1. Since the impedance function is given by

$$Z(\omega) = -\frac{j}{\omega C_2}\frac{\omega^2 - 1/L_1C_1}{\omega^2 - \dfrac{C_1+C_2}{L_1C_1C_2}} \text{ or } Z(s) = \frac{1}{sC_2}\frac{s^2 + 1/L_1C_1}{s^2 + \dfrac{C_1+C_2}{L_1C_1C_2}}.$$

To construct the series resonant circuit, first notice that there are two poles, namely, at s = 0 and s = $j(C_1+C_2)/L_1C_1C_2$ and thus the circuit consists of a series capacitor and a parallel LC resonator. Let C_0 denote the series capacitor, and C_p, L_p denote the capacitor and inductor of the parallel LC resonator, respectively, then

$$C_0 = \frac{1}{k_0} = \frac{1}{sZ(s)}\bigg|_{s=0} = \left[\frac{1}{C_2}\frac{s^2+1/L_1C_1}{s^2+\dfrac{C_1+C_2}{L_1C_1C_2}}\right]^{-1}\Bigg|_{s=0} = C_1 + C_2$$

$$C_p = \frac{1}{2k_i} = \frac{s}{\left(s^2+\omega_i^2\right)Z(s)}\bigg|_{s=j\omega_i} = \left[\frac{s^2C_2}{s^2+1/L_1C_1}\right]\bigg|_{s=j\omega_2} = -\frac{C_1+C_2}{L_1C_1}\left[-\frac{C_1+C_2}{L_1C_1C_2}+\frac{1}{L_1C_1}\right]^{-1} = \frac{(C_1+C_2)C_2}{C_1}$$

$$L_p = \frac{1}{\omega_2^2 C_p} = \left[\frac{C_1+C_2}{L_1C_1C_2}\frac{(C_1+C_2)C_2}{C_1}\right]^{-1} = \frac{L_1C_1^2}{(C_1+C_2)^2}$$

2. Since the admittance function is given by

$$Y(\omega) = -\frac{j}{\omega L_2}\frac{\omega^2 - 1/L_1C_1}{\omega^2 - \dfrac{L_1+L_2}{C_1L_1L_2}} \text{ or } Y(s) = \frac{1}{sL_2}\frac{s^2 + 1/L_1C_1}{s^2 + \dfrac{L_1+L_2}{C_1L_1L_2}}.$$

This has poles at s= 0 and $s = j(L_1+L_2)/L_1L_2C_1$ and thus the circuit consists of a parallel inductor and a series LC resonator. Let L_0 denote the parallel inductor, and C_s, L_s denote the capacitor and inductor of the series LC resonator, respectively, then

$$L_0 = \frac{1}{h_0} = \frac{1}{sY(s)}\bigg|_{s=0} = \left[\frac{1}{L_2}\frac{s^2+1/L_1C_1}{s^2+\dfrac{L_1+L_2}{L_1L_2C_1}}\right]^{-1}\Bigg|_{s=0} = L_1 + L_2$$

$$L_s = \frac{1}{2h_i} = \frac{s}{\left(s^2+\omega_i^2\right)Y(s)}\bigg|_{s=j\omega_i} = \left[\frac{s^2L_2}{s^2+1/L_1C_1}\right]\bigg|_{s=j\omega_2} = -\frac{L_1+L_2}{L_1C_1}\left[-\frac{L_1+L_2}{L_1L_2C_1}+\frac{1}{L_1C_1}\right]^{-1} = \frac{(L_1+L_2)L_2}{L_1}$$

$$C_s = \frac{1}{\omega_2^2 L_s} = \left[\frac{L_1+L_2}{L_1L_2C_1}\frac{(L_1+L_2)L_2}{L_1}\right]^{-1} = \frac{C_1L_1^2}{(L_1+L_2)^2}$$

3. Let V_3 be the voltage at the node between Z_1 and Z_2, then use KCL,

$$\frac{V_1-V_3}{Z_1} = \frac{V_3}{Z_2'} + \frac{V_3-V_2}{Z_2} \rightarrow -\frac{V_2}{Z_2} + \left(\frac{1}{Z_1}+\frac{1}{Z_2'}+\frac{1}{Z_2}\right)V_3 = \frac{V_1}{Z_1}$$

$$\frac{V_2-V_3}{Z_2} = \frac{V_1-V_2}{Z_1'} \rightarrow \left(\frac{1}{Z_1'}+\frac{1}{Z_2}\right)V_2 - \frac{V_3}{Z_2} = \frac{V_1}{Z_1'}$$. Thus,

$$\begin{bmatrix} -\dfrac{1}{Z_2} & \dfrac{1}{Z_1}+\dfrac{1}{Z_2'}+\dfrac{1}{Z_2} \\ \dfrac{1}{Z_1'}+\dfrac{1}{Z_2} & -\dfrac{1}{Z_2} \end{bmatrix}\begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = V_1\begin{bmatrix} \dfrac{1}{Z_1} \\ \dfrac{1}{Z_1'} \end{bmatrix}$$

Using Cramer's rule yields

$$V_2 = V_1 \frac{\begin{vmatrix} \dfrac{1}{Z_1} & \dfrac{1}{Z_1}+\dfrac{1}{Z_2'}+\dfrac{1}{Z_2} \\ \dfrac{1}{Z_1'} & -\dfrac{1}{Z_2} \end{vmatrix}}{\begin{vmatrix} -\dfrac{1}{Z_2} & \dfrac{1}{Z_1}+\dfrac{1}{Z_2'}+\dfrac{1}{Z_2} \\ \dfrac{1}{Z_1'}+\dfrac{1}{Z_2} & -\dfrac{1}{Z_2} \end{vmatrix}} = V_1 \frac{-\dfrac{1}{Z_1Z_2}-\left(\dfrac{1}{Z_1}+\dfrac{1}{Z_2'}+\dfrac{1}{Z_2}\right)\dfrac{1}{Z_1'}}{\dfrac{1}{Z_2{}^2}-\left(\dfrac{1}{Z_1}+\dfrac{1}{Z_2'}+\dfrac{1}{Z_2}\right)\left(\dfrac{1}{Z_1'}+\dfrac{1}{Z_2}\right)} = V_1 \frac{Z_1Z_2+Z_2'(Z_1+Z_1'+Z_2)}{Z_1Z_2'+(Z_1+Z_2')(Z_1'+Z_2)}$$

With port 2 short-circuited,

$$Z_{in} = Z_1' // \left[Z_1 + \left(Z_2 // Z_2'\right)\right] = Z_1' // \left[Z_1 + \frac{Z_2Z_2'}{Z_2+Z_2'}\right] = \frac{Z_1'\left(Z_1+\dfrac{Z_2Z_2'}{Z_2+Z_2'}\right)}{Z_1'+Z_1+\dfrac{Z_2Z_2'}{Z_2+Z_2'}} = \frac{Z_1Z_1'\left(Z_2+Z_2'\right)+Z_1'Z_2Z_2'}{\left(Z_1'+Z_1\right)\left(Z_2+Z_2'\right)+Z_2Z_2'}$$