Chapter 2

Optical Fibers: Structures, Waveguiding & Fabrication

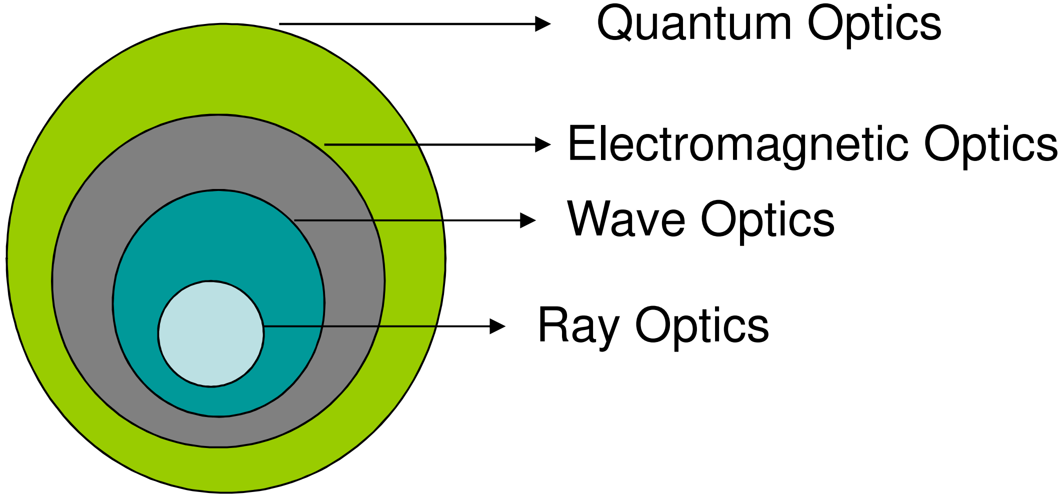
Outline

- Review of Maxwell's Equations and Plane waves
- Nature of Light
- Parallel Slab Waveguide
- Optical Fiber
- Optical Fiber Material and Fabrication

Theories of Optics

- Light is an electromagentic phenomenon described by the same theoretical principles that govern all forms of electromagnetic radiation. **Maxwell's equations** are in the hurt of electromagnetic theory & is fully successful in providing treatment of light propagation. **Electromagnetic optics** provides the most complete treatment of light phenomena in the context of **classical optics.**
- Turning to phenomena involving the interaction of **light & matter,** such as **emission** & **absorptio**n of light, **quantum theory** provides the successful explanation for light-matter interaction. These phenomena are described by quantum electrodynamics which is the marriage of electromagnetic theory with quantum theory. For optical phenomena, this theory also referred to as **quantum optics.** This theory provides an explanation of virtually all optical phenomena.

- In the context of classical optics, electromagentic radiation propagates in the form of two mutually coupled vector waves, an **electric field-wave** & **magnetic field wave**. It is possible to describe many optical phenomena such as diffraction, by **scalar** wave theory in which light is described by a single scalar wavefunction. This approximate theory is called scalar wave optics or simply **wave optics**. When light propagates through & around objects whose dimensions are much greater than the optical wavelength, the wave nature of light is not readily discerned, so that its behavior can be adequately described by rays obeying a set of geometrical rules. This theory is called **ray optics.** Ray optics is the limit of wave optics when the wavelength is very short.



Engineering Model

- In engineering discipline, we should choose the appropriate & easiest physical theory that can handle our problems. Therefore, specially in this course we will use different optical theories to describe & analyze our problems. In this chapter we deal with optical transmission through fibers, and other optical waveguiding structures. Depending on the structure, we may use ray optics or electromagnetic optics, so we begin our discussion with a brief introduction to electromagnetic optics, ray optics & their fundamental connection, then having equipped with basic theories, we analyze the propagation of light in the optical fiber structures.

