Spectrum Analyzer and Sampling Theorem

1 Objectives

1. Learn how to use a spectrum analyzer.
2. Study and understand the sampling theorem.

2 Theory

2.1 Spectrum Analyzer

Like an oscilloscope, a spectrum analyzer produces a visible display on a screen. Unlike an oscilloscope, however, the spectrum analyzer has only one function-to produce a display of the frequency content of an input signal. (But it is possible to display the time waveform on the spectrum analyzer screen with the proper settings.) And also like an oscilloscope, the spectrum analyzer will always produce a picture on the screen; but if you do not know how to properly use the spectrum analyzer, that picture may be completely meaningless.

CAUTION: The input of the spectrum analyzer **cannot** tolerate large signals; before you connect a signal to the input, be sure you know that the signal will not exceed the maximum allowable input rating of the spectrum analyzer.

2.1.1 Signal Acquisition in a Spectrum Analyzer

Most spectrum analyzers are heterodyne[1] spectrum analyzers (also called scanning spectrum analyzers). A heterodyne analyzer is essentially a radio receiver (a very sensitive and selective receiver). Given a voltage signal x(t), we need to somehow extract the frequency content out of it. As we know, the digital storage oscilloscope provides one solution as it can calculate the FFT of the signal from stored samples. Another solution would be to pass x(t) through a long series of very narrow bandpass filters, having adjacent passbands, and then plot the amplitudes of the filter outputs. That is, if filter 1 has passband f1 - BW/2 < f < f1 + BW/2, and filter 2 has passband f2 - BW/2 < f < f2 + BW/2, where f1 + BW/2 = f2 - BW/2, and so on, and if BW (the bandwidth) is small enough, then the filter outputs give us the frequency components X(f1), X(f 2), . . . and so on . This is, of course, not a practical solution. A better solution is suggested by a simple property of Fourier transforms: recall that if we multiply (in the time domain) a signal by a sinusoid, the spectrum of the signal is shifted in frequency by an amount equal to the frequency of the sinusoid. That is,

$$x(t)\cos(2\pi f_0 t) \xrightarrow{\text{Fourier Transform}} \frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$$

Now instead of a bank of narrow filters, we shall have one narrow filter centered at a fixed frequency, say fI, and we shall scan the signal spectrum across this filter by multiplying x(t) by a sinusoid of varying frequency f_0 (See Figure 1). The filter is a narrow bandpass filter at a fixed center frequency, fI, (called the intermediate frequency); in a spectrum analyzer, its bandwidth is selected by the user. The oscillator frequency, f_0, is adjustable, as indicated in Figure 1. In an ordinary AM or FM radio, when you tune the receiver you are selecting this frequency so that the desired signal will pass through the filter; in a spectrum analyzer, this frequency is automatically scanned (repeatedly) over a range, which must be selected so that the frequency component X(f) is shifted to fI and passed by the filter. For example, if we want to view the frequency content of x(t) from f1 to f2 , then we must select f_0 to scan from f1 + fI to f2 + fI.

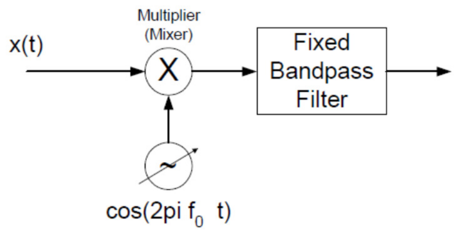

Figure 1: Frequency Mixing, or Heterodyning

Of course, much more signal conditioning is going on inside the spectrum analyzer than is indicated in Figure 1; but the frequency mixing is the fundamental step. In particular, the signal

[1] Heterodyne is derived from a Greek word meaning mixing different frequencies

first is passed through a lowpass filter whose bandwidth is chosen to eliminate image frequencies. Also, most scanning spectrum analyzers are multiple conversion analyzers - they have multiple intermediate frequency stages, at successively lower frequencies. The reason is that we have two conflicting goals to achieve; we would like to have the filter bandwidth as small as feasible, and we would like to be able to scan over large frequency ranges. It is hard to build sharp narrow filters at high frequencies, but it is also hard to build multipliers that will work over large frequency ranges. Therefore, we achieve narrow filters at low intermediate frequencies by shifting the frequency down in several steps.

