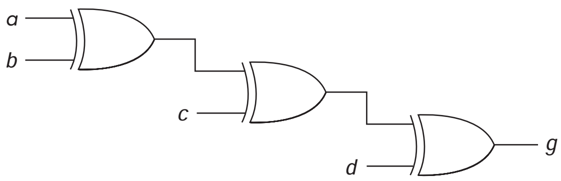
***a.**



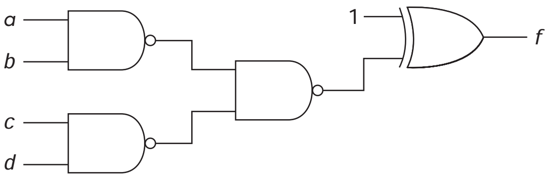
b.



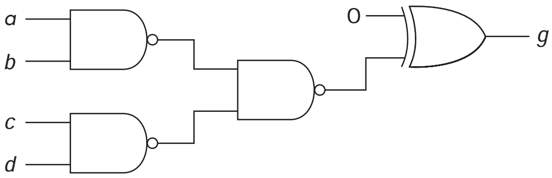
c.



d.



★**e.**



f.

19. Show a block diagram corresponding to each of the expressions below using only NAND gates. Assume all inputs are available both uncomplemented and complemented. Do not manipulate the functions to simplify the algebra.

a. $f = wy' + wxz' + xy'z + w'x'z$

b. $g = wx + (w' + y)(x + y')$

c. $h = z(x'y + w'x') + w(y' + xz')$

★d. $F = D[B'(A' + E') + AE(B + C)] + BD'(A'C' + AE')$

20. Reduce the following expressions to a minimum sum of products form, using P1 through P12. Show each step (number of terms and number of literals in minimum shown in parentheses).

a. $h = ab'c + bd + bcd' + ab'c' + abc'd$ (3 terms, 6 literals)

b. $h = ab' + bc'd' + abc'd + bc$ (3 terms, 5 literals)

★c. $f = ab + a'bd + bcd + abc' + a'bd' + a'c$
(2 terms, 3 literals)

d. $g = abc + abd + bc'd'$ (2 terms, 5 literals)

e. $f = xy + w'y'z + w'xy' + wxyz' + w'yz + wz$
(3 terms, 5 literals)

21. Reduce the following expressions to a minimum sum of products form. Show each step and the property used (number of terms and number of literals in minimum shown in parentheses).

a. $f = x'yz + w'x'z + x'y + wxy + w'y'z$
(3 terms, 7 literals)

b. $G = A'B'C' + AB'D + BCD' + A'BD + CD + A'D$
(4 terms, 9 literals)

★c. $F = W'YZ' + Y'Z + WXZ + WXYZ' + XY'Z + W'Y'Z$
(3 terms, 7 literals)

d. $g = wxz + xy'z + wz' + xyz + wxy'z + w'y'z'$ (3 terms, 6 literals)

e. $F = ABD' + B'CE + AB'D' + B'D'E + ABCD'E + B'C'D'$ (3 terms, 8 literals)

f. $f = b'c + abc + b'cd + a'b'd + a'c'd$ (3 terms, 7 literals)

*g. $G = B'C'D + BC + A'BD + ACD + A'D$ (3 terms, 6 literals)

h. $f = ab + bcd + ab'c' + abd + bc + abc'$ (2 terms, 4 literals)

i. $h = abc' + ab'd + bcd + a'bc$ (3 terms, 8 literals)

*j. $g = a'bc' + bc'd + abd + abc + bcd' + a'bd'$ (2 solutions, 3 terms, 9 literals)

22. i. For the following functions, use consensus to add as many new terms to the sum of product expression given.

ii. Then reduce each to a minimum sum of products, showing each step and the property used.

*a. $f = a'b'c' + a'bd + a'cd' + abc$ (3 terms, 8 literals)

b. $g = wxy + w'y'z + xyz + w'yz'$ (3 terms, 8 literals)

23. Expand the following functions to sum of minterms form:

a. $f(a, b, c) = ab' + b'c'$

*b. $g(x, y, z) = x' + yz + y'z'$

c. $h(a, b, c, d) = ab'c + bd + a'd'$

24. Convert each of the following expressions to sum of products form:

a. $(a + b + c + d')(b + c' + d)(a + c)$

b. $(a' + b + c')(b + c' + d)(b' + d')$

*c. $(w' + x)(y + z)(w' + y)(x + y' + z)$

d. $(A + B + C)(B' + C + D)(A + B' + D)(B + C' + D')$

25. Convert each of the following expressions to product of sums form:

a. $AC + A'D'$

b. $w'xy' + wxy + xz$

*c. $bc'd + a'b'd + b'cd'$

26. Implement each of the following expressions (which are already in minimum sum of products form) using only two-input NAND gates. No gate may be used as a NOT. All inputs are available both uncomplemented and complemented. (The number of gates required is shown in parentheses.)

*a. $f = wy' + wxz' + y'z + w'x'z$ (7 gates)

b. $ab'd' + bde' + bc'd + a'ce$ (10 gates)

c. $H = A'B'E' + A'B'CD' + B'D'E' + BDE' + BC'E + ACE'$ (14 gates)

*d. $F = A'B'D' + ABC' + B'CD'E + A'B'C + BC'D$ (11 gates)

e. $G = B'D'E' + A'BC'D + ACE + AC'E' + B'CE$ (12 gates, one of which is shared)

f. $h = b'd'e' + ace + c'e' + bcde$ (9 gates)

27. Each of the following is already in minimum sum of products form. All variables are available both uncomplemented and complemented. Find two solutions each of which uses no more than the number of integrated circuit packages of NAND gates (4 two-input or 3 three-input or 2 four-input gates per package) listed. One solution must use only two and three input gates; the other must use at least 1 four-input gate package.

*a. $F = ABCDE + B'E' + CD'E' + BC'D'E + A'B'C + A'BC'E$ (3 packages)

b. $G = ABCDEF + A'B'D' + C'D'E + AB'CE' + A'BC'DF + ABE'F'$ (4 packages)

2.12 CHAPTER 2 TEST (100 MINUTES, OR TWO 50-MINUTE TESTS)

1. The inputs of this system A and B represent one binary number in the range 0:3. The inputs C and D represent a second binary number (also in the range 0:3). There are three outputs, X, Y, and Z.

Show a truth table such that Y and Z represent a number equal to the magnitude of the difference of the two inputs and X is 1 if and only if the first is larger. Two lines of the table are filled in.

A	*B*	*C*	*D*	*X*	*Y*	*Z*
0	0	0	0			
0	0	0	1			
0	0	1	0			
0	0	1	1			
0	1	0	0			
0	1	0	1			
0	1	1	0	0	0	1
0	1	1	1			
1	0	0	0			
1	0	0	1	1	0	1
1	0	1	0			
1	0	1	1			
1	1	0	0			
1	1	0	1			
1	1	1	0			
1	1	1	1			