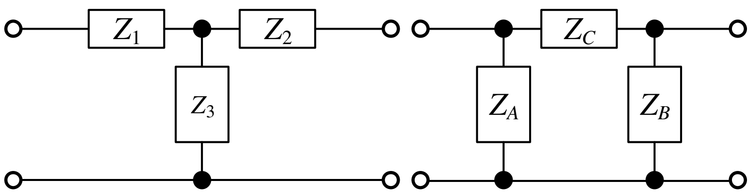

T-network to π-network

1. Use Y Matrix

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_A + Y_C; Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -Y_C = Y_{21}; Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_B + Y_C$$

$$\therefore \mathbf{Y} = \begin{bmatrix} Y_A + Y_C & -Y_C \\ -Y_C & Y_B + Y_C \end{bmatrix}$$

Since the Z matrix of the T-network is given by

$$\mathbf{Z} = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \text{->} \ \mathbf{Y} = \mathbf{Z}^{-1} = \frac{1}{\Delta_Z}\begin{bmatrix} Z_2 + Z_3 & -Z_3 \\ -Z_3 & Z_1 + Z_3 \end{bmatrix}; \Delta_Z = Z_1Z_2 + Z_1Z_3 + Z_2Z_3$$

Thus,

$Y_C = \dfrac{Z_3}{\Delta_Z} \rightarrow Z_C = \dfrac{\Delta_Z}{Z_3}$. It follows that

$$Y_A = \frac{Z_2}{\Delta_Z} \rightarrow Z_A = \frac{\Delta_Z}{Z_2}; Y_B = \frac{Z_1}{\Delta_Z} \rightarrow Z_B = \frac{\Delta_Z}{Z_1}.$$

2. Use ABCD matrix

The ABCD matrices of T- and π-networks are given by

$$\mathbf{T}_T = \begin{bmatrix} 1 + \dfrac{Z_1}{Z_3} & Z_1 + Z_2 + \dfrac{Z_1Z_2}{Z_3} \\ \dfrac{1}{Z_3} & 1 + \dfrac{Z_2}{Z_3} \end{bmatrix}; \mathbf{T}_\pi = \begin{bmatrix} 1 + \dfrac{Y_B}{Y_C} & \dfrac{1}{Y_C} \\ Y_A + Y_B + \dfrac{Y_AY_B}{Y_C} & 1 + \dfrac{Y_A}{Y_C} \end{bmatrix}$$

Thus,

$$\frac{1}{Y_C} = Z_C = Z_1 + Z_2 + \frac{Z_1Z_2}{Z_3} = \frac{\Delta_Z}{Z_3};$$

$$1 + \frac{Y_B}{Y_C} = 1 + \frac{Y_B\Delta_Z}{Z_3} = 1 + \frac{Z_1}{Z_3} \rightarrow Z_B = \frac{\Delta_Z}{Z_1}; 1 + \frac{Y_A}{Y_C} = 1 + \frac{Y_A\Delta_Z}{Z_3} = 1 + \frac{Z_2}{Z_3} \rightarrow Z_A = \frac{\Delta_Z}{Z_2}.$$

π-network to T-network

1. Use Z Matrix

$$\mathbf{Y} = \begin{bmatrix} Y_A + Y_C & -Y_C \\ -Y_C & Y_B + Y_C \end{bmatrix} \rightarrow \mathbf{Z} = \frac{1}{\Delta_Y}\begin{bmatrix} Y_B + Y_C & Y_C \\ Y_C & Y_A + Y_C \end{bmatrix}; \Delta_Y = Y_AY_B + Y_AY_C + Y_BY_C$$

Thus,

$Z_3 = \dfrac{Y_C}{\Delta_Y} = \dfrac{Z_AZ_B}{\Sigma_Z}; \Sigma_Z = Z_A + Z_B + Z_C$. It follows that

$$Z_1 = \frac{Y_B}{\Delta_Y} = \frac{Z_AZ_C}{\Sigma_Z}; Z_2 = \frac{Y_A}{\Delta_Y} = \frac{Z_BZ_C}{\Sigma_Z}.$$

2. Use ABCD matrix

From the ABCD matrices of two networks,

$\frac{1}{Z_3} = Y_A + Y_B + \frac{Y_A Y_B}{Y_C} = \frac{\Sigma_Z}{Z_A Z_B} \rightarrow Z_3 = \frac{Z_A Z_B}{\Sigma_Z}$. It follows that

$$1+\frac{Z_1}{Z_3} = 1+\frac{\Sigma_Z Z_1}{Z_A Z_B} = 1+\frac{Y_B}{Y_C} = 1+\frac{Z_C}{Z_B} \rightarrow Z_1 = \frac{Z_A Z_C}{\Sigma_Z};$$

$$1+\frac{Z_2}{Z_3} = 1+\frac{\Sigma_Z Z_2}{Z_A Z_B} = 1+\frac{Y_A}{Y_C} = 1+\frac{Z_C}{Z_A} \rightarrow Z_2 = \frac{Z_B Z_C}{\Sigma_Z}.$$