Two-Port Symmetry

The open-circuit and short-circuit impedances of a two-port are defined as

$$Z_{1o}=\left.\frac{V_1}{I_1}\right|_{I_2=0};\ Z_{2o}=\left.\frac{V_2}{I_2}\right|_{I_1=0};\ Z_{1s}=\left.\frac{V_1}{I_1}\right|_{V_2=0};\ Z_{2s}=\left.\frac{V_2}{I_2}\right|_{V_1=0}.$$

The subscripts *o* and *s* denote the open-circuit and short-circuit, respectively, which specifies the condition at the other port. For example, Z_{1o} is an impedance measurement at port 1 with port 2 open-circuited.

Transmission Parameters in Terms of Open-circuit and Short-circuit Parameters

Given the transmission parameters *A*, *B*, *C*, and *D*, the following relationships hold:

$V_1=AV_2-BI_2;\ I_1=CV_2-DI_2$.

Therefore,

$$Z_{1o}=\left.\frac{V_1}{I_1}\right|_{I_2=0}=\frac{A}{C};\ Z_{2o}=\left.\frac{V_2}{I_2}\right|_{I_1=0}=\frac{D}{C};\ Z_{1s}=\left.\frac{V_1}{I_1}\right|_{V_2=0}=\frac{B}{D};\ Z_{2s}=\left.\frac{V_2}{I_2}\right|_{V_1=0}=\frac{B}{A}.$$

Assume that the network is reciprocal (i.e., *AD*-*BC* = 1), then one can obtain

$$A=\pm\sqrt{\frac{Z_{1o}}{Z_{2o}-Z_{2s}}};B=\pm Z_{2o}\sqrt{\frac{Z_{1o}}{Z_{2o}-Z_{2s}}};C=\pm\sqrt{\frac{1}{Z_{1o}(Z_{2o}-Z_{2s})}};D=\pm\frac{Z_{2o}}{\sqrt{Z_{1o}(Z_{2o}-Z_{2s})}}.$$

Input Impedance in Terms of Two-Port Parameters

Here, assume that the output port (port 2) is terminated by a load impedance, Z_L, then the input impedance is given by the ratio V_1/I_1, which can be determined in terms of two-port parameters and Z_L.

(i) In terms of *Z*-parameters

Recall that

$V_1=Z_{11}I_1+Z_{12}I_2;\ V_2=Z_{21}I_1+Z_{22}I_2$ and $V_2=-Z_LI_2$.

Hence, $-Z_LI_2=Z_{21}I_1+Z_{22}I_2$ or $Z_{21}I_1=-(Z_L+Z_{22})I_2$. It follows that

$$I_2=\frac{-Z_{21}I_1}{Z_L+Z_{22}}\text{ and }V_1=Z_{11}I_1+Z_{12}I_2=Z_{11}I_1-\frac{Z_{12}Z_{21}I_1}{Z_L+Z_{22}}=\left(Z_{11}-\frac{Z_{12}Z_{21}}{Z_L+Z_{22}}\right)I_1.$$

Therefore,

$$Z_{in}=\frac{V_1}{I_1}=Z_{11}-\frac{Z_{12}Z_{21}}{Z_L+Z_{22}}=\frac{Z_{11}Z_L-Z_{12}Z_{21}+Z_{11}Z_{22}}{Z_L+Z_{22}}.$$

If the output port is open-circuited, then

$$Z_{in,oc}=\left.\frac{V_1}{I_1}\right|_{Z_L=\infty}=\left.\frac{Z_{11}Z_L-Z_{12}Z_{21}+Z_{11}Z_{22}}{Z_L+Z_{22}}\right|_{Z_L=\infty}=Z_{11}.$$

Likewise, if the output port is short-circuited, then

$$Z_{in,sc}=\left.\frac{V_1}{I_1}\right|_{Z_L=0}=\left.\frac{Z_{11}Z_L-Z_{12}Z_{21}+Z_{11}Z_{22}}{Z_L+Z_{22}}\right|_{Z_L=0}=\frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{22}}=\frac{\Delta Z}{Z_{22}}.$$

(ii) In terms of *ABCD* parameters

From $V_1=AV_2-BI_2$, $I_1=CV_2-DI_2$ and $V_2=-Z_LI_2$,

$I_1=C(-Z_LI_2)-DI_2=-(CZ_L+D)I_2$.

It follows that

$$I_2=\frac{-I_1}{CZ_L+D}\text{ and }V_1=AV_2-BI_2=A(-Z_LI_2)-BI_2=-(AZ_L+B)I_2=\frac{AZ_L+B}{CZ_L+D}I_1.$$

Therefore,

$$Z_{in} = \frac{V_1}{I_1} = \frac{AZ_L + B}{CZ_L + D}.$$

$Z_{in,oc}$ and $Z_{in,sc}$ then become

$$Z_{in,oc} = \left.\frac{V_1}{I_1}\right|_{Z_L=\infty} = \frac{A}{C};\ Z_{in,sc} = \left.\frac{V_1}{I_1}\right|_{Z_L=0} = \frac{B}{D}.$$

Output Impedance in Terms of Two-Port Parameters

Here, assume that the input port (port 1) is terminated by a load impedance, Z_L, then the output impedance is given by the ratio V_2/I_2, which can be determined in terms of two-port parameters and Z_L.

(i) In terms of Z-parameters

From $V_1 = Z_{11}I_1 + Z_{12}I_2;\ V_2 = Z_{21}I_1 + Z_{22}I_2$ and $V_1 = -Z_L I_1$, it can be shown that

$$Z_{out} = \frac{V_2}{I_2} = \frac{Z_{22}Z_L - Z_{12}Z_{21} + Z_{11}Z_{22}}{Z_L + Z_{11}}.$$

$Z_{out,oc}$ and $Z_{out,sc}$ then become

$$Z_{out,oc} = Z_{22};\ Z_{out,sc} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{11}} = \frac{\Delta Z}{Z_{11}}.$$

(ii) In terms of *ABCD* parameters

From $V_1 = AV_2 - BI_2,\ I_1 = CV_2 - DI_2$ and $V_1 = -Z_L I_1$, it can be shown that

$$Z_{out} = \frac{V_2}{I_2} = \frac{DZ_L + B}{CZ_L + A}.$$

$Z_{out,oc}$ and $Z_{out,sc}$ then become

$$Z_{out,oc} = \frac{D}{C};\ Z_{out,sc} = \frac{B}{A}.$$