Chapter 10

WDM Concepts and Components

Outline

- Operational Principles of WDM
- Passive Coupler Components
  - The 2×2 Fiber Coupler
  - Scattering Matrix Representation
  - The 2×2 Waveguide Coupler
  - The $N \times N$ Star Couplers
  - Mach-Zehnder Interferometer
- Isolators and Circulators
- Fiber Grating Filters

Operational Principles of WDM

- Figure 10-1 depicts the attenuation of light in silica SMF. Two low-loss regions are extended over the wavelengths ranging from 1270 to 1350 nm (the 1310-nm window) and from 1480 to 1600 nm (the 1550-nm window).

- Differentiating the relationship $c = \nu\lambda$, we have, for $\Delta\lambda << \lambda^2$

$$|\Delta \nu| = \frac{c}{\lambda^2}|\Delta\lambda| \qquad (10\text{-}1)$$

  where the deviation in frequency $\Delta\nu$ corresponds to the wavelength deviation $\Delta\lambda$ around λ.

- From Eq. (10-1), the optical bandwidth is $\Delta\nu$ = 14-THz for a usable spectral band $\Delta\lambda$ = 80-nm in the 1310-nm window. Similarly, $\Delta\nu$ = 15-THz for a usable spectral band $\Delta\lambda$ = 120-nm in the 1550-nm window.

Operational Principles of WDM (2)

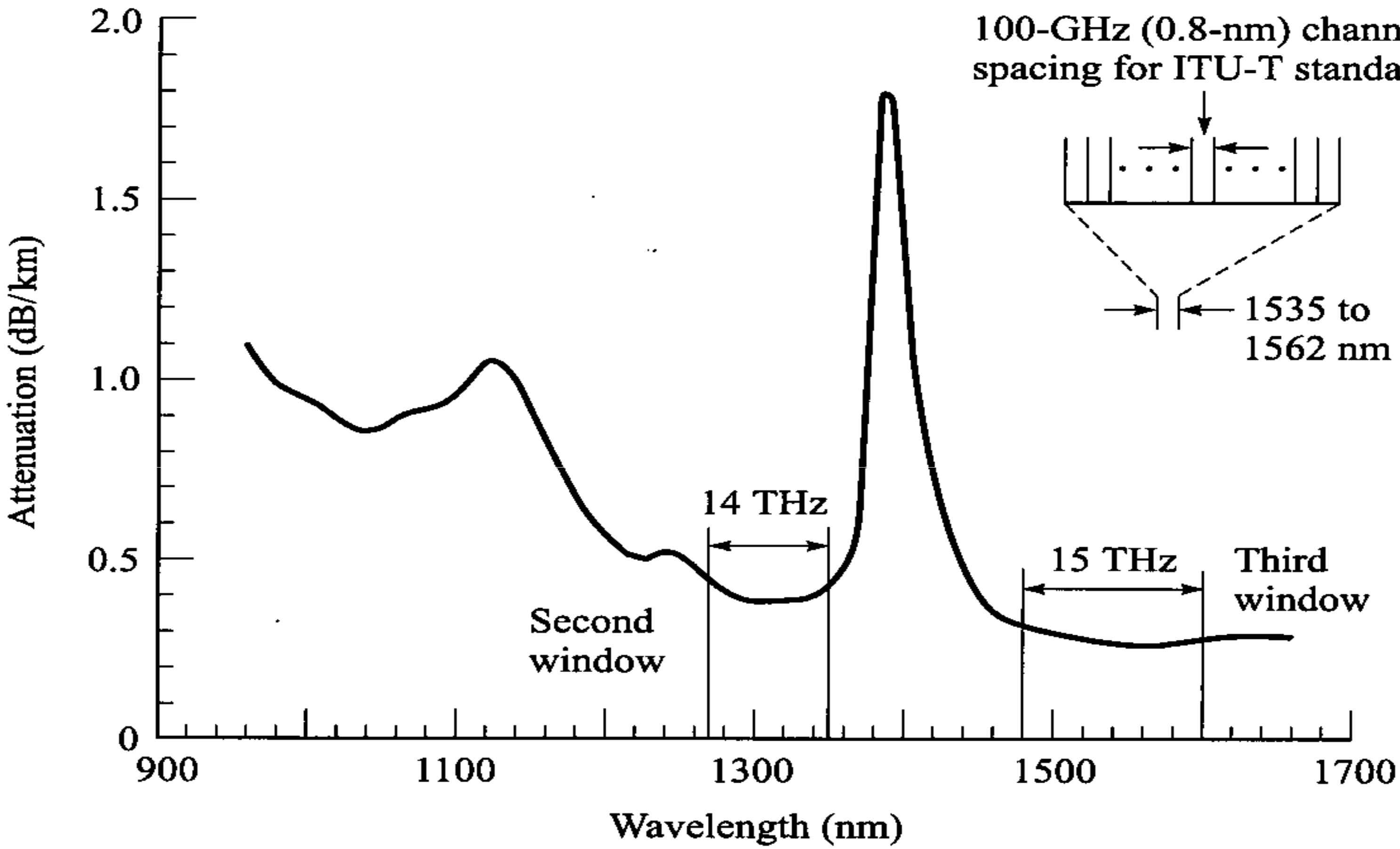


Figure 10-1. The transmission bandwidths in the 1310-nm and 1550-nm windows. The ITU-T standard for WDM specifies channels with 100-GHz spacings.

Operational Principles of WDM (3)

Example 10-2 If one takes a spectral band of 0.8 nm (or equivalently, a mean frequency spacing of 100 GHz) within which narrow-linewidth lasers are transmitting, then one can send 50 independent signals in the 1525-to-1565-nm band on a single fiber.

- The ITU-T Recommendation G.692 specifies selecting the channels from a grid of frequencies referenced to 193.100-THz (1552.524-nm) and spacing them 100-GHz (0.8-nm at 1552-nm) apart. Alternative spacings include 50-GHz (0.4-nm) and 200-GHz (1.6-nm).

Operational Principles of WDM (4)

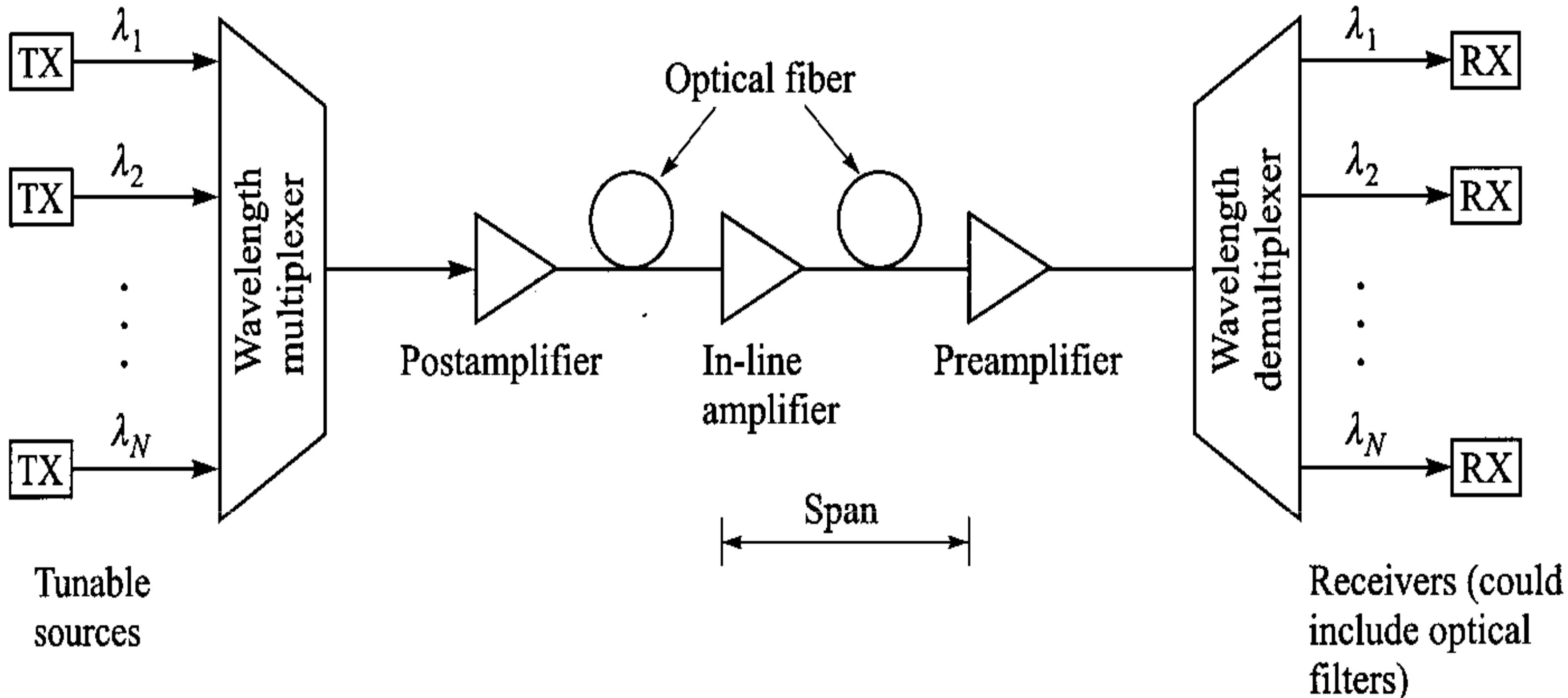


Figure 10-2. A typical WDM network containing various types of optical amplifier.

• At the transmitting end, a *multiplexer* is needed to combine several optical outputs into a serial spectrum of closely spaced wavelength signals and couple them onto a single fiber.
• At the receiving end, a *demultiplexer* is required to separate the optical signals into appropriate detection channels for signal processing.

Operational Principles of WDM (5)

- At the transmitting end, the basic design challenge is to have the multiplexer provide a low-loss path from each optical source to the multiplexer output.
- The optical signals that are combined generally do not emit significant amount of optical power outside the designated channel spectral width, interchannel crosstalk factors therefore are relatively unimportant at the transmitting end.
- To prevent spurious signals from entering a receiving channel (i.e., to give good channel isolation of the different wavelengths being used), the demultiplexer must exhibit narrow spectral operation, or very stable optical filters with sharp wavelength cutoffs must be used. In general, a -10 dB level is not satisfactory, whereas a level of -30 dB is acceptable.

Passive Coupler Components

- Figure 10-3 shows a generic star coupler. A common fabrication method for an *N* x *N* splitter is to fuse together the cores of *N* single-mode fibers over a length of a few millimeters.
- The optical power inserted through one of the *N* fiber entrance ports gets divided uniformly into the cores of the *N* output fibers through evanescent power

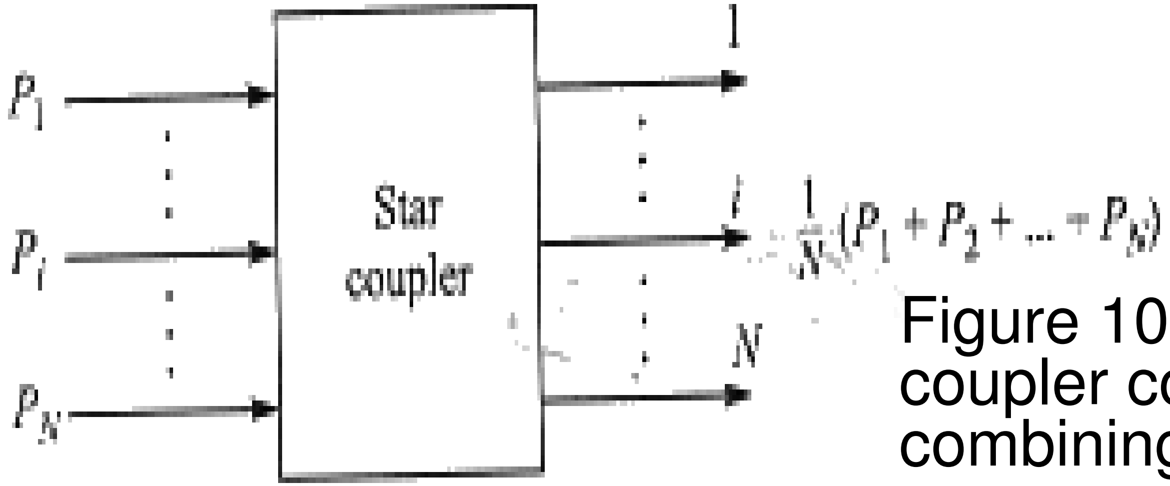


Figure 10-3. Basic star coupler concept for combining or splitting optical powers.

