Chapter 3

Microwave Power Measurement

3.1 Power Definitions

At lower frequencies, voltage and current can be measured easily, and the power is usually obtained from voltage and current. At microwave frequencies, it is hard to measure voltages and currents for several reasons: the voltages and currents change along a transmission line, and in waveguides currents and voltages do not make physical sense. On the other hand, the power flow is the same at any point in the transmission line or waveguide, and this becomes the logical quantity to measure. From an application point of view, power is critical factor. For example, for a communication system transmitter, twice the power means twice the geographic coverage (or 40% more range). Power is expensive at high frequencies, since it is difficult to make fast low-loss active devices (transistors), and tube sources are large and have limited lifetimes. It becomes therefore important to keep track of the generated power and develop means to optimize the amount that is delivered to the load (e.g. an antenna in a wireless system).

Often, microwave engineers refer to some application as a "low-power", "medium-power" or "high-power" one. Usually, what is meant is a range of power levels for each case, approximately equal to 0 – 1 mW, 1mW – 10 W and larger than 10 W, respectively. Power is most commonly expressed in decibels which are relative units. In the low-power experiments in our lab, it will be expressed relative to 1 mW: $P_{dB} = 10\log(P_{mW}/1mW)$. (Why is the logarithm (base 10) multiplied by 10 and not 20?)

Starting from fields, in terms of the electric and magnetic field, the power flow is defined by the Poynting vector:

$$P = \oint_s \left(\vec{E} \times \vec{H}\right) \cdot d\vec{s} = \oint_s \vec{S} \cdot d\vec{s}. \tag{3.1}$$

We can again notice the relationship between fields and circuits—the formula corresponds to $P = VI$.

Usually at microwave frequencies one measures and calculates *average power* (the rate of variation is so rapid that it is difficult or impossible to directly measure instantaneous power or phase at RF frequencies or higher). Starting from the definition of power at lower frequencies as the product of current and voltage, we notice that this product varies during an AC cycle. The average power represents the DC component of the power waveform. What is this power averaged over? The definition of power you have learned in your physics courses is rate of flow (or transfer) of energy in time, so it makes sense to average power over time. Now the question is: how long do we need to average? Usually, the averaging time is equal to many periods of the lowest frequency component of a signal we are measuring. This can be written as

$$P_{\text{av}} = \frac{1}{nT_L} \int_0^{nT_L} v(t)i(t)\, dt, \tag{3.2}$$

where T_L is the period of the lowest frequency component, and n is much larger than 1. In the case of a CW (continuous wave) signal, there is only one frequency, and if V and I are the RMS phasors corresponding to the voltage and current, we find that

$$P_{\text{av}} = \text{Re}\,(VI^*) \tag{3.3}$$

where * denotes the complex conjugate. In the case of an amplitude modulated signal, T_L is the period of the modulation signal, and for a pulse modulated signal, T_L is the repetition rate of the pulse.

In radar applications, pulsed modulation is used very often, and in this case it is often convenient to use the *pulse power* instead of average power. The pulse power is obtained when the energy transfer rate is averaged over the duration of the pulse, or the pulse width T_d:

$$P_{\text{pulse}} = \frac{1}{T_d} \int_0^{T_d} v(t)i(t)\, dt. \tag{3.4}$$

Usually, T_d is defined as the time between the one-half amplitude points. For rectangular pulses, we can measure the average power, and get the pulse power if we know the duty cycle (duty cycle = pulse width × repetition frequency):

$$P_{\text{pulse}} = \frac{P_{\text{av}}}{\text{Duty Cycle}}. \tag{3.5}$$

How is average power measured? There are three devices for measuring average power used in modern instrumentation: *the thermistor, the thermocouple, and the diode detector.* The basic operation of all three devices is that they turn microwave power into a measurable DC or low frequency signal. Typically, there is a sensor (which contains one of the three devices from above) connected with a coaxial cable or waveguide to a power meter, which does the post-processing of the received DC signal.

3.2 The Thermistor

The thermistor falls into the category of *bolometers.* A bolometer is a device that converts RF power into heat, and this changes its resistance, so the power can be measured from the change in resistance. In the early days of microwaves, so called baretters, another type of bolometers, were used. A baretter consists of a thin metal wire (usually platinum) that heats up and its resistance changes. Baretters are usually very small, and as a consequence they are able to detect very low power levels, but they also burn out easily. A thermistor is in principle the same as a baretter, but instead of metal, a semiconductor is used for power detection. The main difference is that for a baretter, the temperature coefficient is positive, which means that the resistance grows with temperature, whereas for a thermistor it is negative.

Thermistors are usually made as a bead about 0.4 mm in diameter with 0.03 mm wire leads. The hard part in mounting it in a coax or waveguide is that the impedances have to be matched, so that the thermistor absorbs as much power as possible over the frequency range that is desired. Fig. 3.1 shows characteristic curves of a typical thermistor. You can see that the dependence of resistance versus power is very nonlinear, and this makes direct measurements hard. How are power meters then built? You will build a bulky thermistor-based power meter in the lab and calibrate the nonlinear responsivity of the thermistor.