Electromagnetic Optics

- Electromagnetic radiation propagates in the form of two mutually coupled **vector** waves, an **electric field wave** & a **magnetic field wave**. Both are vector functions of position & time.
- In a source-free, linear, homogeneous, isotropic & non-dispersive media, such as free space, these electric & magnetic fields satisfy the following partial differential equations, known as **Maxwell' equations:**

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \cdot \vec{E} = 0$$

$$\nabla \cdot \vec{H} = 0$$

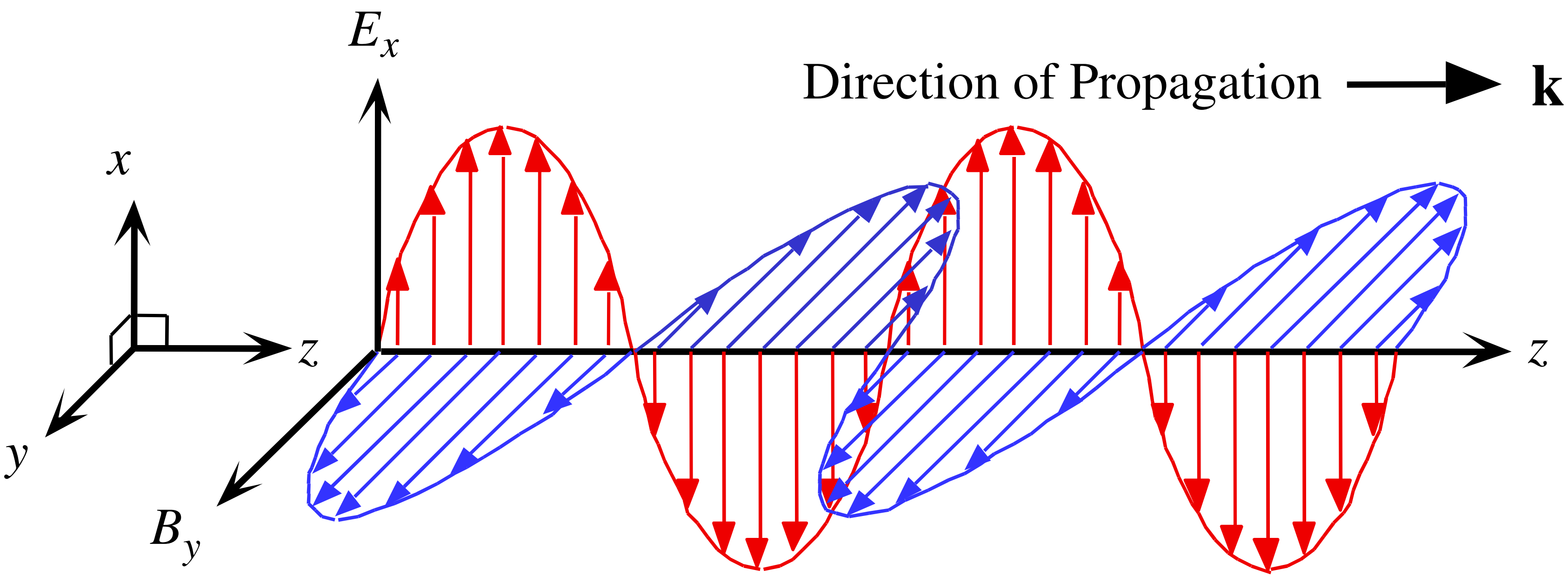
- In Maxwell's equations, **E** is the electric field expressed in [V/m], **H** is the magnetic field expressed in [A/m].

$$\varepsilon\ [\mathrm{F/m}] : \text{Electric permittivity}$$
$$\mu\ [\mathrm{H/m}] : \text{Magnetic permeability}$$

$$\nabla\cdot : \text{is divergence operation}$$
$$\nabla\times : \text{is curl operation}$$

- The solution of Maxwell's equations in free space, through the **wave equation,** can be easily obtained for **monochromatic** electromagnetic wave. All electric & magnetic fields are harmonic functions of time of the same frequency. Electric & magnetic fields are perpendicular to each other & both perpendicular to the direction of propagation, **k**, known as **transverse wave (TEM)**. **E, H** & **k** form a set of orthogonal vectors.

Electromagnetic Plane wave in Free space



An electromagnetic wave is a travelling wave which has time varying electric and magnetic fields which are perpendicular to each other and the direction of propagation, *z*.