The 2 × 2 Fiber Coupler

- The 2 × 2 coupler is fabricated by fusing together two SMFs over a uniform section of length W, as shown in Fig. 10-5.

- Each input and output fiber has a long tapered section of length L. The total draw length is $\mathcal{L} = 2L + W$. This device is known as a *fused biconical tapered coupler*.

- As the input light P_o propagates into the coupling region W, there is a significant decrease in the V number owing to the reduction in the ratio r/λ [see Eq. (2-58)], where r is the reduced fiber radius.

- By making the tapers very gradual, only a negligible fraction of the incoming optical power is reflected back into either of the input ports. Thus, these devices are also known as *directional couplers*.

The 2 × 2 Fiber Coupler

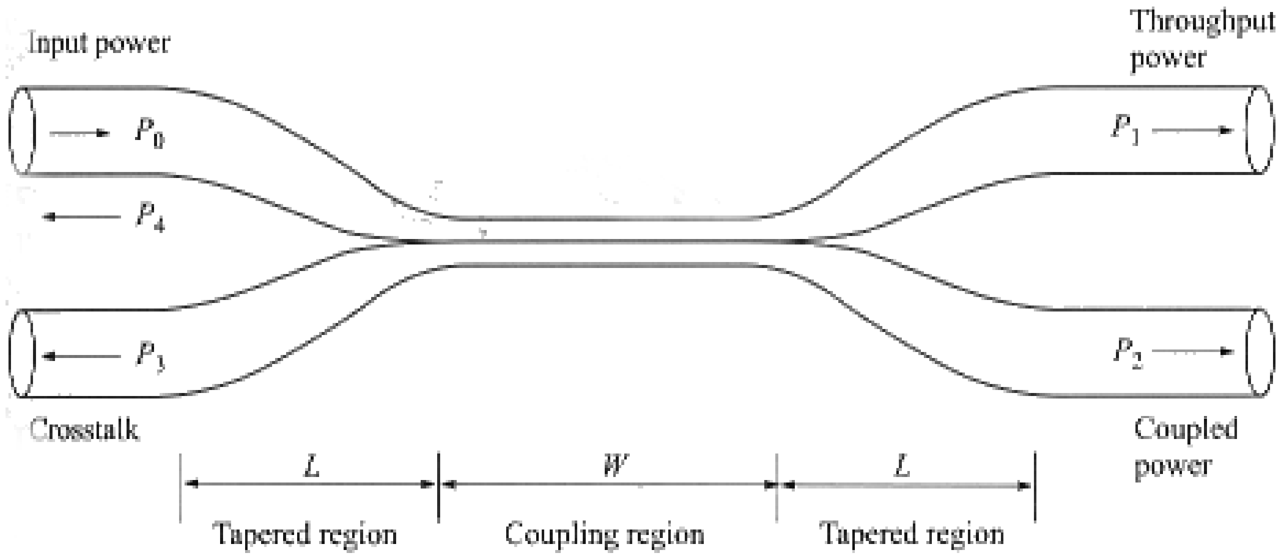


Figure 10-5. Cross-sectional view of a fused-fiber coupler having a coupling region *W* and two tapered regions of length *L*. The total span $\mathcal{L}$ = 2*L*+*W* is the coupler draw length.

The 2 × 2 Fiber Coupler (2)

- Coupled optical power varied through three parameters:

    a). the axial length of the coupling region;
    b). the size of the reduced radius *r* in the coupling region; and
    c). the difference Δr in the radii of the two fibers in the coupling region.

- In making a fused fiber coupler, only *L* and *r* change as the coupler is elongated.

The 2 × 2 Fiber Coupler (3)

- The power P_2 coupled from one fiber to another over an axial distance z is

$$P_2 = P_0 \sin^2(\kappa z)$$

where κ is the *coupling coefficient*.

- By conservation of power, for identical-core fibers we have

$$\begin{aligned} P_1 &= P_0 - P_2 \\ &= P_0[1\text{-}\sin^2(\kappa z)] \\ &= P_0 \cos^2(\kappa z) \end{aligned}$$

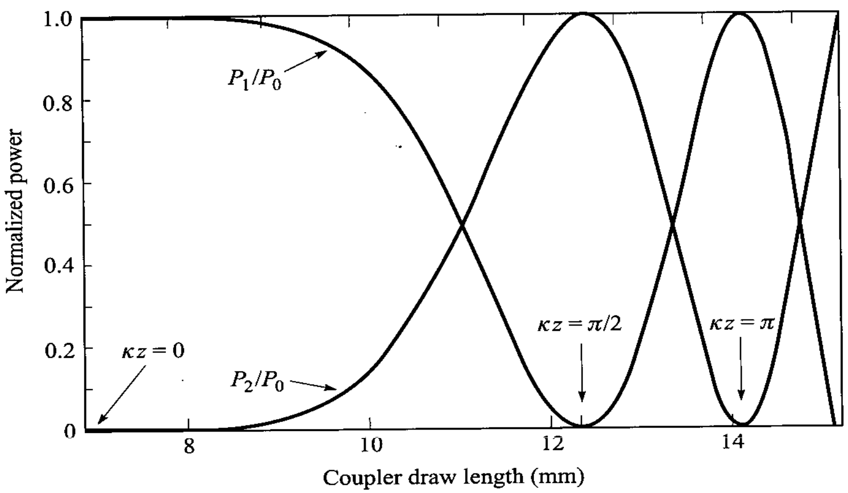
The 2 × 2 Fiber Coupler (4)

- The phase of the driven fiber always lags 90° behind the phase of the driving fiber, as Fig. 10-6a illustrates.

- The lagging phase relationship continues until at a distance that satisfies $\kappa z = \pi/2$, all of the power has been transferred from fiber 1 to fiber 2.
- Now fiber 2 becomes the driving fiber, so that for $\pi/2 \leq \kappa z \leq \pi$ the phase in fiber 1 lags behind that in fiber 2, and so on.

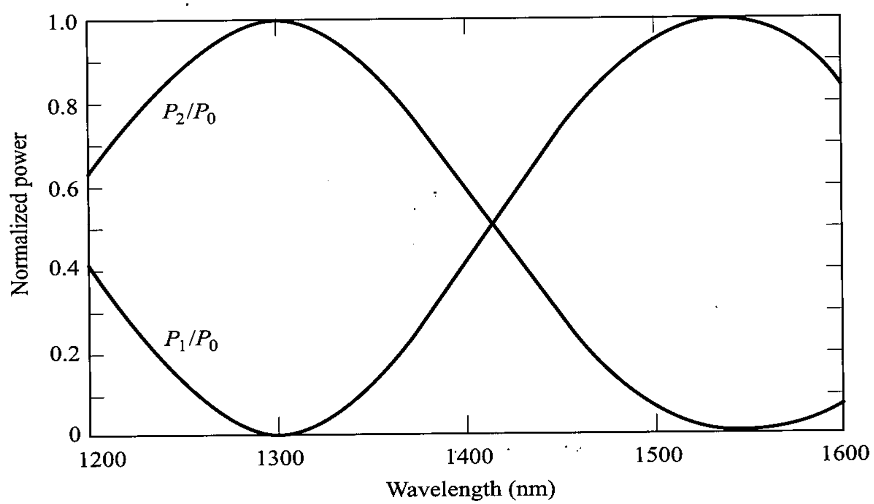
- Figure 10-6b shows how κ varies with wavelength for the final 15mm-long coupler.
- Different-performance couplers can be made by varying the parameters W, L, r, and Δr for a specific wavelength λ.

The 2 × 2 Fiber Coupler (5)



(*a*)

Figure 10-6(a). Normalized coupled powers P_2/P_0 and P_1/P_0 as a function of the coupler draw length for a 1300-nm power level P_0 launched into fiber 1.



(*b*)

Figure 10-6(b). Dependence on wavelength of the coupled powers in the completed 15mm-long coupler.

The 2×2 Fiber Coupler (6)

- Referring to Fig. 10-5, with P_0 being the input power and P_1 and P_2 the output powers, then

$$\text{Splitting Ratio (coupling ratio)} = \frac{P_2}{P_1 + P_2} \times 100\%$$

- By adjusting the parameters so that power is divided evenly, with half of the input power going to each output, one creates a 3-*dB coupler*.
- A coupler could also be made in which almost all the optical power at 1500-nm goes to one port and almost all the energy around 1300-nm goes to the other port.

The 2 × 2 Fiber Coupler (7)

- The *excess loss* is defined as the ratio of the input power to the total output power. In decibels, the excess loss for a 2x2 coupler is

$$\text{Excess loss} = 10\log\left(\frac{P_0}{P_1 + P_2}\right) \qquad (10.5)$$

- The *insertion loss* refers to the loss for a particular port-to-port path. For the path from input port i to output port j, we have

$$\text{Insertion loss} = 10\log\left(\frac{P_i}{P_j}\right) \qquad (10.6)$$

- *Crosstalk* or *return loss* measures the degree of isolation between the input at one port and the optical power scattered back into the other input port. It is a measure of the optical power level P_3 shown in Fig. 10-4:

$$\text{Return loss} = 10\log\left(\frac{P_3}{P_0}\right) \qquad (10.7)$$

The 2 × 2 Fiber Coupler (8)

Example 10-3 A 2 x 2 biconical tapered coupler has an input optical power of $P_o = 200\ \mu W$. The output power at the other three ports are $P_1 = 90\ \mu W$, $P_2 = 85\ \mu W$, and $P_3 = 6.3\ nW$.

$$\text{Coupling ratio} = \left(\frac{85}{90+85}\right)\times 100\% = 48.6\%$$

$$\text{Excess loss} = 10\log\left(\frac{200}{90+85}\right) = 0.58\text{dB}$$

$$\text{Insertion loss(port 0 to port 1)} = 10\log\left(\frac{200}{90}\right) = 3.47\text{dB}$$

$$\text{Insertion loss(port 0 to port 2)} = 10\log\left(\frac{200}{85}\right) = 3.72\text{dB}$$

$$\text{Crosstalk} = 10\log\left(\frac{6.3\times 10^{-3}}{200}\right) = -45\text{dB}$$

Scattering Matrix

- As shown in Fig. 10-6, all-fiber or integrated-optics devices can be analyzed in terms of the *scattering matrix* S, which defines the relationship between input field $a = [a_1\ a_2]^T$ and output field $b = [b_1\ b_2]^T$: $b = Sa$,

where $S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$. (10-8)

- Here, $s_{ij} = |s_{ij}|\exp(j\phi_{ij})$ represents the *coupling coefficient* of optical power transfer from input port j to output port i, with $|s_{ij}|$ being the magnitude of s_{ij} and ϕ_{ij} being its phase at port i relative to port j.
- It follows that $s_{12} = s_{21}$. (10-9)

Scattering Matrix (2)