2.1.2 Spectrum Analyzer Controls

In this section we shall describe some of the basic controls on the spectrum analyzer that you will frequently use. More details on these, and descriptions of the more obscure controls, can be found in the user manual. Mainly, you will use the three large buttons labeled FREQUENCY, SPAN, and AMPLITUDE, the various MARKER buttons for making measurements, and the BW/Avg button for selecting the resolution bandwidth. In addition, you will use the control knob, the up and down buttons labeled with large arrows (above the control knob), and the numerical keypad for entering values that will control the display. When you use the spectrum analyzer, always pay attention to the in- formation about the instrument state given in the top, left, and bottom margins of the screen.

FREQUENCY Control

In normal operation the frequency control selects the range that the variable oscillator in Figure 1 sweeps through. Pressing the FREQUENCY button causes the frequency menu to appear on the screen. You can select the CENTER frequency (CF) and the START and STOP frequencies. You select the numerical values by turning the control knob, pressing the up/down arrows or by entering the value with the numerical keypad.

SPAN Control

Pressing the SPAN button brings up the frequency span menu. Here you select the frequency span displayed on the screen (as opposed to selecting start and stop frequencies), and you can select span zoom, zero span, and full span.

AMPLITUDE / REF LEVEL Control

Pressing this button displays the amplitude menu. Here you select the reference level, whether the amplitude units are power (dBm) or linear (mV), and the scale in dB/division (when using the logarithmic scale).

Here is where the spectrum analyzer seems strange compared to an oscilloscope: you measure signal levels from the top of the screen, or down from the reference level. For example, on power-up, the reference level is 0 dBm, meaning that the top line on the screen is at 0 dBm and you measure the amplitudes of lines in the spectrum down from that level. For example, if the REF LEVEL is set to +10dBm, a signal peak that reaches the top of the display is +10dBm.

The spectrum analyzer tends to provide more accurate readings when the input signal is placed in the upper two or so divisions of the display. Also, smaller REF LEVEL step sizes will provide more accurate measurements.

Once again, you are cautioned to be careful about applying signals to the spectrum analyzer; it is easy to cause extensive and expensive damage.

Resolution Bandwidth Control

The resolution bandwidth is essentially the bandwidth of the fixed narrowband filter in Figure 1. (In reality, there are several stages of filtering.) Pressing the BW/Avg button displays the menu from which you can select the resolution bandwidth, the video bandwidth, and associated controls. Note that you cannot select a continuous range of RBW-there is only a finite selection available.

The resolution bandwidth determines how close frequency components in the signal spectrum can be and still be displayed as distinct components on the screen. A large RBW may reveal only one signal, say at 900MHz. However, if the RBW is decreased, another signal at 899.5MHz may also be present (and thus will show up on the display). The AUTO button allows the spectrum analyzer to automatically select the RBW – manual selection is done with the UP/DOWN arrows – the AUTO button allows you to toggle between auto and manual RBW selection. Video Bandwidth (VBW) is basically a smoothing filter with a bandwidth equal to the RBW. VBW essentially reduces the noise displayed, making the power levels easier to see. VID FLTR button allows you to change the VBW. The sweep control is usually controlled by the spectrum analyzer (the rate at which different frequencies, f_0 in Figure 1, are changed) but you can change this by using the arrows under the SWEEP heading.

Markers

Just as the oscilloscope has markers, the spectrum analyzer also has markers to help you make measurements. You select markers, difference markers, or no markers with the MARKER control buttons and their menus.

2.2 Sampling Theorem

Sampling is the process of converting a signal (for example, a function of continuous time or space) into a numeric sequence (a function of discrete time or space). Shannon's version of the theorem states:

If a function $x(t)$ contains no frequencies higher than B cps, it is completely determined by giving its ordinates at a series of points spaced $1/(2B)$ seconds apart.

A sufficient sample-rate is therefore $2B$ samples/second, or anything larger. Conversely, for a given sample rate f_s the bandlimit for perfect reconstruction is $B \leq f_s/2$. When the bandlimit is too high (or there is no bandlimit), the reconstruction exhibits imperfections known as aliasing. Modern statements of the theorem are sometimes careful to explicitly state that $x(t)$ must contain no sinusoidal component at exactly frequency B, or that B must be strictly less than ½ the sample rate. The two thresholds, $2B$ and $f_s/2$ are respectively called the **Nyquist rate** and **Nyquist frequency**. And respectively, they are attributes of $x(t)$ and of the sampling equipment. The condition described by these inequalities is called the **Nyquist criterion**, or sometimes the *Raabe condition*. The theorem is also applicable to functions of other domains, such as *space,* in the case of a digitized image. The only change, in the case of other domains, is the units of measure applied to t, f_s, and B.