S.O.Kasap, optoelectronics and Photonics Principles and Practices, prentice hall, 2001

Linearly Polarized Electromagnetic Plane wave

$$\vec{E} = \hat{e}_x E_{0x} \cos(\omega t - kz)$$

$$\vec{H} = \hat{e}_y H_{0y} \cos(\omega t - kz)$$

where :

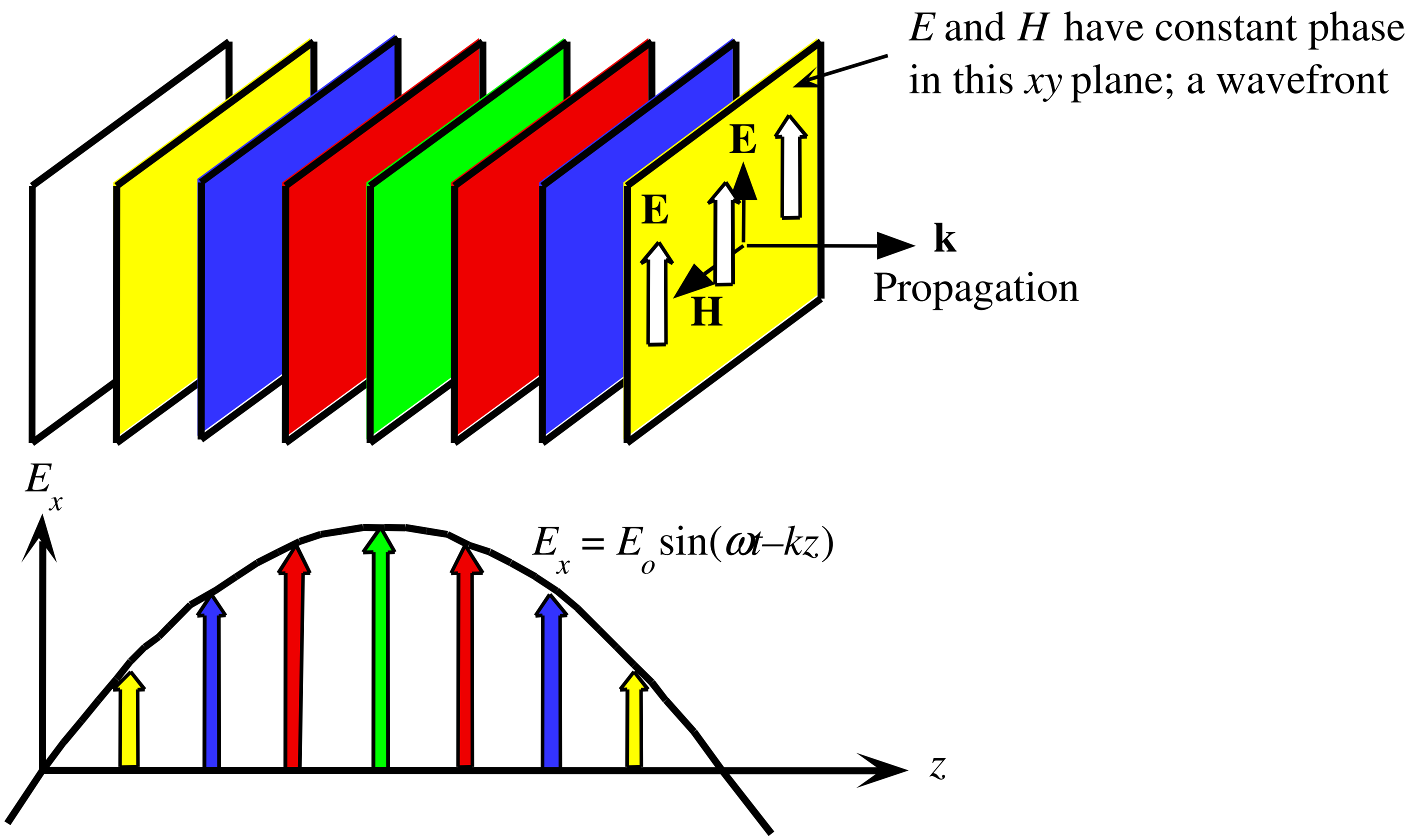
$\omega = 2\pi f$: Angular frequency [rad/m]

k : Wavenumber or wave propagation constant [1/m]

$\lambda = \dfrac{2\pi}{k}$: Wavelength [m]

$\eta = \dfrac{E_{0x}}{H_{0y}} = \sqrt{\dfrac{\mu}{\varepsilon}}\,[\Omega]$: intrinsic (wave) impedance

$v = \dfrac{1}{\sqrt{\mu\varepsilon}}\,[\text{m/s}]$: velocity of wave propagation



A plane EM wave travelling along z, has the same E_x (or B_y) at any point in a given xy plane. All electric field vectors in a given xy plane are therefore in phase. The xy planes are of infinite extent in the x and y directions.

