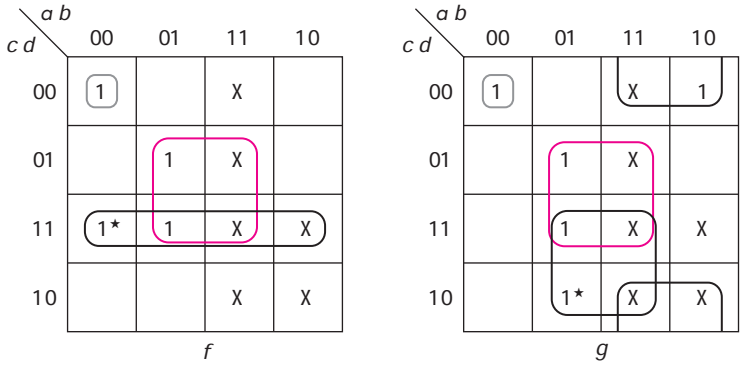

3.8 EXERCISES

1. Plot the following functions on the Karnaugh map:

a. $f(a, b, c) = \Sigma m(1, 2, 3, 4, 6)$

*b. $g(w, x, y, z) = \Sigma m(1, 3, 5, 6, 7, 13, 14) + \Sigma d(8, 10, 12)$

c. $F = WXYZ + WXYZ + WXYZ + WXYZ + WXYZ$

*d. $g = a'c + a'bd' + bc'd + ab'd + ab'cd'$

e. $h = x + yz' + x'z$

2. For each of the following, find all minimum sum of products expressions. (If there is more than one solution, the number of solutions is given in parentheses.)

a. $f(a, b, c) = \Sigma m(1, 2, 3, 6, 7)$

*b. $g(w, x, y) = \Sigma m(0, 1, 5, 6, 7)$ (2 solutions)

c. $h(a, b, c) = \Sigma m(0, 1, 2, 5, 6, 7)$ (2 solutions)

d. $f(a, b, c, d) = \Sigma m(1, 2, 3, 5, 6, 7, 8, 11, 13, 15)$

*e. $G(W, X, Y, Z) = \Sigma m(0, 2, 5, 7, 8, 10, 12, 13)$

f. $h(a, b, c, d) = \Sigma m(2, 4, 5, 6, 7, 8, 10, 12, 13, 15)$ (2 solutions)

g. $f(a, b, c, d) = \Sigma m(1, 3, 4, 5, 6, 11, 12, 13, 14, 15)$ (2 solutions)

h. $g(w, x, y, z) = \Sigma m(2, 3, 6, 7, 8, 10, 11, 12, 13, 15)$ (2 solutions)

*i. $h(p, q, r, s) = \Sigma m(0, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15)$ (3 solutions)

j. $F(W, X, Y, Z) = \Sigma m(0, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 15)$ (4 solutions)

k. $f(w, x, y, z) = \Sigma m(0, 1, 2, 4, 5, 6, 9, 10, 11, 13, 14, 15)$ (2 solutions)

l. $g(a, b, c, d) = \Sigma m(0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15)$

*m. $H(W, X, Y, Z) = \Sigma m(0, 2, 3, 5, 7, 8, 10, 12, 13)$ (4 solutions)

*n. $f(a, b, c, d) = \Sigma m(0, 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15)$ (6 solutions)

o. $g(w, x, y, z) = \Sigma m(0, 1, 2, 3, 5, 6, 7, 8, 9, 10, 13, 14, 15)$ (6 solutions)

*p. $f(a, b, c, d) = \Sigma m(0, 3, 5, 6, 7, 9, 10, 11, 12, 13, 14)$ (32 solutions)

3. For the following functions,

i. List all prime implicants, indicating which are essential.

ii. Show the minimum sum of products expression(s).

a. $f(a, b, c, d) = \Sigma m(0, 3, 4, 5, 8, 11, 12, 13, 14, 15)$

*b. $g(w, x, y, z) = \Sigma m(0, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15)$