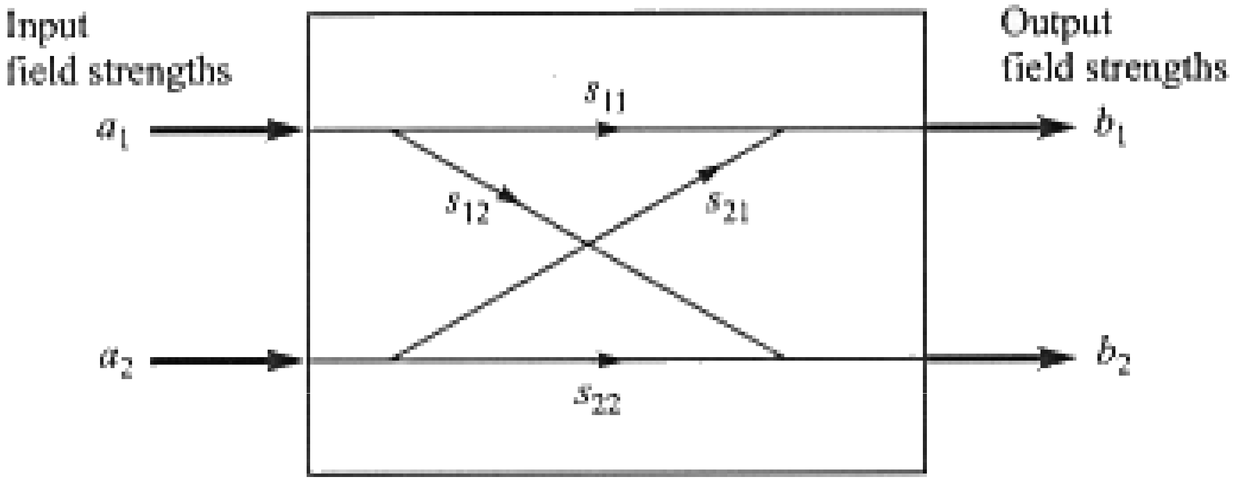


Figure 10-6. Generic 2x2 guided-wave coupler. Here, a_i and b_j represent the field strengths of input port i and output port j, respectively, and the s_{ij} are the scattering matrix parameters.

Scattering Matrix (3)

- Also, the sum of the output intensities I_o must equal the sum of the input intensities I_i :

$$I_o = b_1{}^*b_1 + b_2{}^*b_2 = I_i = a_1{}^*a_1 + a_2{}^*a_2$$

or

$$\underline{b}^+\underline{b} = \underline{a}^+\underline{a} \qquad (10\text{-}10)$$

- where the superscript '*' means the complex conjugate and the superscript '+' indicates the transpose conjugate.

- Substituting Eqs. (10-8) and (10-9) into (10-10) yields

$$s_{11}{}^*s_{11} + s_{12}{}^*s_{12} = 1 \qquad (10\text{-}11)$$

$$s_{11}{}^*s_{12} + s_{12}{}^*s_{22} = 0 \qquad (10\text{-}12)$$

$$s_{22}{}^*s_{22} + s_{12}{}^*s_{12} = 1 \qquad (10\text{-}13)$$

Scattering Matrix (4)

- Assume that the fraction (1-ε) of the optical power from input port 1 appears at output port 1, with the remainder ε going to output port 2, then we have

$$s_{11} = (1\text{-}\varepsilon)^{1/2} \ \text{ with } \phi_{11} = 0$$
$$s_{22} = (1\text{-}\varepsilon)^{1/2} \ \text{ with } \phi_{22} = 0.$$

- Inserting s_{11} and s_{22} into Eq. (10-12) and
  letting $s_{12} = |s_{12}|\exp(j\phi_{12})$,
  we have $\exp(j2\phi_{12}) = -1$ (10-14)
  which holds when

$$\phi_{12} = (2n+1)\pi/2, \quad \text{where } n = 0, 1, 2, \ldots \tag{10-15}$$

- so that the scattering matrix from Eq. (10-8) becomes

$$\mathbf{S} = \begin{bmatrix} \sqrt{1-\varepsilon} & j\sqrt{\varepsilon} \\ j\sqrt{\varepsilon} & \sqrt{1-\varepsilon} \end{bmatrix} \tag{10-16}$$

Scattering Matrix (5)

<u>Example 10-5 :</u> For a 3-dB coupler, we have $\varepsilon = 0.5$ and the output field intensities $E_{\text{out},1}$ and $E_{\text{out},2}$ can be found from the input intensities $E_{\text{in},1}$ and $E_{\text{in},2}$ and the scattering matrix in Eq. (10-16):

$$\begin{bmatrix} E_{out,1} \\ E_{out,2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \begin{bmatrix} E_{in,1} \\ E_{in,2} \end{bmatrix}$$

Letting $E_{\text{in},2} = 0$, we have $E_{\text{out},1} = (1/2^{1/2})E_{\text{in},1}$, and $E_{\text{out},2} = (j/2^{1/2})E_{\text{in},1}$. The output powers are given by

$$P_{\text{out},1} = E_{\text{out},1} \cdot E^*_{\text{out},1} = E^2_{\text{in},1}/2 = P_{\text{o}}/2$$
$$P_{\text{out},2} = E_{\text{out},2} \cdot E^*_{\text{out},2} = E^2_{\text{in},2}/2 = P_{\text{o}}/2$$

so that half the input power appears at each output of the coupler.

The 2 × 2 Waveguide Coupler

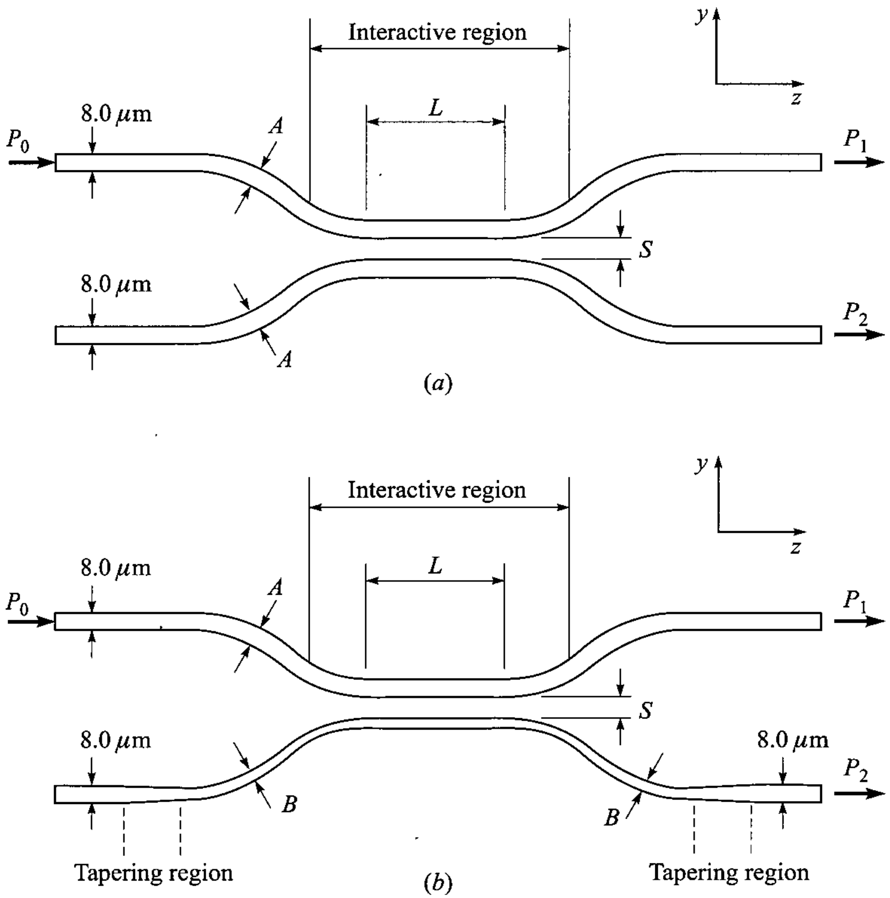


Figure 10-9(a).
Cross-sectional top views of a uniformly symmetric directional waveguide coupler with both guides having a width A = 8μm,

Figure 10-9(b).
A uniformly asymmetric directional coupler in which one guide has a narrower width B in the coupling region.

The 2×2 Waveguide Coupler (2)

- Figure 10-9 shows two types of 2×2 waveguide couplers. Waveguide devices have intrinsic wavelength dependence in the coupling region.
- The degree of interaction between the guides can be varied through the guide width w, the gap s between the guides, and the refractive index n_1 between the guides.
- Due to the scattering and absorption losses, the propagation constant β_z of real waveguides is given by $\beta_r + j\alpha/2$. The power in both guides decreases by the factor exp($-\alpha z$).
- Losses in semiconductor waveguide devices fall in the range $0.2 < \alpha < 1$ dB/cm, which substantially is higher than the nominal 0.1-dB/km losses in fused-fiber couplers.

The 2×2 Waveguide Coupler (3)

- The transmission characteristics of a symmetric coupler can be expressed through the coupled-mode theory approach to yield

$$P_2 = P_o \sin^2(\kappa z) \exp(-\alpha z) \qquad (10\text{-}18)$$

- where the coupling coefficient is

$$\kappa = \frac{2\beta_y^2 q e^{-qs}}{\beta_z w(q^2 + \beta_y^2)} \qquad (10\text{-}19)$$

- This is a function of the waveguide propagation constants β_y and β_z, the gap width w and separation s,
  and the extinction coefficient q in the y direction outside the waveguide.

$$q^2 = \beta_z^2 - k_1^2 \qquad (10\text{-}20)$$

The 2 × 2 Waveguide Coupler (4)

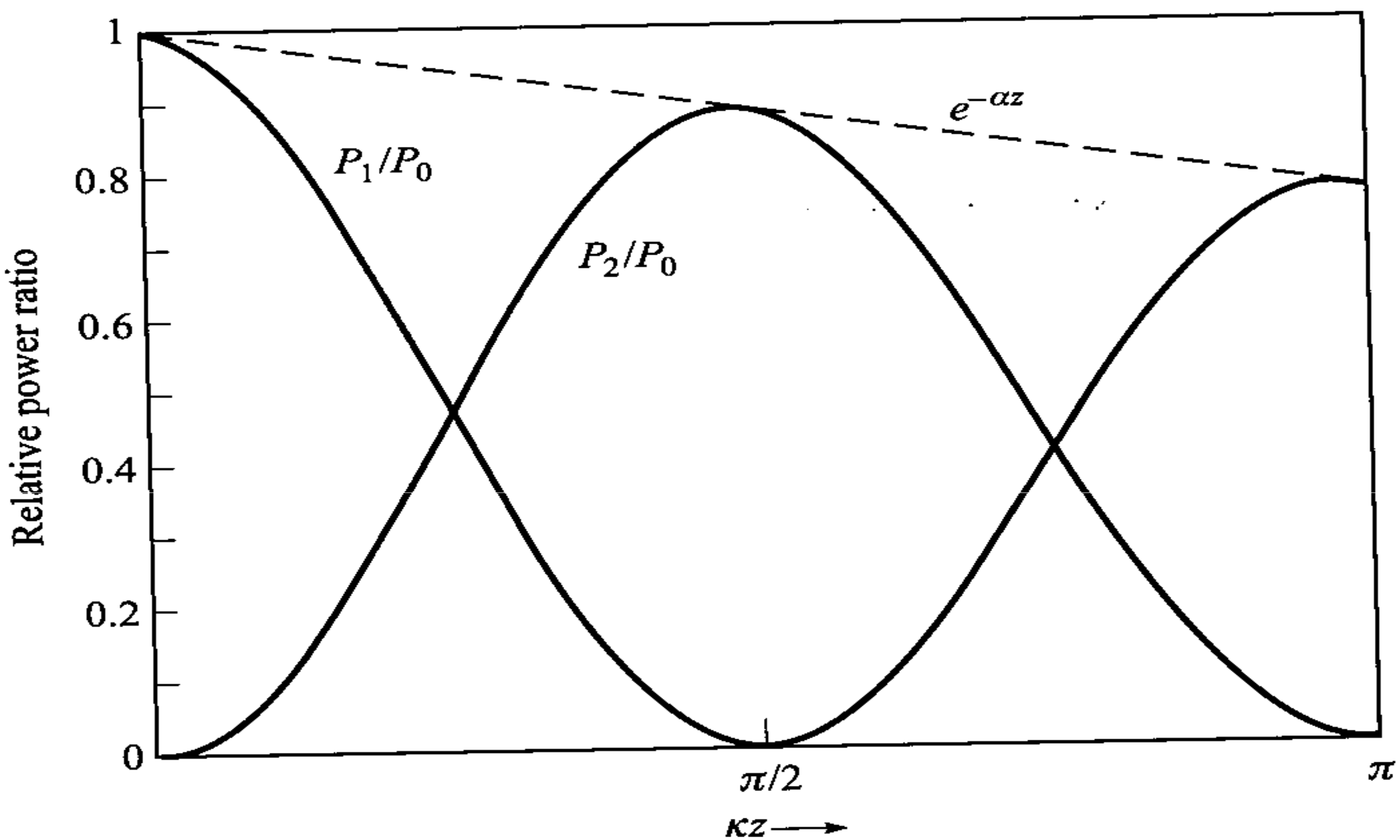


Figure 10-10. Through-path and coupled power distributions as a function of the guide length in a symmetric 2x2 guided-wave coupler with κ = 0.6 mm^{-1} and α = 0.02 mm^{-1}.