S.O.Kasap, optoelectronics and Photonics Principles and Practices, prentice hall, 2001

Some Wave Parameters

- **Wavelength** is the distance over which the phase changes by 2π.

$$\lambda = \frac{v}{f}$$

- In vacuum (free space):

$$\varepsilon_0 = \frac{10^{-9}}{36\pi} \quad [\text{F/m}] \qquad \mu_0 = 4\times 10^{-7} \quad [\text{H/m}]$$

$$v = c \cong 3\times 10^{8} \text{ m/s} \qquad \eta_0 = 120\pi \ [\Omega]$$

- **Refractive index** of <u>a medium</u> is defined as:

$$n = \frac{c}{v} = \frac{\text{velocity of light (EM wave) in vacuum}}{\text{velocity of light (EM wave) in medium}} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} = \sqrt{\mu_r\varepsilon_r}$$

μ_r : Relative magnetic permeability

ε_r : Relative electric permittivity

- For non-magnetic media $(\mu_r = 1)$:

$$n = \sqrt{\varepsilon_r}$$

Intensity & power flow of TEM wave

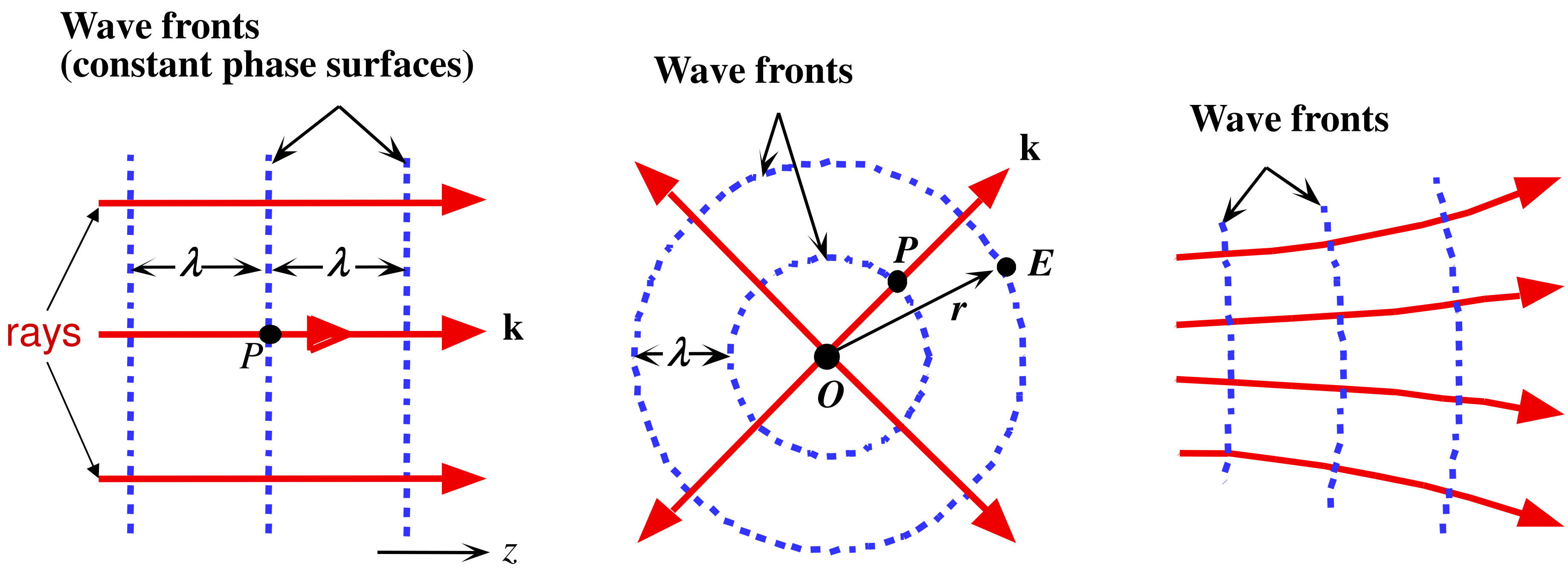
- The Poynting vector $\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}} \times \vec{\mathbf{H}}$ for TEM wave is parallel to the wavevector **k** so that the power flows along in a direction normal to the **wavefront** or parallel to **k**. The magnitude of the Poynting vector is the intensity of TEM wave as follows:

$$I = \frac{|E_0|^2}{2\eta} \quad [\mathrm{W/m^2}]$$

Connection between EM wave optics & Ray optics

According to wave or physical optics viewpoint, the EM waves radiated by a small optical source can be represented by a train of spherical wavefronts with the source at the center. A **wavefront** is defined as the locus of all points in the wave train which exhibit the same phase. Far from source wavefronts tend to be in a plane form. Next page you will see different possible phase fronts for EM waves.