The change in resistance of a thermistor due to the presence of RF or microwave power can be used to make a power meter for measuring that RF power level. A clever way to do this is to keep the resistance of the thermistor constant with a balanced resistive bridge circuit, the basic idea of which is shown in Fig. 3.2. The thermistor (R_T) is DC-biased through the bridge to a known value of resistance. When no RF power is incident on the thermistor, its resistance is determined solely by the DC power P_{T0} dissipated in it, which can be determined from a knowledge of the DC voltage V_0 across the bridge, and the value of the resistance R in the upper arm of the bridge:

$$P_{T0} = \frac{V_T^2}{R_T} = \frac{V_0^2 R_T}{(R + R_T)^2} \tag{3.6}$$

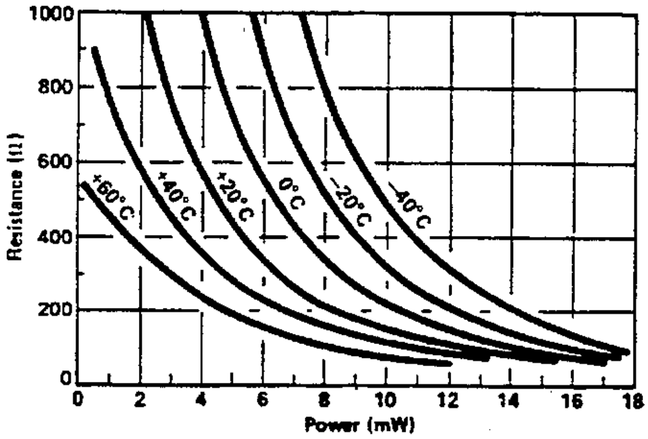

Figure 3.1: Characteristic curves of a typical thermistor bead.

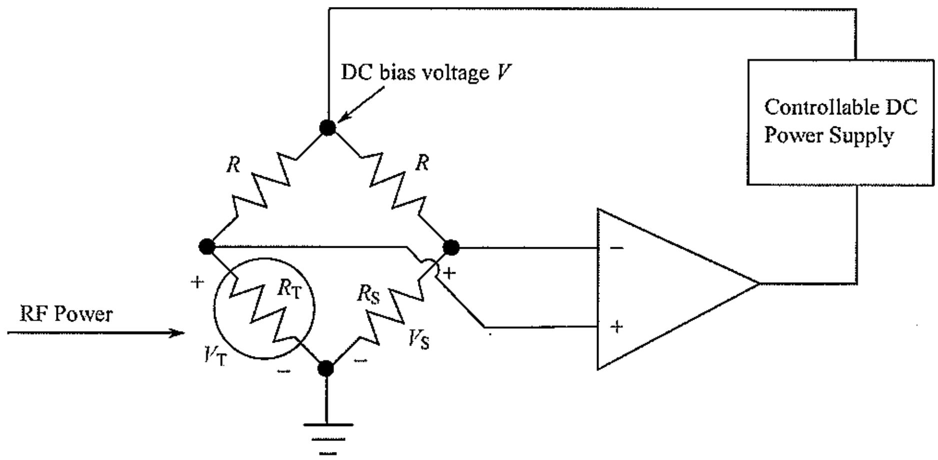

Figure 3.2: Diagram of a self-balanced Wheatstone bridge.

A particular desired value of R_T can be assured if we select the resistance R_S in the opposite arm of the bridge to be equal to this desired value, and then adjust the bias voltage V to the value V_0 that balances the bridge (so that $V_T = V_S$ in Fig. 3.2).

When there is incident RF power applied to the thermistor, this power added to the DC power already present will tend to decrease R_T, thereby unbalancing the bridge. When this happens, a voltage difference appears across the input terminals of an op-amp, whose output is used to change the DC voltage applied across the bridge to a new value V in such a way as to re-balance the bridge. The new DC bias voltage will be lower than before, such that there is less DC power dissipated in the thermistor, by an amount equal to the RF power incident on it, since the balance of the bridge was maintained. The RF power is readily calculated from knowledge of the old and new DC voltages across the bridge:

$$P_{\rm RF} = P_{T0} - P_T = \frac{V_0^2 R_T}{(R+R_T)^2} - \frac{V^2 R_T}{(R+R_T)^2} = \frac{(V_0^2 - V^2) R_T}{(R+R_T)^2} \tag{3.7}$$

and is translated into a power level displayed on the power meter scale. The main drawback of this scheme is that a change in ambient temperature (for example, the engineer touching the thermistor mount) would change the resistance, and make the measurement invalid. In modern thermistors this problem is solved in the mount itself, using an additional thermistor that senses the temperature of the mount and corrects the bridge reading.

This process is automated in commercial thermistor-based power meters such as the HP 432A, a diagram of which is shown in Fig. 3.3. The main parts are two self-balanced bridges (an "RF" bridge