The 2 × 2 Waveguide Coupler (5)

- The theoretical power distribution as a function of the guide length is as shown in Fig. 10-10, where $\kappa = 0.6$ mm^{-1} and $\alpha = 0.02$ mm^{-1}.

- Analogous to the fused-fiber coupler, complete power transfer to the second guide occurs when the guide length L is

$$L = [(m+1)/\kappa](\pi/2), \quad \text{with } m = 0, 1, 2, \ldots \quad (10\text{-}21)$$

- Since κ is almost monotonically proportional to wavelength, the coupling ratio P_2/P_0 rises and falls sinusoidally from 0 to 100% as a function of wavelength, as Fig. 10-11 illustrates generically.

The 2 × 2 Waveguide Coupler (6)

- When the two guides do not have the same widths, as shown in Fig. 10-7b, the amplitude of the coupled power is dependent on wavelength, and the coupling ratio becomes

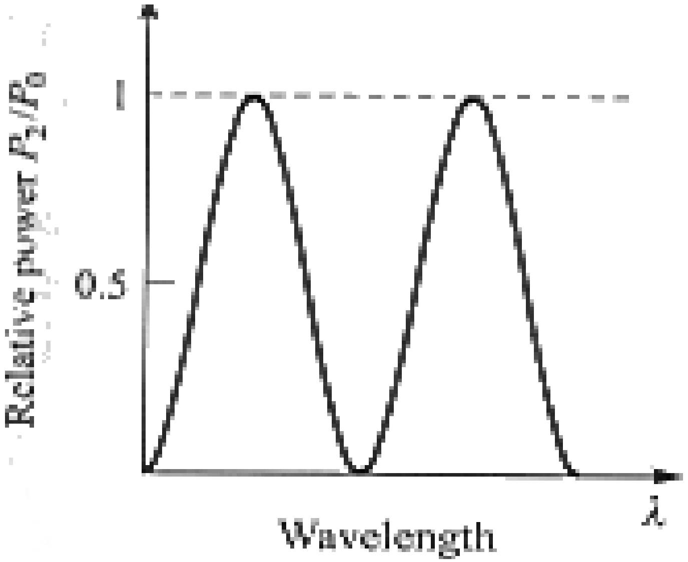
$$P_2/P_o = (\kappa^2/g^2)\sin^2(gz)\exp(-\alpha z) \qquad (10\text{-}22)$$

- where

$$g^2 = \kappa^2 + (\Delta\beta/2)^2 \qquad (10\text{-}23)$$

with $\Delta\beta$ being the phase difference between the two guides in the z direction.

The 2 × 2 Waveguide Coupler (7)



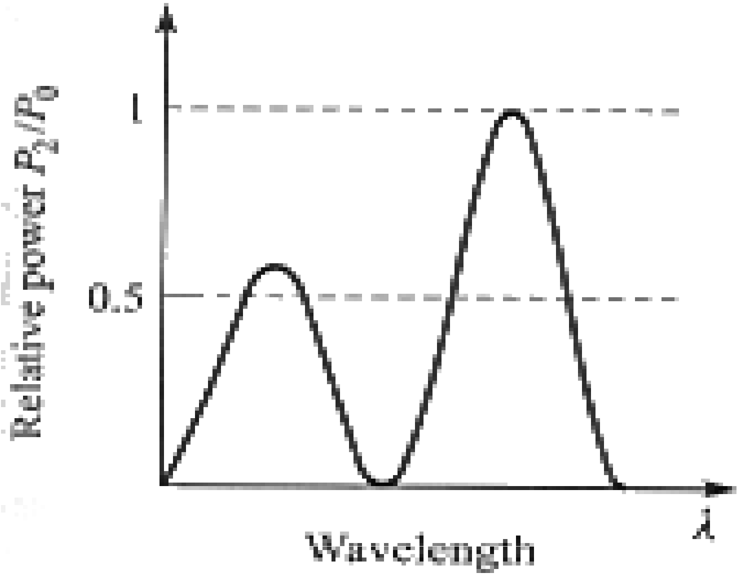


Figure 10-11 **Figure 10-12**

Wavelength response of the coupled power P_2/P_0 in the symmetric and asymmetric 2x2 guided-wave coupler

Example 10-6: A symmetric waveguide coupler has a coupling coefficient $\kappa = 0.6\ \text{mm}^{-1}$. Using Eq. (10-21), we find the coupling length for $m = 1$ to be $L =$ 5.24 mm.

The 2×2 Waveguide Coupler (8)

- One can fabricate devices that have a flattened response in which the coupling ratio is < 100% in a specific desired wavelength range, as shown in Fig. 10-12.
- The wave-flattened response at the lower wavelength results from suppression by the amplitude term κ^2/g^2.
- Note that when $\Delta\beta = 0$, Eq. (10-22) reduces to the symmetric case given by Eq. (10-18).

The $N \times N$ Star Couplers

- The principal role of star couplers is to combine the powers from N inputs and divide them equally among M output ports.
- The fiber-fusion technique has been a popular construction method for $N \times N$ star couplers.
- Figure 10-13 shows a generic 4×4 fused-fiber star coupler.
- In a star coupler, the *splitting loss* is given by

$$\text{Splitting Loss} = -10\log(1/N) = 10\log N \qquad (10\text{-}24)$$

- For a single input power P_{in} and N output powers, the *excess loss* is given by

$$\text{excess loss} = 10\log\left(\frac{P_{in}}{\sum_{i=1}^{N} P_{out,i}}\right) \qquad (10\text{-}25)$$

The $N \times N$ Star Couplers (2)

- The insertion loss and crosstalk can be found from Eqs. (10-6) and (10-7), respectively.
- Figure 10-14 shows an 8×8 device formed by twelve 3-dB couplers. N has to be the form of $N = 2^n$ with $n \geq 1$.
- If an extra node needs to be added to a fully connected $N \times N$ network, the $N \times N$ star needs to be replaced by a $2N \times 2N$ star, thereby leaving $2(N-1)$ new ports being unused.
- Alternatively, one extra 2×2 coupler can be used at one port to get $N + 1$ outputs. However, these two new ports have an additional 3-dB loss.

The $N \times N$ Star Couplers (3)

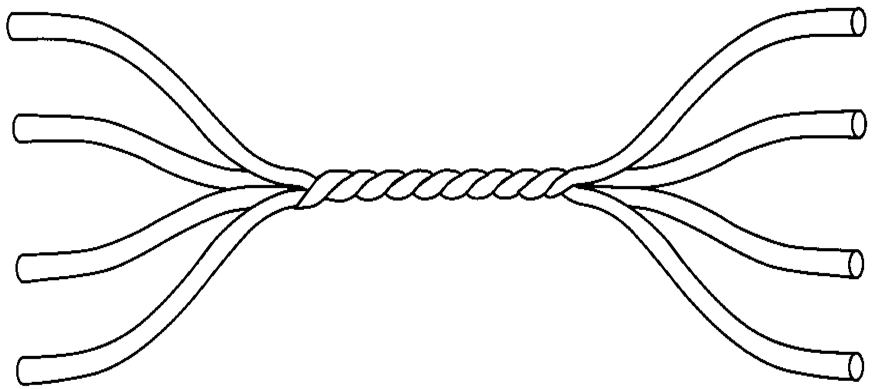
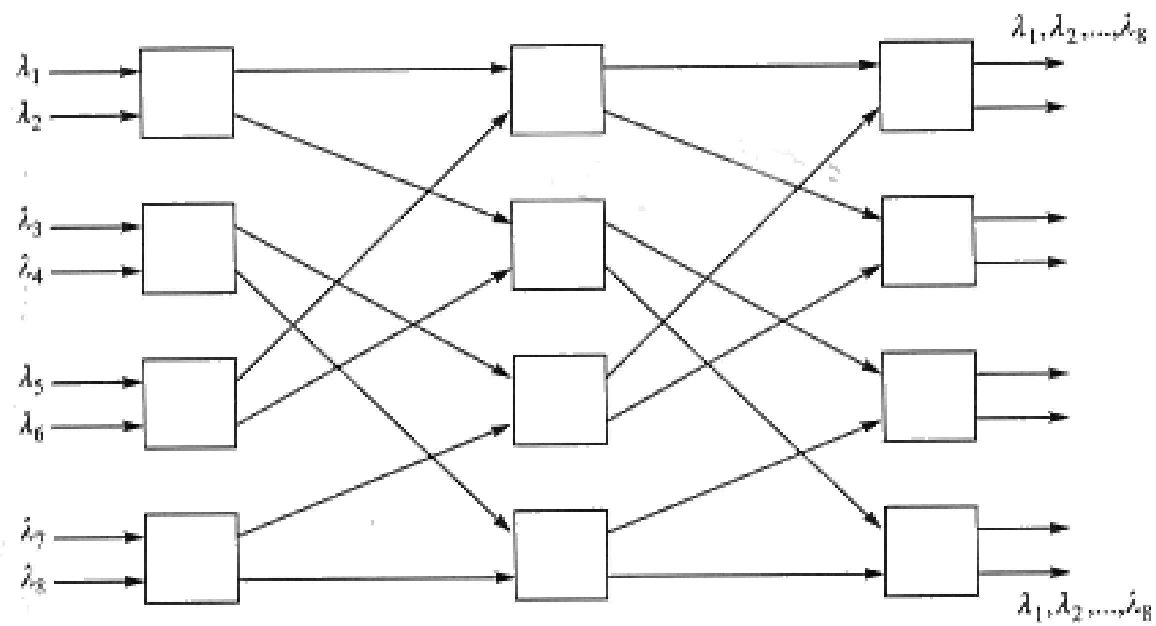


Figure 10-13. Generic 4 × 4 fused-fiber star coupler fabricated by twisting, heating, and pulling on four fibers to fuse them together.

Figure 10-14. Example of an 8x8 star coupler formed by interconnecting twelve 2x2 couplers.



The $N \times N$ Star Couplers (4)

- As can be deduced from Fig. 10-14, the number of 3-dB couplers needed to construct an $N\times N$ star is

$$N_c = (N/2)\log_2 N$$
$$= (N/2)(\log N/\log 2) \qquad (10\text{-}26)$$

  since there are $N/2$ elements in the vertical direction and $\log_2 N$ elements horizontally.