4. Map each of the following functions and find the minimum sum of products expression:

a. $F = AD + AB + A'CD' + B'CD + A'BC'D'$

*b. $g = w'yz + xy'z + wy + wxy'z' + wz + xyz'$

5. For each of the following, find all minimum sum of products expressions. (If there is more than one solution, the number of solutions is given in parentheses.) Label the solutions $f_1, f_2, \ldots$.

a. $f(w, x, y, z) = \Sigma m(1, 3, 6, 8, 11, 14) + \Sigma d(2, 4, 5, 13, 15)$ (3 solutions)

b. $f(a, b, c, d) = \Sigma m(0, 3, 6, 9, 11, 13, 14) + \Sigma d(5, 7, 10, 12)$

*c. $f(a, b, c, d) = \Sigma m(0, 2, 3, 5, 7, 8, 9, 10, 11) + \Sigma d(4, 15)$ (3 solutions)

d. $f(w, x, y, z) = \Sigma m(0, 2, 4, 5, 10, 12, 15) + \Sigma d(8, 14)$ (2 solutions)

e. $f(a, b, c, d) = \Sigma m(5, 7, 9, 11, 13, 14) + \Sigma d(2, 6, 10, 12, 15)$ (4 solutions)

*f. $f(a, b, c, d) = \Sigma m(0, 2, 4, 5, 6, 7, 8, 9, 10, 14) + \Sigma d(3, 13)$ (3 solutions)

g. $f(w, x, y, z) = \Sigma m(1, 2, 5, 10, 12) + \Sigma d(0, 3, 4, 8, 13, 14, 15)$ (7 solutions)

6. For each of the functions of problem 5, indicate which solutions are equal.

7. For each of the following functions, find all of the minimum sum of products expressions and all of the minimum product of sums expressions:

*a. $f(A, B, C, D) = \Sigma m(1, 4, 5, 6, 7, 9, 11, 13, 15)$

b. $f(W, X, Y, Z) = \Sigma m(2, 4, 5, 6, 7, 10, 11, 15)$

c. $f(A, B, C, D) = \Sigma m(1, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15)$
(1 SOP and 2 POS solutions)

*d. $f(a, b, c, d) = \Sigma m(0, 2, 4, 6, 7, 9, 11, 12, 13, 14, 15)$
(2 SOP and 1 POS solutions)

e. $f(w, x, y, z) = \Sigma m(0, 4, 6, 9, 10, 11, 14) + \Sigma d(1, 3, 5, 7)$

f. $f(a, b, c, d) = \Sigma m(0, 1, 2, 5, 7, 9) + \Sigma d(6, 8, 11, 13, 14, 15)$
(4 SOP and 2 POS solutions)

g. $f(w, x, y, z) = \Sigma m(4, 6, 9, 10, 11, 13) + \Sigma d(2, 12, 15)$
(2 SOP and 2 POS solutions)

h. $f(a, b, c, d) = \Sigma m(0, 1, 4, 6, 10, 14) + \Sigma d(5, 7, 8, 9, 11, 12, 15)$
(13 SOP and 3 POS solutions)

*i. $f(w, x, y, z) = \Sigma m(1, 3, 7, 11, 13, 14) + \Sigma d(0, 2, 5, 8, 10, 12, 15)$
(6 SOP and 1 POS solutions)

j. $f(a, b, c, d) = \Sigma m(0, 1, 6, 15) + \Sigma d(3, 5, 7, 11, 14)$
(1 SOP and 2 POS solutions)

8. Label the solutions of each part of problem 7 as f_1, f_2, . . . and indicate which solutions are equal.

9. For each of the following five-variable functions, find all minimum sum of products expressions. (If there is more than one solution, the number of solutions is given in parentheses.)

a. $F(A, B, C, D, E) = \Sigma m(0, 1, 5, 7, 8, 9, 10, 11, 13, 15, 18, 20, 21, 23, 26, 28, 29, 31)$

b. $G(A, B, C, D, E) = \Sigma m(0, 1, 2, 4, 5, 6, 10, 13, 14, 18, 21, 22, 24, 26, 29, 30)$