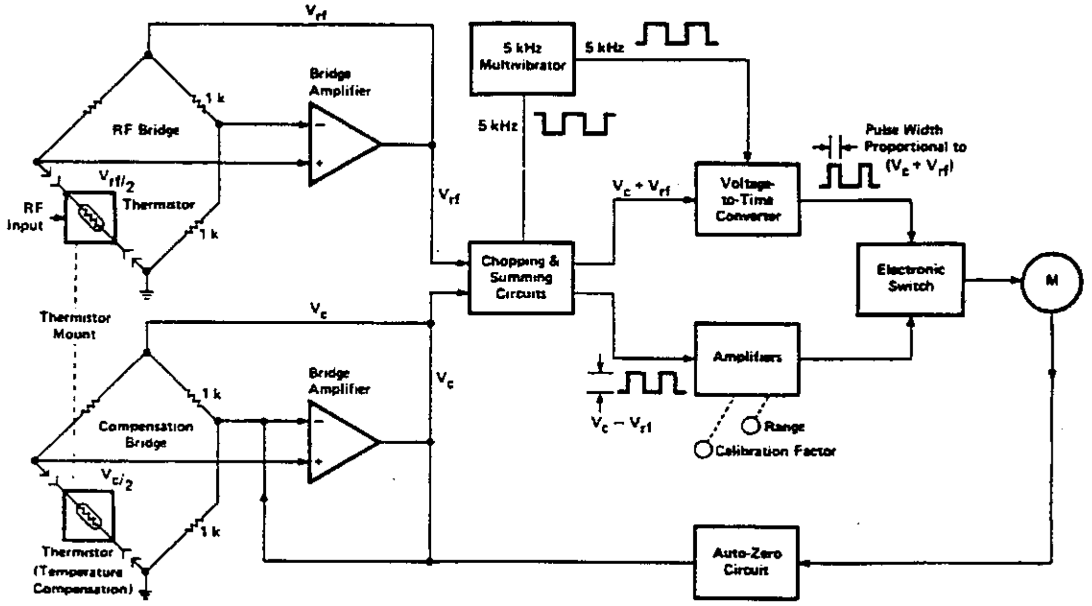

Figure 3.3: Diagram of the HP 432A power meter.

and a compensating bridge), a logic section, and an auto-zeroing circuit. The RF bridge, which contains the thermistor that detects power, is balanced by automatically varying the DC voltage V_{rf} which appears across the thermistor in this bridge. The compensating bridge, which takes care of temperature compensation, is balanced with the DC voltage V_c. If one of the bridges is unbalanced, an error voltage is applied to the top of the bridge, which causes the thermistor to change resistance in the direction required to keep the balance. The power meter is zero-set by making V_c equal to $V_{\mathrm{rf}0}$, which is the value of V_{rf} in the absence of RF input power. When RF power is applied to the detecting thermistor, V_{rf} decreases, so that

$$P_{\mathrm{rf}} = \frac{{V_{\mathrm{rf}0}}^2}{4R} - \frac{{V_{\mathrm{rf}}}^2}{4R} = \frac{{V_c}^2 - {V_{\mathrm{rf}}}^2}{4R} = \frac{1}{4R}\left(V_c - V_{\mathrm{rf}}\right)\left(V_c + V_{\mathrm{rf}}\right). \tag{3.8}$$

where R is the value of the fixed resistors in the bridge circuits ($= 1\ \mathrm{k}\Omega$ in Fig. 3.3). The meter logic circuit monitors the value of the right side of (3.8). Ambient temperature changes cause changes in both V_c and V_{rf}, such there is no change in P_{rf}. In the lab experiment, you will be balancing the bridge manually.

3.3 The Thermocouple

The thermocouple detector is used in the HP8481 sensor series, which you will use in the lab together with the HP437 power meters. Thermocouples generate rather low DC signals compared to thermistors. Due to progress in thin-film technology over the last few decades, they are now parts of most modern microwave power-measurement instruments. Standards are still, however, traced to thermistor power sensors, due to their stability.

A thermocouple generates a voltage due to temperature differences between its two ends. A *very* simplified physical explanation is the following: when one end of a piece of metal is heated, more electrons are made free to move. Due to diffusion, they will move away from the heated end and towards the cold end of the piece of metal, and leave extra positive charges behind. This separation of charges causes and electric field. Thermal equilibrium is reached when the Coulomb force on the charges is of equal amplitude to the force caused by diffusion. The electric field in steady-state can be integrated to find a voltage between the two ends of the piece of metal. This voltage is called the Thomson EMF. The same principle applies at a junction of two metals with different free-electron densities. Here diffusion causes

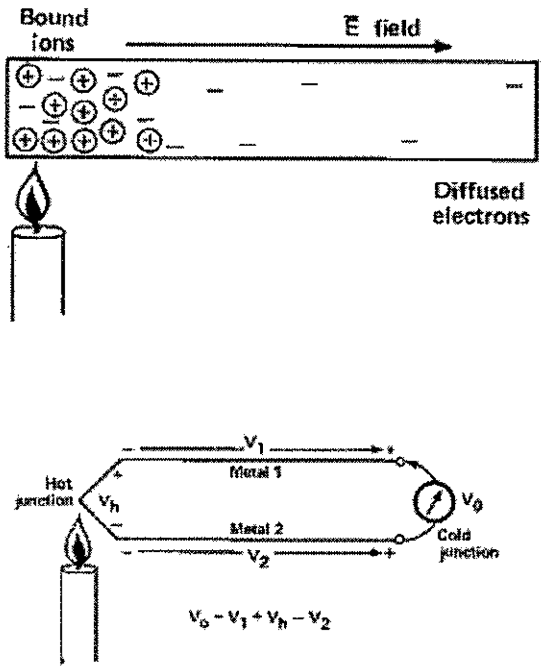

Figure 3.4: Simplified explanation of thermocouple operation. The total measured EMF is a resultant of several thermoelectric voltages generated along the circuit (V_1 and V_2 are Thomson EMFs, while V_h is a Peltier EMF).

the Peltier EMF. A schematic of a thermocouple, Fig. 3.4 shows that it combines the Thomson and Peltier effects. It consists of a loop of two different materials. One junction of the materials is exposed to heat, and the other is kept cold. A sensitive voltmeter is inserted into the loop. In order to have a larger value of the EMF, several thermocouples can be connected in series, or a thermopile. However, at microwave frequencies, large thermocouples also have large parasitics (inductances and capacitances), so thin film thermocouples have been developed for the microwave frequency range. A photograph of a thermocouple silicon chip is shown in Fig. 3.5.