- If the fraction of power traversing each 3-dB coupler element is F_T, with $0 \le F_T \le 1$, then the *excess loss* in decibels is

$$\text{Excess Loss} = -10\log[F_T^{\log_2 N}] \qquad (10\text{-}27)$$

The $N \times N$ Star Couplers (5)

- The splitting loss for this star is given by Eq. (10-24).
- The total loss a signal passes through the $\log_2 N$ stages of the $N\times N$ star and gets divided into N outputs is

$$
\begin{aligned}
\text{total loss} &= \text{splitting loss} + \text{excess loss} \\
&= -10\log\left[F_T^{\log_2 N} / N\right] \\
&= -10[\log N(\log F_T/\log 2) - \log N] \\
&= \;10\log(1 - 3.322\log F_T)\log N \qquad (10\text{-}28)
\end{aligned}
$$

- This shows that the loss increases logarithmically with N.

The $N \times N$ Star Couplers (6)

Example 10-5 Consider a commercially available 32 x 32 single-mode coupler made from a cascade of 3-dB fused-fiber 2 x 2 couplers, where 5 % of the power is lost in each element.

- From Eq. (10-27), the excess loss is

$$\text{Excess Loss} = -10 \log(0.95^{\log 32/\log 2}) = 1.1 \text{ dB}$$

- From Eq. (10-24), the splitting loss is

$$\text{Splitting Loss} = 10 \log 32 = 15 \text{ dB.}$$

- Hence, the total loss is 16.1 dB.

Mach-Zehnder Interferometer (MZI) Multiplexers

- In Figure 10-15, the 2 x 2 Mach-Zehnder interferometer (MZI) consists of three stages:
- a 3-dB directional coupler which splits the input signals,
- a central section where one of the waveguides is longer by ΔL to give a wavelength-dependent phase shift between the two arms,
- and another 3-dB coupler which recombines the signals at the output.

Mach-Zehnder Interferometer (MZI) Multiplexers (2)

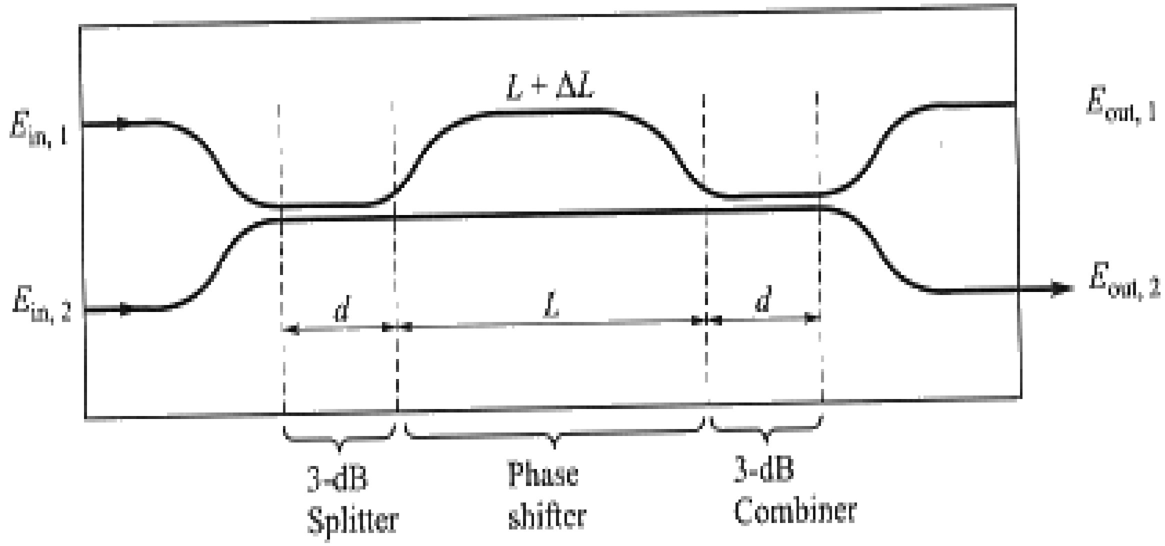


Figure 10-15. Layout of a basic 2 x 2 Mach-Zehnder Interferometer.

- The function of this arrangement is that, by splitting the input beam and introducing a phase shift in one of the paths, the recombined signals will interfere constructively at one output and destructively at the other. The signals then finally emerge from only one output port.

Mach-Zehnder Interferometer (MZI) Multiplexers (3)

- The propagation matrix M_{coupler} for a coupler of length d is

$$\mathbf{M}_{coupler} = \begin{bmatrix} \cos\kappa d & j\sin\kappa d \\ j\sin\kappa d & \cos\kappa d \end{bmatrix} \qquad (10\text{-}29)$$

  where κ is the coupling coefficient.

- Since we are considering 3-dB couplers which divide the power equally, then $2\kappa d = \pi/2$, so that

$$\mathbf{M}_{coupler} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \qquad (10\text{-}30)$$

Note that this matrix is a scattering matrix.

Mach-Zehnder Interferometer (MZI) Multiplexers (4)

- In the central region, the outputs from the two guides have a phase difference $\Delta\phi$ given by

$$\Delta\phi = k_1 L - k_2(L + \Delta L)$$
$$= 2\pi n_1 L/\lambda - 2\pi n_2 (L + \Delta L)/\lambda \qquad (10\text{-}31)$$

- This phase difference can arise either from a different path length or through a refractive index difference.

Mach-Zehnder Interferometer (MZI) Multiplexers (5)

- Take both arms to have the same index $n_1 = n_2 = n_{\text{eff}}$, we can rewrite Eq. (10-31) as

$$\Delta\phi = k\Delta L, \tag{10-32}$$

where $k = 2\pi n_{\text{eff}} / \lambda$.

- For a given phase difference $\Delta\phi$, the propagation matrix $\mathbf{M}_{\Delta\phi}$ for the phase shifter is

$$\mathbf{M}_{\Delta\phi} = \begin{bmatrix} \exp(jk\Delta L/2) & 0 \\ 0 & \exp(-jk\Delta L/2) \end{bmatrix} \tag{10-33}$$

Mach-Zehnder Interferometer (MZI) Multiplexers (6)

- The optical output fields $E_{\text{out},1}$ and $E_{\text{out},2}$ from the two central arms can be related to the input fields $E_{\text{in},1}$ and $E_{\text{in},2}$ *by*

$$\begin{bmatrix} E_{out,1} \\ E_{out,2} \end{bmatrix} = \mathbf{M} \begin{bmatrix} E_{in,1} \\ E_{in,2} \end{bmatrix} \qquad (10\text{-}34)$$

where

$$\mathbf{M} = \mathbf{M}_{coupler} \mathbf{M}_{\Delta\phi} \mathbf{M}_{coupler}$$

$$= j \begin{bmatrix} \sin(k\Delta L/2) & \cos(k\Delta L/2) \\ \cos(k\Delta L/2) & -\sin(k\Delta L/2) \end{bmatrix} \qquad (10\text{-}35)$$

Mach-Zehnder Interferometer (MZI) Multiplexers (7)

- The inputs to the MZI are at different wavelengths: $E_{\mathrm{in},1}$ at λ_1 and $E_{\mathrm{in},2}$ at λ_2.

- From Eq. (10-34), the output fields $E_{\mathrm{out},1}$ and $E_{\mathrm{out},2}$ are each the sum of the contributions from the two input fields:

$$E_{\mathrm{out},1} = j[E_{\mathrm{in},1}(\lambda_1)\sin(k_1\Delta L/2) + E_{\mathrm{in},2}(\lambda_2)\cos(k_2\Delta L/2)] \quad (10\text{-}36)$$

$$E_{\mathrm{out},2} = j[E_{\mathrm{in},1}(\lambda_1)\cos(k_1\Delta L/2) - E_{\mathrm{in},2}(\lambda_2)\sin(k_2\Delta L/2)] \quad (10\text{-}37)$$

where $k_{\mathrm{j}} = 2\pi n_{\mathrm{eff}}/\lambda_{\mathrm{j}}$.

Mach-Zehnder Interferometer (MZI) Multiplexers (8)

- The output powers are found from the light intensity, which is the square of the field strengths. Thus,

$$P_{out,1} = E_{out,1} \cdot E^*_{out,1}$$
$$= \sin^2(k_1 \Delta L/2) P_{in,1} + \cos^2(k_2 \Delta L/2) P_{in,2} \qquad (10\text{-}38)$$

$$P_{out,2} = E_{out,2} \cdot E^*_{out,2}$$
$$= \cos^2(k_1 \Delta L/2) P_{in,1} + \sin^2(k_2 \Delta L/2) P_{in,2} \qquad (10\text{-}39)$$

where $P_{in,i} = |E_{in,i}|^2$.

- In deriving Eqs. (10-38) and (10-39), the cross terms are dropped because the twice optical carrier frequency is beyond the response capability of the photo-detector.

Mach-Zehnder Interferometer (MZI) Multiplexers (9)

- From Eqs. (10-38) and (10-39), we see that if we want all the power from both inputs to leave the same output port (port 2), we need to have

  $k_1\Delta L/2 = \pi$ and $k_2\Delta L/2 = \pi/2$, or

  $$(k_1 - k_2)\Delta L = 2\pi n_{\text{eff}}[(1/\lambda_1)\text{-}(1/\lambda_2)]\Delta L = \pi \qquad (10\text{-}40)$$

- Hence, the length difference in the interferometer arms should be

  $$\Delta L = \{2n_{\text{eff}}[(1/\lambda_1) - (1/\lambda_2)]\}^{-1}$$

  $$= c/(2n_{\text{eff}}\Delta\nu) \qquad (10\text{-}41)$$

  where $\Delta\nu$ is the frequency separation of the two wavelengths.

Mach-Zehnder Interferometer (MZI) Multiplexers (10)

Example 10-8:

- (a) Assume that the input wavelengths of a 2 x 2 silicon MZI are separated by 10-GHz (i.e., $\Delta\lambda$= 0.08nm at 1550nm).

- With $n_{\text{eff}} = 1.5$ in a silicon waveguide, we have that (Eq. 10-41) the waveguide length difference must be

$$\Delta L = (3\text{x}10^8 \text{ m/s})/[2(1.5)10^{10}/\text{s}] = 10 \text{ mm}.$$

- (b) If the frequency separation is 130-GHz (i.e., $\Delta\lambda$= 1nm), then $\Delta L = 0.77$ mm.