*c. $H(A, B, C, D, E) = \Sigma m(5, 8, 12, 13, 15, 17, 19, 21, 23, 24, 28, 31)$

d. $F(V, W, X, Y, Z) = \Sigma m(2, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 24, 25, 29, 30, 31)$

e. $G(V, W, X, Y, Z) = \Sigma m(0, 1, 4, 5, 8, 9, 10, 15, 16, 18, 19, 20, 24, 26, 28, 31)$

*f. $H(V, W, X, Y, Z) = \Sigma m(0, 1, 2, 3, 5, 7, 10, 11, 14, 15, 16, 18, 24, 25, 28, 29, 31)$ (2 solutions)

g. $F(A, B, C, D, E) = \Sigma m(0, 4, 6, 8, 12, 13, 14, 15, 16, 17, 18, 21, 24, 25, 26, 28, 29, 31)$ (6 solutions)

h. $G(A, B, C, D, E) = \Sigma m(0, 3, 5, 7 12, 13, 14, 15, 19, 20, 21, 22, 23, 25, 26, 29, 30)$ (3 solutions)

*i. $H(A, B, C, D, E) = \Sigma m(0, 1, 5, 6, 7, 8, 9, 14, 17, 20, 21, 22, 23, 25, 28, 29, 30)$ (3 solutions)

j. $F(V, W, X, Y, Z) = \Sigma m(0, 4, 5, 7, 10, 11, 14, 15, 16, 18, 20, 21, 23, 24, 25, 26, 29, 31)$ (4 solutions)

k. $G(V, W, X, Y, Z) = \Sigma m(0, 2, 5, 6, 8, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 26, 29, 31)$
(3 solutions)

l. $H(V, W, X, Y, Z) = \Sigma m(0, 1, 2, 3, 5, 8, 9, 10, 13, 17, 18, 19, 20, 21, 26, 28, 29)$
(3 solutions)

m. $F(A, B, C, D, E) = \Sigma m(1, 2, 5, 8, 9, 10, 12, 13, 14, 15, 16, 18, 21, 22, 23, 24, 26, 29, 30, 31)$
(18 solutions)

*n. $G(V, W, X, Y, Z) = \Sigma m(0, 1, 5, 7, 8, 13, 24, 25, 29, 31) + \Sigma d(9, 15, 16, 17, 23, 26, 27, 30)$
(2 solutions)

o. $H(A, B, C, D, E) = \Sigma m(0, 4, 12, 15, 27, 29, 30) + \Sigma d(1, 5, 9, 10, 14, 16, 20, 28, 31)$
(4 solutions)

p. $F(A, B, C, D, E) = \Sigma m(8, 9, 11, 14, 28, 30) + d(0, 3, 4, 6, 7, 12, 13, 15, 20, 22, 27, 29, 31)$
(8 solutions)

10. For each of the following six-variable functions, find all minimum sum of products expressions. (The number of terms and literals, and, if there is more than one solution, the number of solutions is given in parentheses.)

a. $G(A, B, C, D, E, F) = \Sigma m(4, 5, 6, 7, 8, 10, 13, 15, 18, 20, 21, 22, 23, 26, 29, 30, 31, 33, 36, 37, 38, 39, 40, 42, 49, 52, 53, 54, 55, 60, 61)$
(6 terms, 21 literals)

*b. $G(A, B, C, D, E, F) = \Sigma m(2, 3, 6, 7, 8, 12, 14, 17, 19, 21, 23, 25, 27, 28, 29, 30, 32, 33, 34, 35, 40, 44, 46, 49, 51, 53, 55, 57, 59, 61, 62, 63)$
(8 terms, 30 literals)

c. $G(A, B, C, D, E, F) = \Sigma m(0, 1, 2, 4, 5, 6, 7, 9, 13, 15, 17, 19, 21, 23, 26, 27, 29, 30, 31, 33, 37, 39, 40, 42, 44, 45, 46, 47, 49, 53, 55, 57, 59, 60, 61, 62, 63)$
(8 terms, 28 literals, 2 solutions)

11. Find a minimum two-level circuit (corresponding to sum of products expressions) using AND and one OR gate per function for each of the following sets of functions.