The thermocouple used in the HP8481A sensor head is shown in Fig. 3.6. It consists of a silicon chip which has one part that is n-doped and acts as one of the metals of the thermocouple. The other metal is a thin film of tantalum nitride. This thin film is actually a resistor, which can be tailored for a good impedance match of the thermocouple to the cable (50 or 75 Ω, for example). The resistor converts RF energy into heat. In the process of fabricating this thin-film sensor, a silicon-dioxide layer is used as an

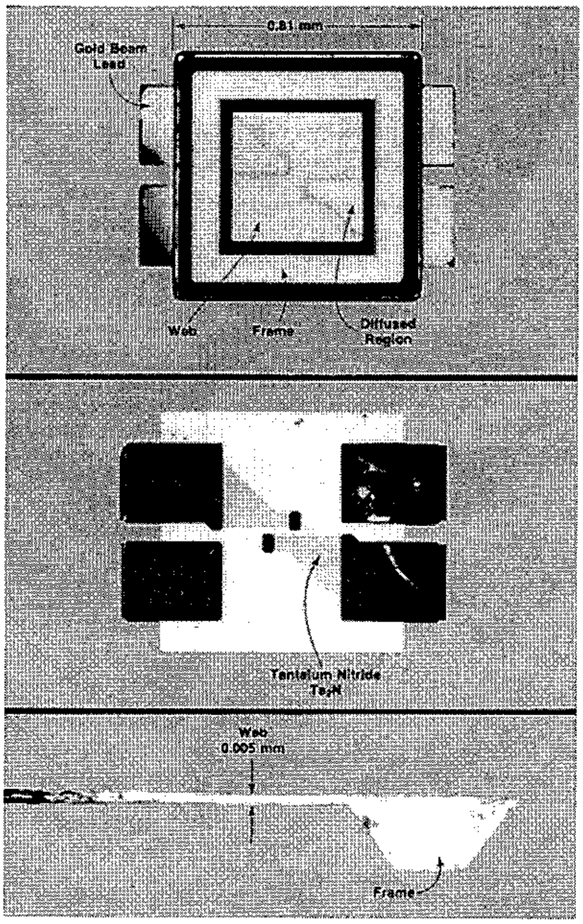

Figure 3.5: Photo-micrograph of the structure of the HP 8481A thermocouple chip on a thin silicon web.

insulator between the silicon and the resistive film. A hole is then made in this insulator so that the resistor contacts the silicon and forms the hot junction. The cold junction is formed by the resistor and the outside edges of the silicon substrate.

As the resistor converts RF energy into heat, the center of the chip gets hotter than the outside edges (why?). Thus, there is a thermal gradient which gives rise to a thermoelectric EMF. The thermocouple chip is attached to a planar transmission line deposited on a sapphire substrate (sapphire is a good thermal conductor). The planar transmission line has a transition to a coaxial connector (outside world). The thermoelectric voltage is very low – microvolts per measured milliwatts of RF power. However, it does not change very much with ambient temperature, which is illustrated in Fig. 3.7.

Power meters which use a thermocouple, such as the HP437 which you will use in the lab, are built to detect very small voltages. The fundamental principle is the same as a lock-in amplifier: the small signal is chopped (amplitude modulated) at a low frequency in the sensor itself, (on the order of a kHz). Then the voltage is synchronously detected (i.e. demodulated) at the other end of the sensor cable (i.e. in the instrument). Because of the low DC voltages, the thermocouple does not have enough sensitivity to measure small RF power levels. For this purpose, a semiconductor diode is usually used.

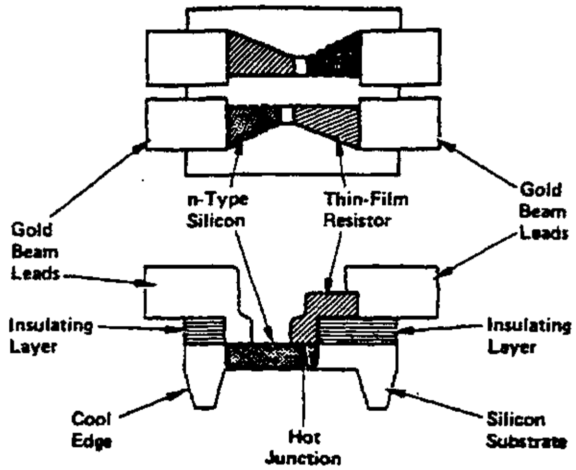

Figure 3.6: Structure of the thin-film thermocouple used in the HP8481A sensor.

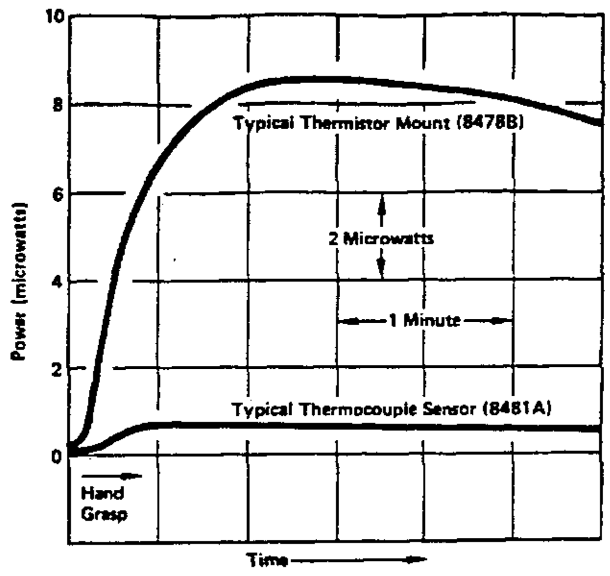

Figure 3.7: Difference in behavior of thermistor and thermocouple when grasped by the hand. (From HP Application Note 64-1.)