Mach-Zehnder Interferometer (MZI) Multiplexers (11)

- Figure 10-15 gives an example for a 4 x 4 multiplexer.
- The inputs to MZI_1 are ν and $\nu+2\Delta\nu$ (which we will call λ_1 and λ_3, respectively), and the inputs to MZI_2 are $\nu+\Delta\nu$ and $\nu+3\Delta\nu$ (which are called λ_2 and λ_4, respectively).
- Since the signals in both interferometers of the 1st stage are separated by $2\Delta\nu$, the path differences satisfy the condition

$$\Delta L_1 = \Delta L_2 = c/2n_{\mathrm{eff}}(2\Delta\nu) \qquad (10\text{-}42)$$

Mach-Zehnder Interferometer (MZI) Multiplexers (12)

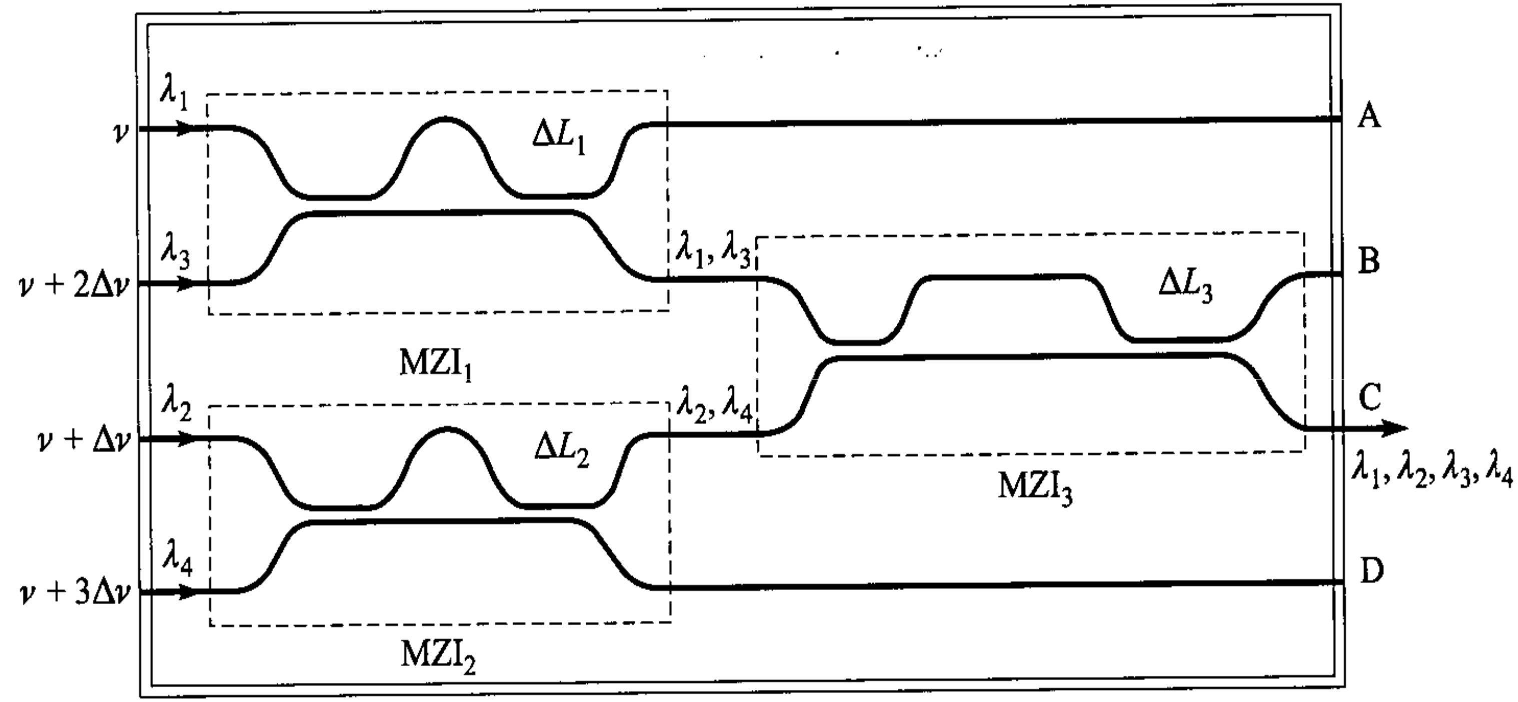


Figure 10-15. Example of a four-channel wavelength multiplexer using three 2 x 2 MZI elements.

Mach-Zehnder Interferometer (MZI) Multiplexers (13)

- In the next stage, the inputs are separated by $\Delta\nu$. Consequently, we need to have

$$\Delta L_3 = c/2n_{\text{eff}}\,\Delta\nu = 2\Delta L_1 \qquad (10\text{-}43)$$

- When these conditions are satisfied, all four input powers will emerge from port C.

Mach-Zehnder Interferometer (MZI) Multiplexers (14)

- For an N-to-1 MZI multiplexer, where $N = 2^n$ with $n \geq 1$, the number of multiplexer stages is n and the number of MZIs in stage j is 2^{n-j}.

- The path difference in an interferometer element of stage j is

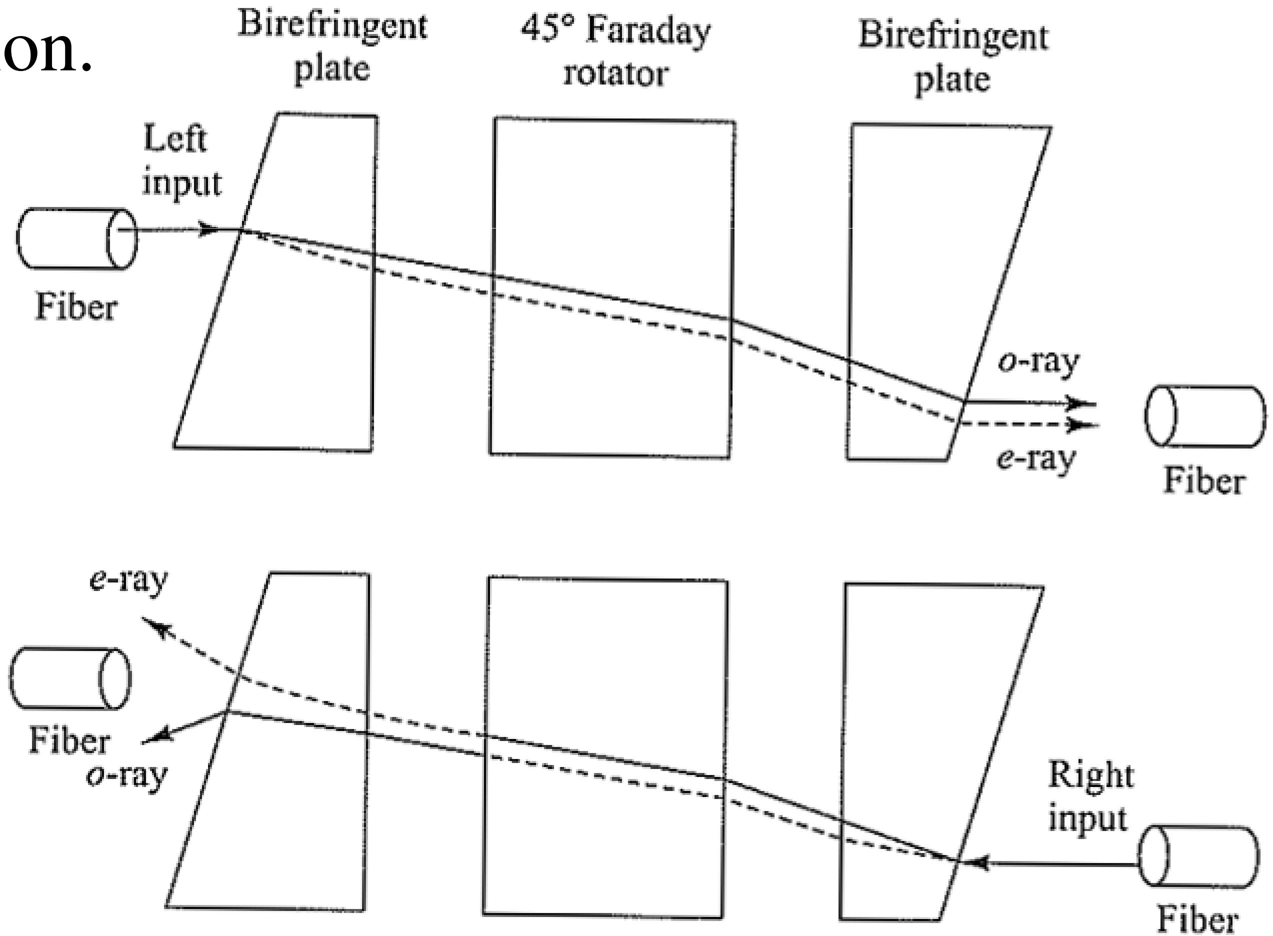
$$\Delta L_{\text{stage-j}} = c/(2^{n-j}\, n_{\text{eff}}\, \Delta\nu) \qquad (10\text{-}44)$$

- The N-to-1 MZI multiplexer can be used as a 1-to-N demultiplexer by reversing the light-propagation direction.

Isolators

- Isolators are devices that allow light to pass through them in only one direction. Note that the Faraday rotator is not reciprocal, resulting in light diverges coming from the other direction.

Product of Leysop



Circulators

- Circulators are devices that direct light sequentially from port to port in only one direction.