The symbol $T = 1/f_s$ is customarily used to represent the interval between samples and is called the **sample period** or **sampling interval**. And the samples of function $x(t)$ are commonly denoted by $x[n] = x(nT)$ (alternatively "x_n" in older signal processing literature), for all integer values of n. The mathematically ideal way to interpolate the sequence involves the use of sinc functions. Each sample in the sequence is replaced by a sinc function, centered on the time axis at the original location of the sample, nT, with the amplitude of the sinc function scaled to the sample value, $x[n]$. Subsequently, the sinc functions are summed into a continuous function. A mathematically equivalent method is to convolve one sinc function with a series of Dirac delta pulses, weighted by the sample values. Neither method is numerically practical. Instead, some type of approximation of the sinc functions, finite in length, is used. The imperfections attributable to the approximation are known as *interpolation error*.

Practical digital-to-analog converters produce neither scaled and delayed sinc functions, nor ideal Dirac pulses. Instead they produce a piecewise-constant sequence of scaled and delayed rectangular pulses (the zero-order hold), usually followed by an "anti-imaging filter" to

3 Equipment

1. Spectrum Analyzer
2. Function Generator
3. Oscilloscope
4. DC power supply
5. Modicom 1 board

4 Experiment

Experiment 1 Spectrum Analyzer

Experiment Procedure

1. Connect the function generator to the oscilloscope and apply the sine wave with frequency 200 kHz and magnitude 1 V_{pp}.

2. Connect the function generator to the spectrum analyzer and apply the sine wave with frequency 200 kHz and magnitude 0.5 V_{pp} and 2 V_{pp}, respectively. Record the results in both V and dBm units.
3. Repeat step 2 by changing the frequency to 100 kHz and 300 kHz, respectively.
4. Apply the square wave with magnitude 1 V_{pp}, duty cycle of 50% and frequency 100 kHz. Record the results in both V and dBm units.
5. Repeat step 4 by changing the frequency to 500 kHz.
6. Repeat step 4 by changing the duty cycle to 20%.
7. Apply the triangular wave with magnitude 1 V_{pp}, and frequency 100 kHz. Record the results in both V and dBm units.
8. Repeat step 7 by changing the frequency to 500 kHz.

Experiment 2 Sampling Theorem

Experiment Procedure **Turn off the power supply during circuit connection**

1. Connect the power supply to the Modicom 1 board using the diagram shown in figure 2. Set the sampling control switch to "internal".
2. Set the frequency of the sampled signal ($p(t)$) to 32 kHz by choosing 320 kHz (after divided-by-10 circuit) and set the duty cycle to 1.
3. Apply a sine wave with V_p = 1 V and f = 12 kHz to the analog input of Modicom 1. Record the signals at sample output and sample/hold output in both time and frequency domains.
4. Change the duty cycle to 5 and repeat step 2-3.
5. Change the duty cycle back to 1 and change the frequency of the sine wave to f = 20 kHz, then repeat step 2-3.
6. Change the frequency of the sampled signal to 16 kHz (160 kHz on board) and repeat step 2-3.
7. Apply 1-kHz sine wave (test point 7) to the analog input.
8. Connect the sample output to second order and fourth order low pass filters (LPF).
9. Set the duty cycle to 1, change the sampling frequency to 2k, 4k, 8k, 16k and 32k, respectively, and record the output.
10. Change from sample output to sample/hold output and repeat step 8-10.

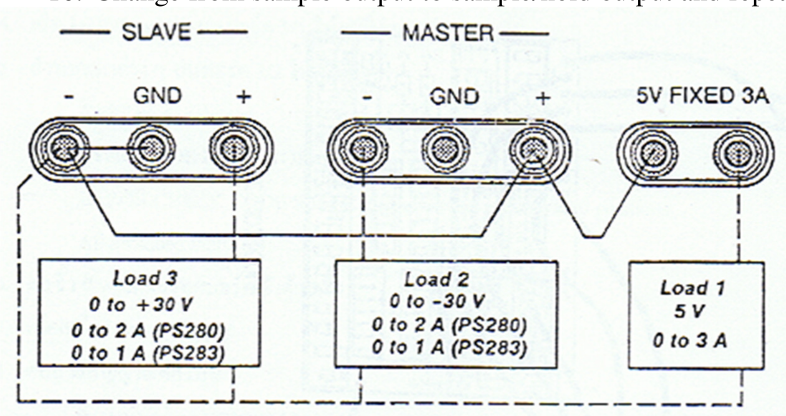

Figure 2: DC power supply configuration

5 Postlab Questions

1. How do spectra of sampled signal differ from those of sample/hold signal? Also explain why.