3.4 The Diode Detector

A semiconductor diode is a rectifier, as you have learned in your circuits classes. At microwave frequencies, special diodes have to be used because low frequency diodes are not fast enough to detect nanosecond changes in time (corresponding to GHz frequencies). These diodes are called Schottky diodes, and they are basically a metal contact on a piece of gallium arsenide (GaAs). They can measure powers as low as -70 dBm (100 pW) up to 18 GHz. Essentially the same circuitry can be used to make a power meter with a Schottky diode as is used with a thermocouple. The schematic symbol for a Schottky diode is shown in Fig. 3.8, but often an ordinary diode symbol is used instead.

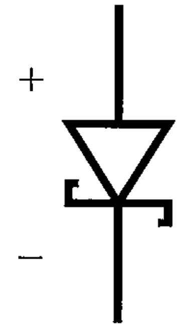

Figure 3.8: Schematic symbol and reference directions for a Schottky diode.

At sufficiently low frequencies, a Schottky diode behaves as a nonlinear resistor for which the current-voltage dependence obeys approximately the following equation, shown graphically in Fig. 3.9:

$$I(V) = I_S\left(e^{\alpha V} - 1\right), \tag{3.9}$$

where I_S is the so called reverse saturation current (or leakage current) whose value is small (between about 10^{-5} A and 10^{-15} A), and $\alpha = n/(25\,\text{mV})$ at room temperature. n is called the ideality factor, and it depends on the diode structure. For a typical Schottky diode, $n = 1.2$, while for point-contact silicon diodes $n = 2$.

When the diode is biased at a DC voltage V_0, the total voltage across the diode terminals is

$$V = V_0 + v, \tag{3.10}$$

where v is the AC voltage that we are trying to detect with this diode. The previous equation can be expanded in a Taylor series about V_0 assuming v is small compared to V_0, and the first and second derivatives evaluated:

$$\begin{aligned} I(V) &= I_0 + v\left.\frac{dI}{dV}\right|_{V_0} + \frac{1}{2}v^2\left.\frac{d^2I}{dV^2}\right|_{V_0} + \ldots \\ \left.\frac{dI}{dV}\right|_{V_0} &= \alpha I_S e^{\alpha V_0} = \alpha(I_0 + I_S) = G_d = \frac{1}{R_j}, \\ \left.\frac{d^2I}{dV^2}\right|_{V_0} &= \alpha^2(I_S + I_0) = \alpha G_d = G_d{}'. \end{aligned} \tag{3.11}$$

Here $I_0 = I(V_0)$ is the DC bias current. R_j is called the junction resistance of the diode, and $G_d = 1/R_j$ is the dynamic conductance. Now the current can be rewritten as

$$I(V) = I_0 + i = I_0 + vG_d + \frac{v^2}{2}G_d{}' + \ldots \tag{3.12}$$

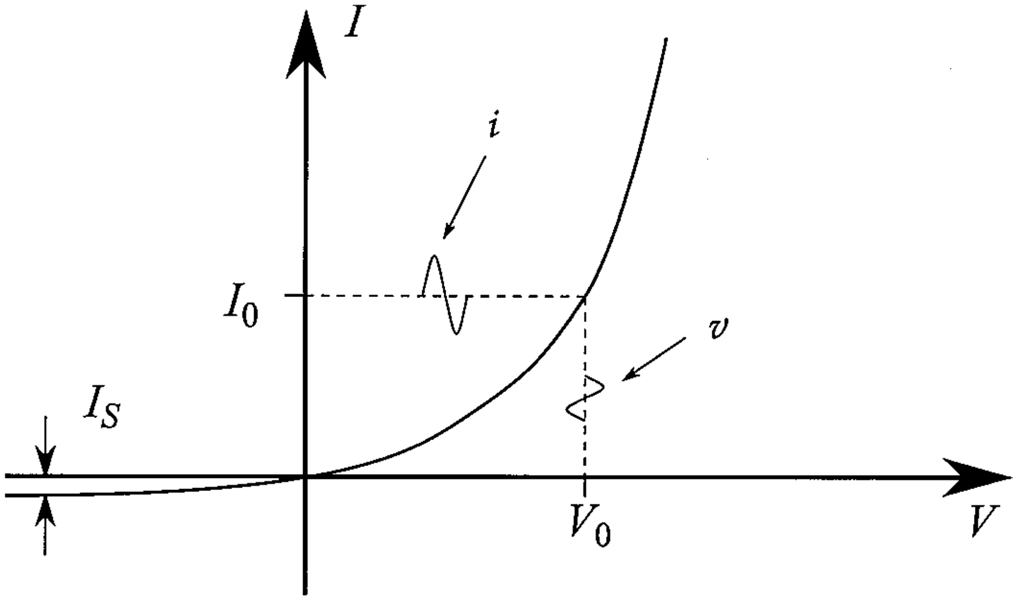

Figure 3.9: Current-voltage characteristic of a diode.

This approximation for the current is called the *small signal* or *quadratic* approximation, and is adequate for many purposes.