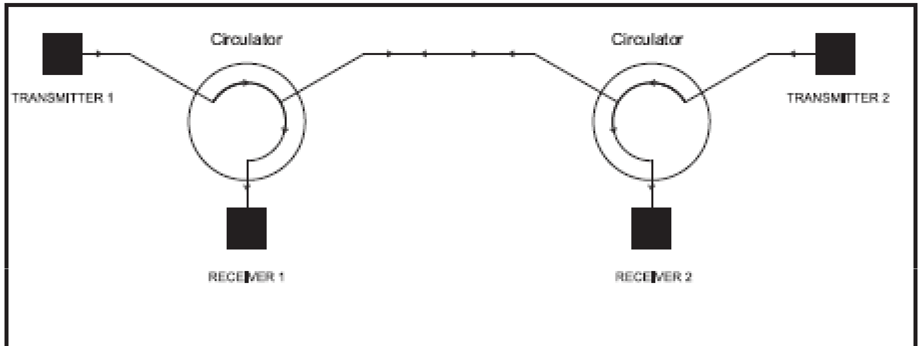


Figure 3: Duplex Transmitter/Receiver System

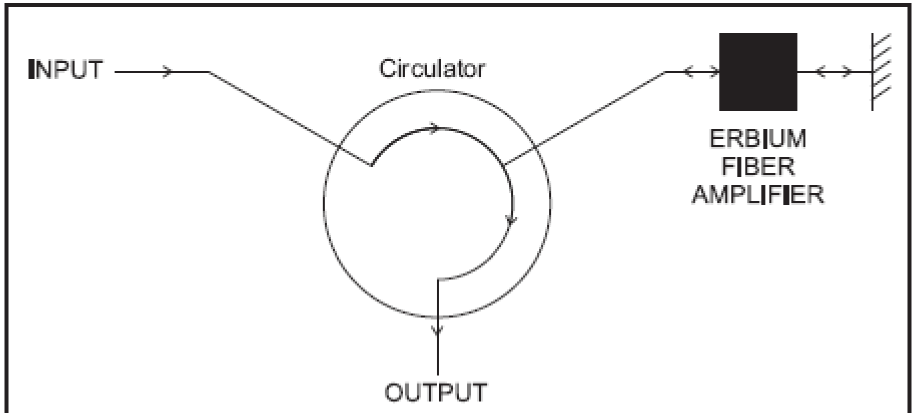
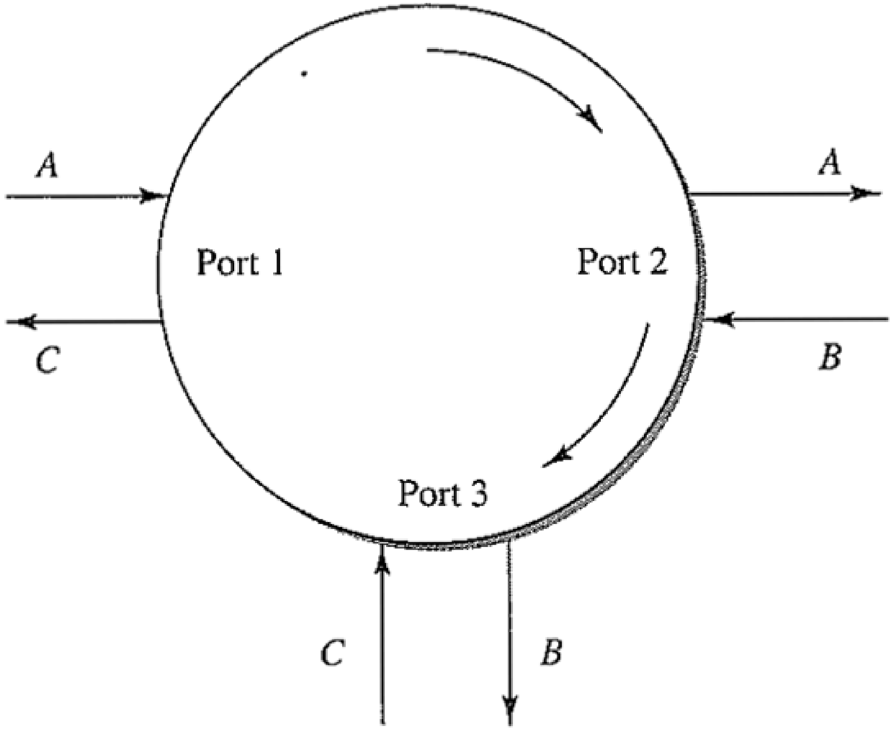
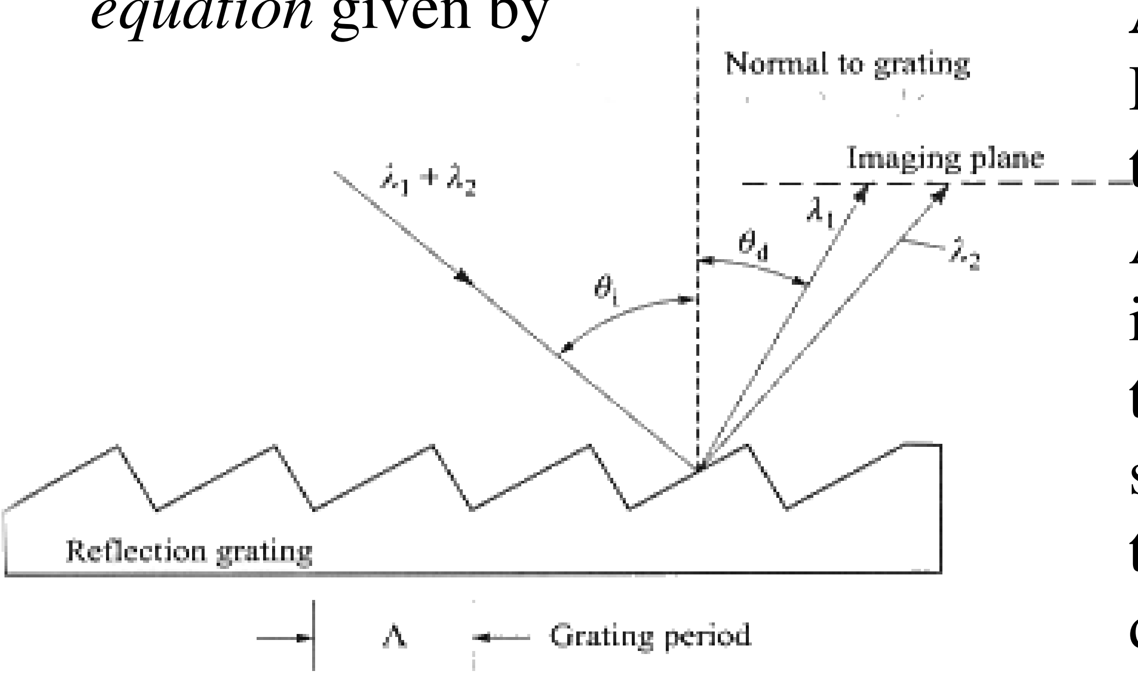


Figure 4: Double Pass Erbium Doped Amplifier



Fiber Grating Filters

- Figure below defines various parameters for a reflection grating.
- Here, θ_i is the incident angle of the light, θ_d is the diffracted angle, and Λ is the *period of the grating*.
- The spacing between two adjacent slits is called the *pitch* of the grating.
- Constructive interference at a wavelength λ occurs in the imaging plane when the rays diffracted at the angle θ_d satisfy the *grating equation* given by



$\Lambda(\sin\theta_i - \sin\theta_d) = m\lambda$ [10-45]

Here, *m* is called the *order* of the grating.

A grating can separate individual wavelengths since the grating equation is satisfied at different points in the imaging plane for different wavelengths.

Fiber Bragg Grating (FBGs)

- The imprinted grating can be represented as a uniform sinusoidal modulation of the refractive index along the core:

$$n(z) = n_{core} + \delta n \cdot [1 + \cos(2\pi z/\Lambda)] \qquad (10\text{-}46)$$

where n_{core} is the unexposed core refractive index and δn is the photo-induced change in the index

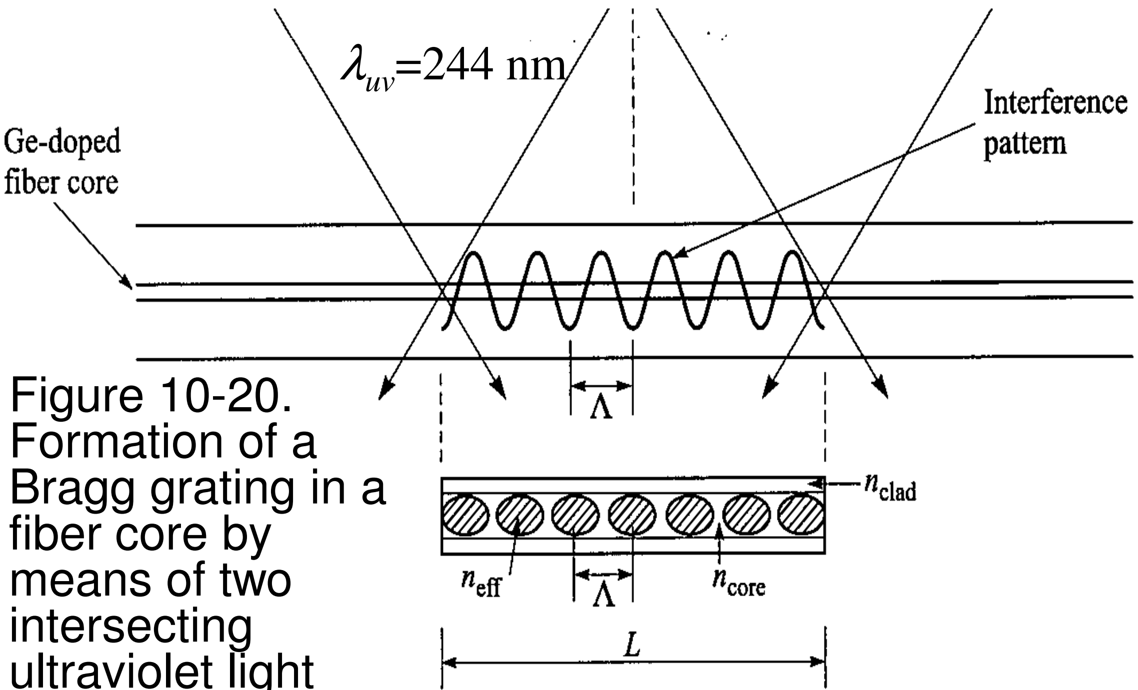


Figure 10-20. Formation of a Bragg grating in a fiber core by means of two intersecting ultraviolet light beams.

The maximum reflectivity occurs when the Bragg condition holds:

$$\lambda_{Bragg} = 2\Lambda n_{eff}, \qquad (10\text{-}47)$$

where n_{eff} is the mode effective index of the core.

Fiber Bragg Grating (FBGs) (2)

At this Bragg wavelength, the peak reflectivity R_{max}, for the grating of length L and coupling coefficient κ, is given by

$$R_{max} = \tanh^2(\kappa L). \tag{10-48}$$

The full bandwidth $\Delta\lambda$ over which R_{max} holds is

$$\Delta\lambda = (\lambda^2{}_{Bragg}/\pi n_{eff}L)[(\kappa L)^2+\pi^2]^{1/2} \tag{10-49}$$

An approximation for the FWHM bandwidth is

$$\Delta\lambda_{FWHM} = \lambda_{Bragg}\, s[(\delta n/2n_{core})^2 + (\Lambda/L)^2]^{1/2} \tag{10-50}$$

where $s = 1$ for strong gratings with near 100% reflectivity, and $s = 0.5$ for weak gratings.

For a uniform sinusoidal modulation of the core index, the coupling coefficient is given by

$$\kappa = \pi\delta n.\eta/\lambda_{Bragg}, \tag{10-51}$$

with η being the fraction of optical power contained in the fiber core.

Fiber Bragg Grating (FBGs) (3)

Under the assumption that the grating is uniform in the core, η can be approximated by

$$\eta = 1 - V^{-2}, \qquad (10\text{-}52)$$

where V is the V-number of the fiber.

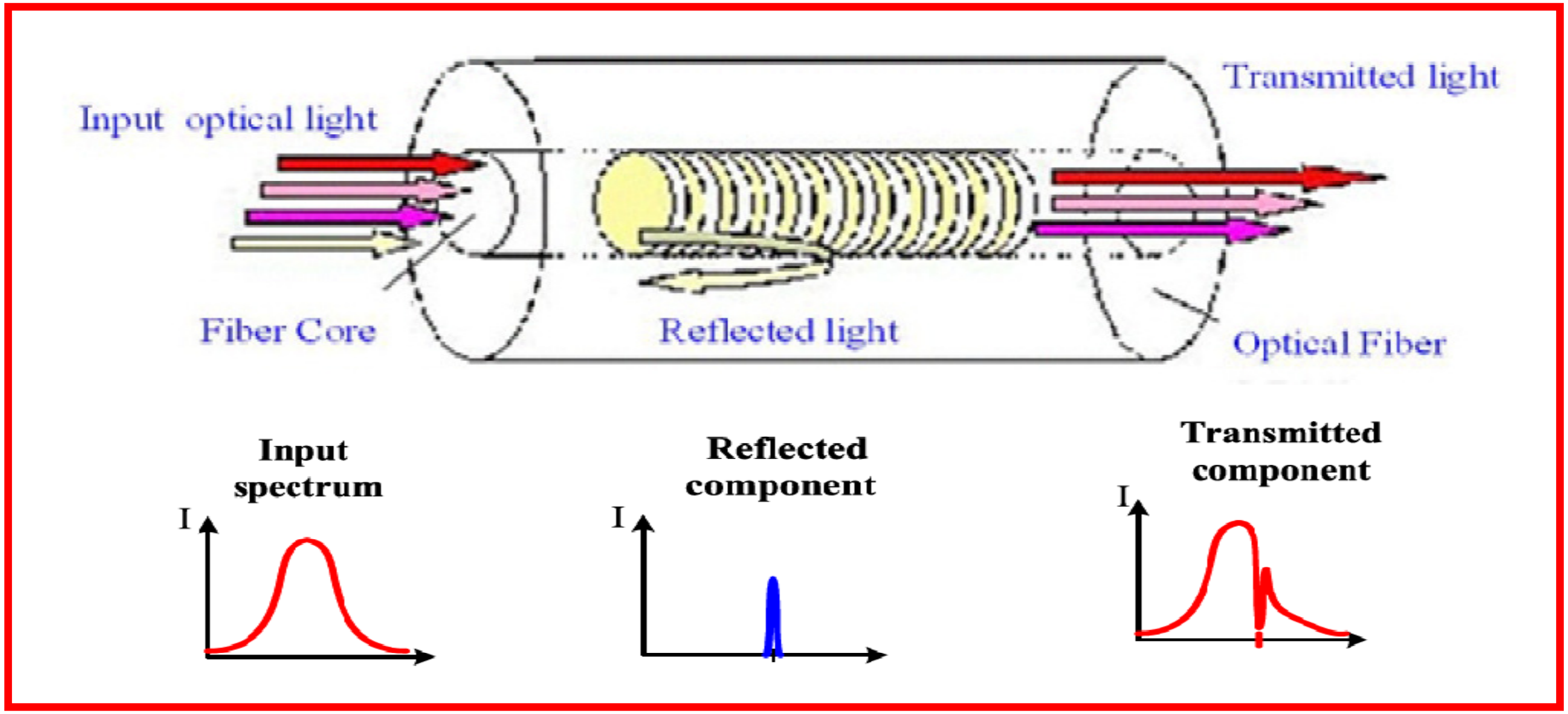
Example 10-9: (a) The table below shows the values of R_{max} as given by Eq. (10-48) for different values of κL:

κL	R_{max} (%)
1	58
2	93
3	98

- (b). Consider a fiber grating with the following parameters:
  $L = 0.5$ cm, $\lambda_{Bragg} = 1530$ nm, $n_{eff} = 1.48$,
  $\delta n = 2.5 \times 10^{-4}$, and $\eta = 82$ %.