*a. $f(a, b, c, d) = \Sigma m(1, 3, 5, 8, 9, 10, 13, 14)$
$g(a, b, c, d) = \Sigma m(4, 5, 6, 7, 10, 13, 14)$ (7 gates, 21 inputs)

b. $f(a, b, c, d) = \Sigma m(0, 1, 2, 3, 4, 5, 8, 10, 13)$
$g(a, b, c, d) = \Sigma m(0, 1, 2, 3, 8, 9, 10, 11, 13)$
(6 gates, 16 inputs)

c. $f(a, b, c, d) = \Sigma m(5, 8, 9, 12, 13, 14)$
$g(a, b, c, d) = \Sigma m(1, 3, 5, 8, 9, 10)$
(3 solutions, 8 gates, 25 inputs)

d. $f(a, b, c, d) = \Sigma m(1, 3, 4, 5, 10, 11, 12, 14, 15)$
$g(a, b, c, d) = \Sigma m(0, 1, 2, 8, 10, 11, 12, 15)$
(9 gates, 28 inputs)

*e. $F(W, X, Y, Z) = \Sigma m(1, 5, 7, 8, 10, 11, 12, 14, 15)$
$G(W, X, Y, Z) = \Sigma m(0, 1, 4, 6, 7, 8, 12)$ (8 gates, 23 inputs)

f. $F(W, X, Y, Z) = \Sigma m(0, 2, 3, 7, 8, 9, 13, 15)$
$G(W, X, Y, Z) = \Sigma m(0, 2, 8, 9, 10, 12, 13, 14)$
(2 solutions, 8 gates, 23 inputs)

g. $f(a, b, c, d) = \Sigma m(1, 3, 5, 7, 8, 9, 10)$
$g(a, b, c, d) = \Sigma m(0, 2, 4, 5, 6, 8, 10, 11, 12)$
$h(a, b, c, d) = \Sigma m(1, 2, 3, 5, 7, 10, 12, 13, 14, 15)$
(2 solutions, 12 gates, 33 inputs)

*h. $f(a, b, c, d) = \Sigma m(0, 3, 4, 5, 7, 8, 12, 13, 15)$
$g(a, b, c, d) = \Sigma m(1, 5, 7, 8, 9, 10, 11, 13, 14, 15)$
$h(a, b, c, d) = \Sigma m(1, 2, 4, 5, 7, 10, 13, 14, 15)$
(2 solutions, 11 gates, 33 inputs)

i. $f(a, b, c, d) = \Sigma m(0, 2, 3, 4, 6, 7, 9, 11, 13)$
$g(a, b, c, d) = \Sigma m(2, 3, 5, 6, 7, 8, 9, 10, 13)$
$h(a, b, c, d) = \Sigma m(0, 4, 8, 9, 10, 13, 15)$
(2 solutions for f and g, 10 gates, 32 inputs)

*j. $f(a, c, b, d) = \Sigma m(0, 1, 2, 3, 4, 9) + \Sigma d(10, 11, 12, 13, 14, 15)$
$g(a, c, b, d) = \Sigma m(1, 2, 6, 9) + \Sigma d(10, 11, 12, 13, 14, 15)$
(3 solutions for f, 6 gates, 15 inputs)

k. $f(a, c, b, d) = \Sigma m(5, 6, 11) + \Sigma d(0, 1, 2, 4, 8)$
$g(a, c, b, d) = \Sigma m(6, 9, 11, 12, 14) + \Sigma d(0, 1, 2, 4, 8)$
(2 solutions for g, 7 gates, 18 inputs)

12. In each of the following sets, the functions have been minimized individually. Find a minimum two-level circuit (corresponding to sum of products expressions) using AND and one OR gate per function for each.

a. $F = B'D' + CD' + AB'C$
$G = BC + ACD$ (6 gates, 15 inputs)

*b. $F = A'B'C'D + BC + ACD + AC'D'$
$G = A'B'C'D' + A'BC + BCD'$
$H = B'C'D' + BCD + AC' + AD$
(2 solutions for H, 10 gates, 35 inputs)

c. $f = a'b' + a'd + b'c'd'$
$g = b'c'd' + bd + acd + abc$
$h = a'd' + a'b + bc'd + b'c'd'$ (10 gates, 31 inputs)