In this case, the current through the diode contains a term proportional to the square of the AC voltage, or, equivalently, to the AC power. This allows the diode to work as a power detector, as we will now detail. We assume the RF voltage is

$$v = \sqrt{2}V_{\mathrm{RF}} \cos \omega t \tag{3.13}$$

where V_{RF} is the RMS value of the RF voltage. Since, by a trig identity,

$$v^2 = 2V_{\mathrm{RF}}^2 \cos^2 \omega t = V_{\mathrm{RF}}^2 (1 + \cos 2\omega t) \tag{3.14}$$

we can gather the terms from (3.12) which are constant in time:

$$I_{DC} = I_0 + \frac{1}{2} G_d' V_{\mathrm{RF}}^2 \tag{3.15}$$

All other terms from (3.12) vary rapidly (at least as fast as the RF voltage itself) and have a time-average value of zero. They are thus not detected by ordinary low-frequency measurement devices such as voltmeters, and typically the diode response is low-pass filtered before connection to the voltmeter. This assures that only (3.15) will register on the instrument, and the RF behavior of the circuit under test is not modified (see the discussion on bias networks in section 3.5 below).

Now, the RF power delivered to the diode junction is

$$P_{j,\mathrm{RF}} = \frac{V_{\mathrm{RF}}^2}{R_j} \tag{3.16}$$

so by (3.15) and (3.11),

$$I_{DC} = I_0 + \frac{\alpha}{2} P_{j,\mathrm{RF}} \tag{3.17}$$

By (3.9), the DC voltage at the junction is then

$$V_{j,DC} = \frac{1}{\alpha}\ln\left(1+\frac{I_{DC}}{I_S}\right) = \frac{1}{\alpha}\ln\left(1+\frac{I_0}{I_S}+\frac{\alpha P_{j,\mathrm{RF}}}{2I_S}\right) \tag{3.18}$$

Since typical values of the part of $V_{j,DC}$ due to $P_{j,\mathrm{RF}}$ are tens or perhaps a few hundreds of millivolts, it is usual to set the DC bias voltage $V_0 = 0$ so that the small DC voltage proportional to the RF power is not lost in noise. If $I_0 = 0$, then

$$V_{j,DC} = \frac{1}{\alpha}\ln\left(1+\frac{\alpha P_{j,\mathrm{RF}}}{2I_S}\right) \tag{3.19}$$

Two limiting cases of (3.19) are of interest. If $P_{j,\mathrm{RF}} \ll P_{d0}$, where

$$P_{d0} = \frac{2I_S}{\alpha} \qquad \text{(typical values would be between -20 dBm and 0 dBm)}$$

then

$$V_{j,DC} \simeq \frac{P_{j,\mathrm{RF}}}{2I_S} \tag{3.20}$$

This is the so-called square-law approximation, valid when the RF voltage v is a small signal, meaning that the input power on the diode is not too large. In this range, $V_{j,DC}$ is proportional to $P_{j,\mathrm{RF}}$ in watts. On the other hand, if $P_{j,\mathrm{RF}} \gg P_{d0}$ we have

$$V_{j,DC} \simeq \frac{1}{\alpha}\ln\left(\frac{\alpha P_{j,\mathrm{RF}}}{2I_S}\right) \tag{3.21}$$

Thus, in this regime $V_{j,DC}$ is proportional to $P_{j,\mathrm{RF}}$ in dBm plus a constant, and for such large input power levels the diode is said to be saturated, the dependence of $V_{j,DC}$ on $P_{j,\mathrm{RF}}$ being a fundamentally nonlinear one. The different regimes of a typical diode detector with respect to the input power level are shown in Fig. 3.10. If it is desired to measure higher levels of RF power, a resistive attenuator can be

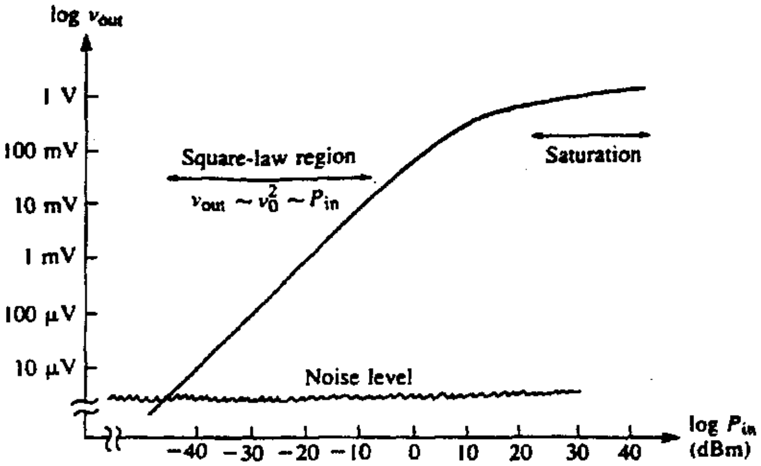

Figure 3.10: Diode detector regimes of operation.

placed before the diode in order to make sure of operation in the square-law region and prevent damage to the diode.

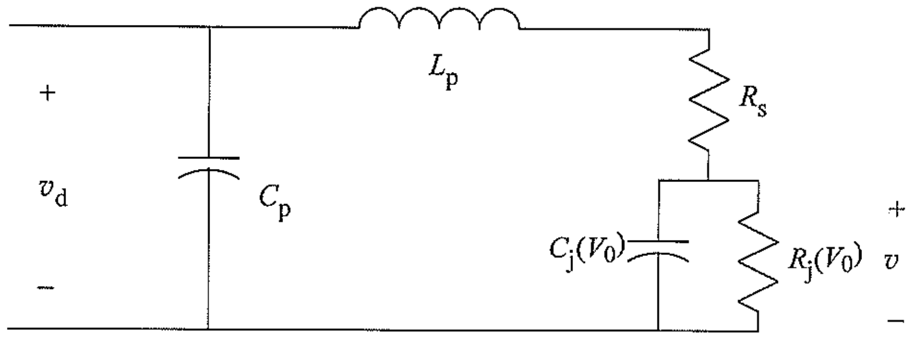

Figure 3.11: Equivalent small-signal AC circuit for a Schottky diode.