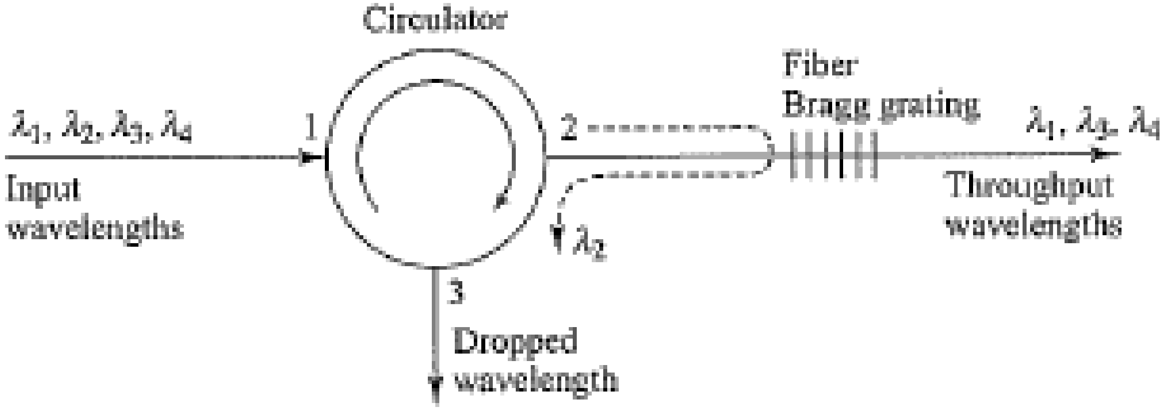
- From Eq. (10-51) we have $\kappa = 4.2$ cm^{-1}.
  Substituting this into Eq. (10-49) then yields $\Delta\lambda = 0.38$ nm.

Development of Fiber Bragg Gratings

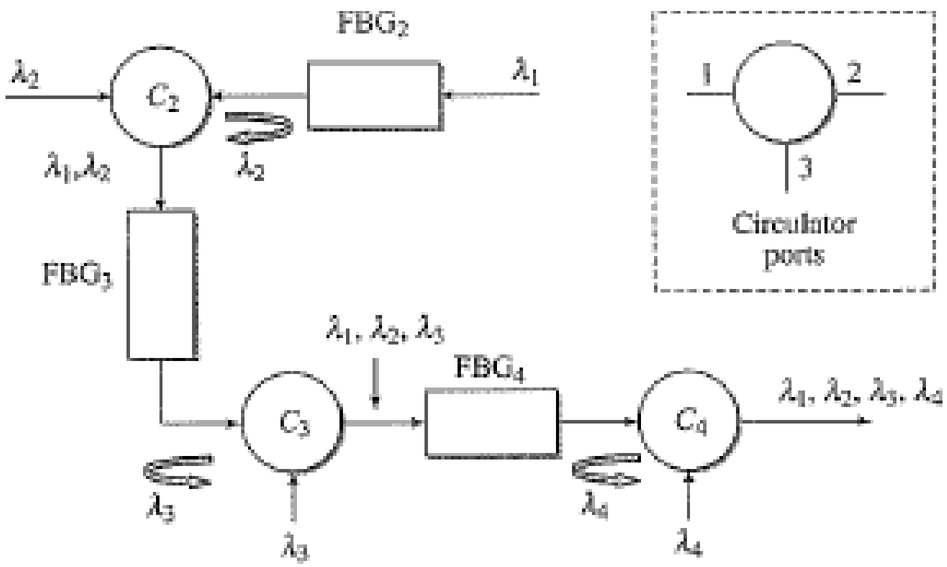


- On filtering the input light source with FBG, the wavelength satisfying the Bragg condition will be reflected, and the other will pass uninfluenced.

FBG Application



Demultiplexing



Multiplexing

Thin-Film Filters (TFF)

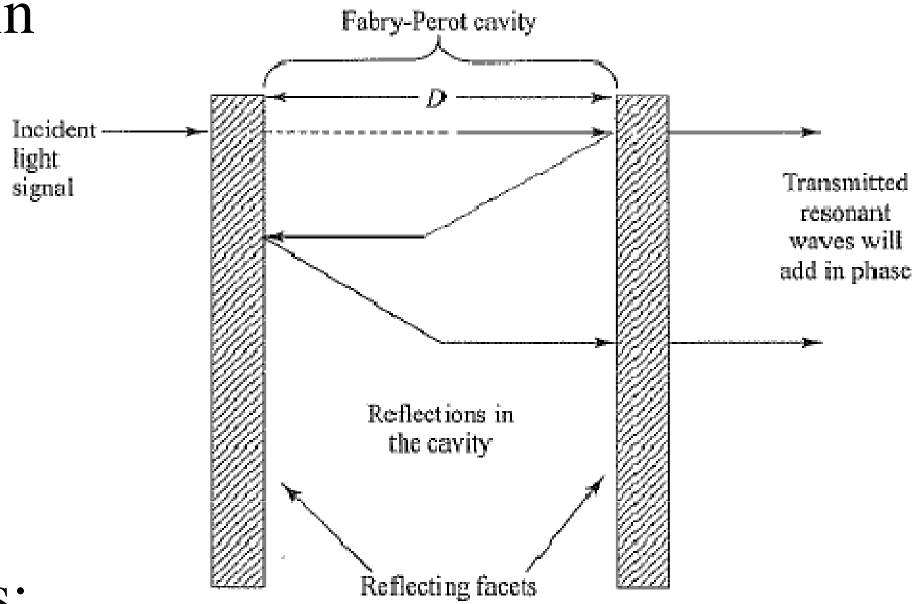
The transmission T of an ideal etalon in which there is no light absorption by the mirrors is an *Airy* function:

$$T = \left[1 + \frac{4R}{(1-R)^2}\sin^2\left(\frac{\phi}{2}\right)\right]^{-1}$$

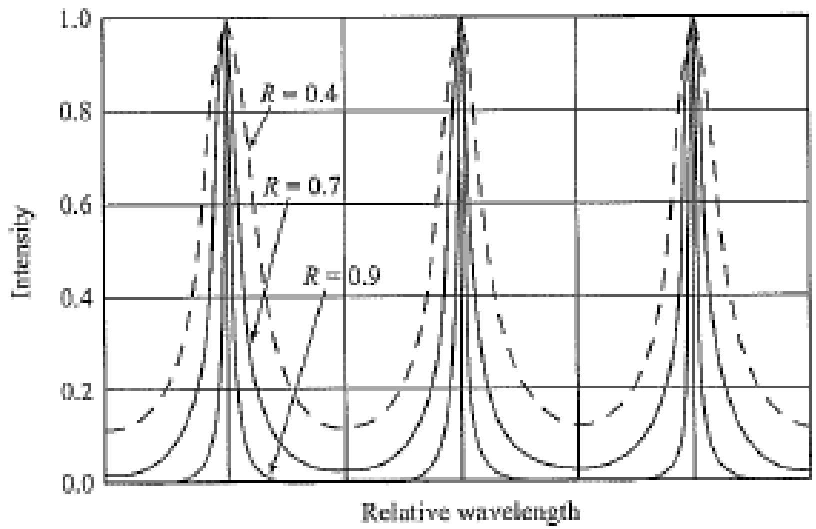
where R is the *reflectivity* of the mirrors. Ignoring any phase change at the mirrors, the phase change becomes:

$$\phi = \frac{2\pi}{\lambda} 2nD\cos\theta$$

where n is the refractive index, D is the distance between mirrors, and θ is the angle to the normal of the incoming light beam.



Fabry-Perot Cavity (Etalon)

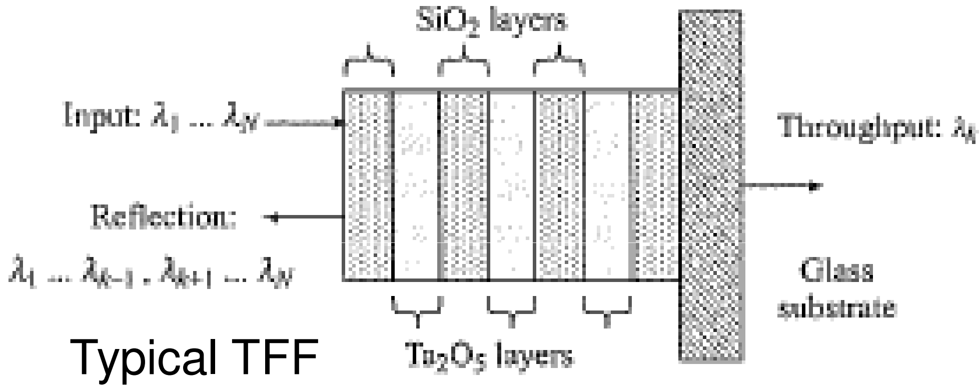


Thin-Film Filters (TFF) (2)

The peaks of the spacings, called the *passbands*, occur at those wavelengths that satisfy condition $N\lambda = 2nD\cos\,\theta$, where N is an integer.

The distance between adjacent peaks is called the *free spectral range* or FSR, is given by

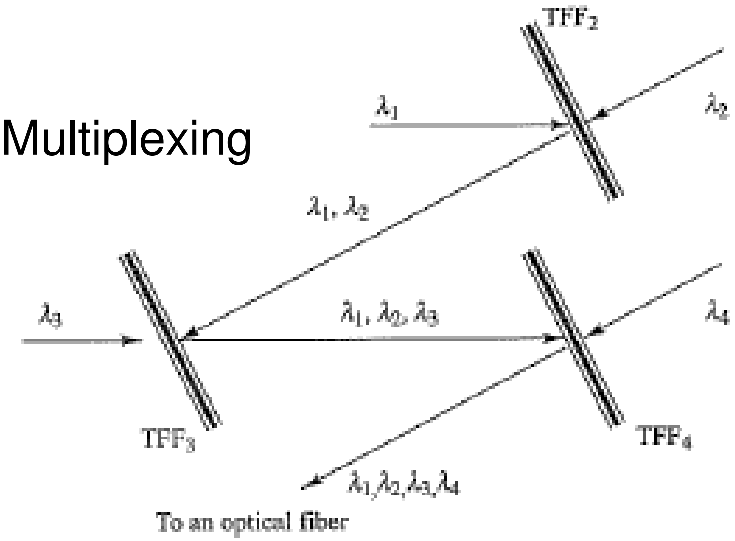
$$\mathrm{FSR} = \frac{\lambda^2}{2nD\cos\theta}$$



Typical TFF

The ratio FSR/FWHM (*full-width half-maximum*) gives an approximation of the number of wavelengths that a filter can accommodate. This is known as the *finesse F* of the filter.

$$F = \frac{\pi\sqrt{R}}{1-R}$$



Multiplexing