As operating frequency increases, other effects must be accounted for when modeling the behavior of a diode. A typical *AC* small-signal equivalent circuit valid at microwave frequencies is shown in Fig. 3.11. L_p and C_p are due to the diode package—L_p is a series inductance due to the wire leads, and C_p is a shunt capacitance of the contacts. The resistance R_s is that of the wire leads of the diode and the spreading of the current at the connection of the leads.

The capacitance C_j due to the diode's *pn* junction is dependent on the total voltage V_j across the junction, which can be approximated by the DC bias voltage V_0. The value of the resulting capacitance is typically a fraction of a pF. The RF resistance of the diode is obtained from the slope of the *I*-*V* curve as $R_j = 1/(\alpha I_s)$ (if $V_0 = 0$), and is usually a few kΩ, for leakage currents I_S on the order of 10 μA. Since this impedance has to be matched to the 50-Ohm impedance of the coax, the sensitivity of the diode as a power measuring device is reduced for larger impedances. The temperature performance is improved, however, for larger values of R_j, so there is a compromise involved in choosing the diode resistance for good power sensors.

It should be noted that the passive impedances in a diode's equivalent circuit will generally be different at higher harmonic frequencies than at the fundamental, and furthermore the DC equivalent circuit will be very different than at RF. In the latter case, a Thévenin or Norton equivalent circuit for the diode junction will apply, as shown in Fig. 3.12. The DC junction resistance of the diode is given by

$$R_{DC} = \frac{V_{j,DC}}{I_{DC}} \tag{3.22}$$

using (3.17) and (3.18).

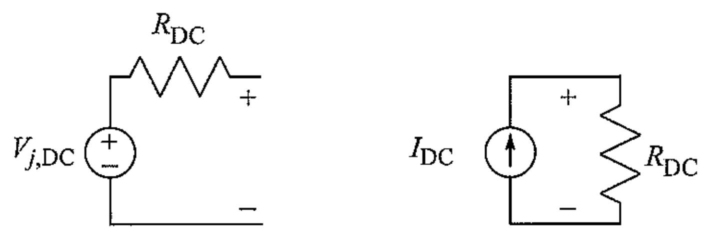

Figure 3.12: DC equivalent circuits for a diode junction.

3.5 DC Biasing Networks

Providing a DC bias voltage to a diode must be done in such a way so as not to disrupt the RF operation of the circuit. The following general principles must be followed in designing a network to provide this bias voltage.

1) The bias network needs to be "invisible" to the RF signals, i. e. be as close to an open circuit as possible. We do not wish to lose any of the RF power to the biasing circuit and power supply.

2) Conversely, the DC bias needs to be isolated from the RF circuit, i. e., we do not want the DC voltage to be present at the RF input to an amplifier or other sensitive device.

3) These constraints must hold over all frequencies that might be important to operation of the circuit, including those outside the primary design frequencies of interest. This is because, as we will see in Lectures 6 and 8, unwanted oscillations may occur when amplification is present in the circuit.

In order to satisfy the first criterion, the DC bias lines need to have inductance in series with the power supply as shown in Fig. 3.13. It is difficult to make an inductor at microwave frequencies due to

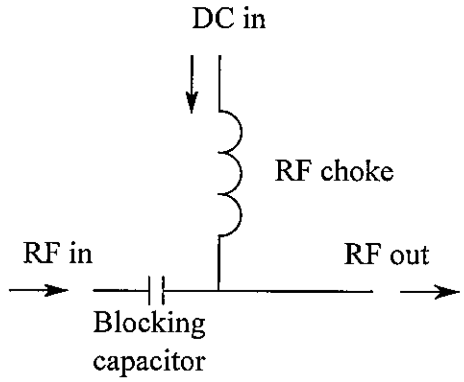

Figure 3.13: A bias Tee equivalent circuit.

parasitics, so another option is to design the bias lines with the characteristics of a low-pass filter, as will be discussed in section 8.6. In order to satisfy the second requirement, a DC blocking capacitor needs to be added to the circuit in series with the RF signal path. A bias tee is similar to a single unit cell of an artificial transmission line of the type discussed in section 1.7, so performance could in principle be improved by adding more sections to this circuit. Capacitors are not ideal shorts, nor are inductors ideal opens at microwave frequencies (they have parasitics), so they need to be taken into account in microwave frequency designs. More details about biasing will be given in Lecture 8.

3.6 Measuring Reflection Coefficients: The Slotted Line

We have seen that time-average power is virtually the only thing about a microwave signal that can be easily measured. In this section, we will see one way that such power measurements can be used indirectly to measure voltages, impedances and reflection coefficients. This technique uses a slotted-line configuration, which enables direct sampling of the electric field amplitude (via diode detection) of a standing wave. In modern labs, network analyzers are used to measure impedances and S-parameters. The slotted line is now mainly of historical interest, except at high millimeter-wave frequencies or when the expense of a network analyzer cannot be met. However, doing a few slotted-line measurements can help you get a better feeling for some quantities than staring at the network analyzer screen.

The slotted line is a coax or waveguide section that has a longitudinal slot into which a movable probe with a diode detector is inserted. There is a generator at one end of the line, and the unknown load terminates the line at the other end. The probe is a needle-like small post that acts as a receiving antenna and samples the electric field. (You will understand better how this works when we study antennas later.)

In slotted-line measurements, we want to find the unknown load impedance, or what is equivalent, its complex reflection coefficient

$$\rho = |\rho|e^{j\theta}. \tag{3.23}$$

We measure the SWR on the line and the distance from the load to the first voltage minimum l_{min}. We need to measure two quantities, since the load impedance is a complex number with both an amplitude and a phase. From the SWR, we obtain the magnitude of the reflection coefficient as

$$|\rho| = \frac{SWR-1}{SWR+1}. \tag{3.24}$$

We know that the voltage minimum occurs for $e^{j(\theta-2\beta l)} = -1$; that is, when

$$\theta = \pi + 2\beta l_{min}. \tag{3.25}$$

(Note: Any multiple of $\lambda/2$ can be added to l_{min} without changing the result, since the voltage minima repeat every $\lambda/2$.) In conclusion, by measuring the SWR and l_{min}, we can find both $|\rho|$ and θ, and therefore find ρ from (3.23). From this, we can find the unknown complex load impedance to be

$$Z = Z_0\frac{1+\rho}{1-\rho}. \tag{3.26}$$

We will illustrate the procedure by means of an example.

3.6.1 Example

We will consider the example of a load connected to a $50\,\Omega$ air-filled coaxial slotted line. Any slotted line measurement must be preceded by a calibration step: the location of the *proxy planes* that are an integer number of half wavelengths from the load position. We do this by placing a short circuit at the load position. This results in a large SWR on the line with sharply defined voltage minima, as shown in Fig. 3.14(a). On some arbitrarily positioned scale along the axis of the line, the voltage minima are observed at $z = 0.2$, 2.2, and 4.2 cm, which are the locations of the proxy planes. [Question: What are the wavelength and the frequency equal to?]

The short is next replaced by the unknown load, for which the SWR is measured to be 1.5 and the voltage minima (not as sharp as when the short was connected) are found at $z = 0.72$, 2.72, and 4.72 cm, as shown in Fig. 3.14(b). From these voltage minima, knowing that they repeat every $\lambda/2$, we can find the distance $l_{min} = (4.2 - 2.72)\,\text{cm} = 1.48\,\text{cm} = 0.37\lambda$. Then

$$\begin{aligned}
|\rho| &= \frac{1.5-1}{1.5+1} = 0.2 \\
\theta &= \pi + 1.48\,\text{cm}\cdot 2\cdot\frac{2\pi}{4\,\text{cm}} = 7.7911\,\text{rad} = 446.4^\circ = 86.4^\circ \\
\rho &= 0.2\, e^{j86.4^\circ} = 0.0126 + j0.1996 \\
Z &= 50\frac{1+\rho}{1-\rho} = 47.3 + j19.7\Omega
\end{aligned}$$

It is clear that the accuracy of this technique will be dependent on how accurately we can measure distance along the slotted line.

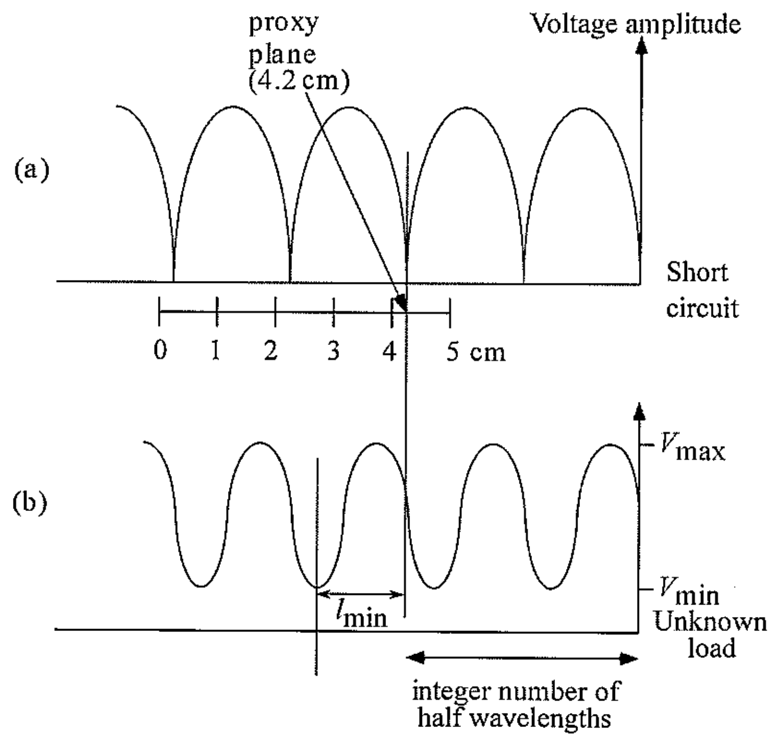

Figure 3.14: Voltage standing waves on a line terminated in a short (a) and an unknown load (b).

3.7 Practice questions

1. You are given the electric and magnetic field vectors of a wave. How do you find the wave power flowing through a given surface? What is the equivalent for a TEM mode on a transmission line?

2. What kind of power levels, in mW, does the range between -110 dBm and +100 dBm correspond to?

3. How do bolometers measure power?

4. How does a bolometer bridge work? Why is a bridge used?

5. What is the difference between a bolometer power measurement and a diode power measurement?

6. Can the diode power measurement give you information about the phase of the wave?

7. When is the square-law approximation for the diode I-V curve valid?

8. Sketch a Schottky diode equivalent circuit and explain what the elements are. Which elements, based on physical reasoning, depend on the DC bias point?

9. In which situations would you use a thermistor, thermocouple or diode for microwave power measurements?

10. What are the advantages and disadvantages of a thermistor over a thermocouple sensor?