When the wavelength of light is much smaller than the object, the wavefronts appear as straight lines to this object. In this case the light wave can be indicated by a **light ray,** which is drawn perpendicular to the phase front and parallel to the Poynting vector, which indicates the flow of energy. Thus, large scale optical effects such as reflection & refraction can be analyzed by simple geometrical process called **ray tracing**. This view of optics is referred to as **ray optics** or **geometrical optics (GO)**.

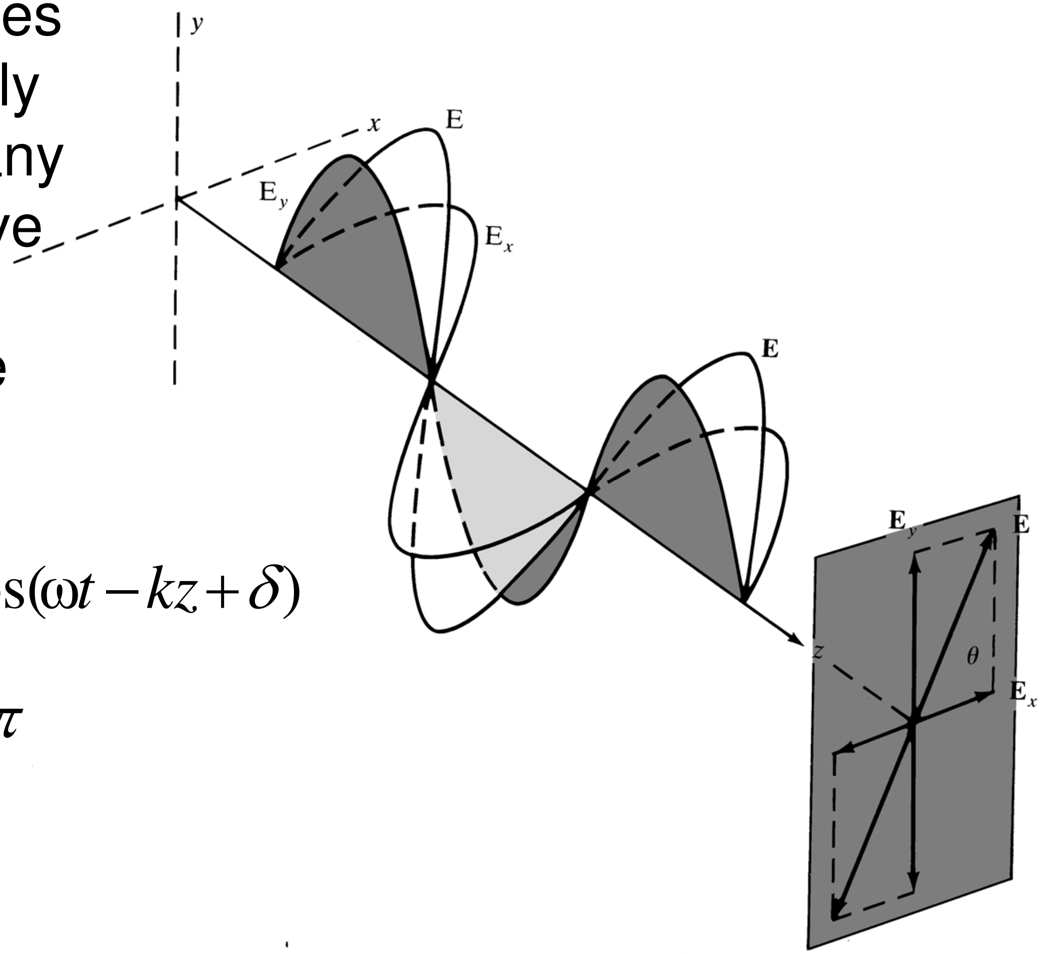


A perfect plane wave | A perfect spherical wave | A divergent beam
(a) | (b) | (c)

Examples of possible EM waves

General form of linearly polarized plane waves

Any two orthogonal plane waves Can be combined into a linearly Polarized wave. Conversely, any arbitrary linearly polarized wave can be resolved into two independent Orthogonal plane waves that are in phase.



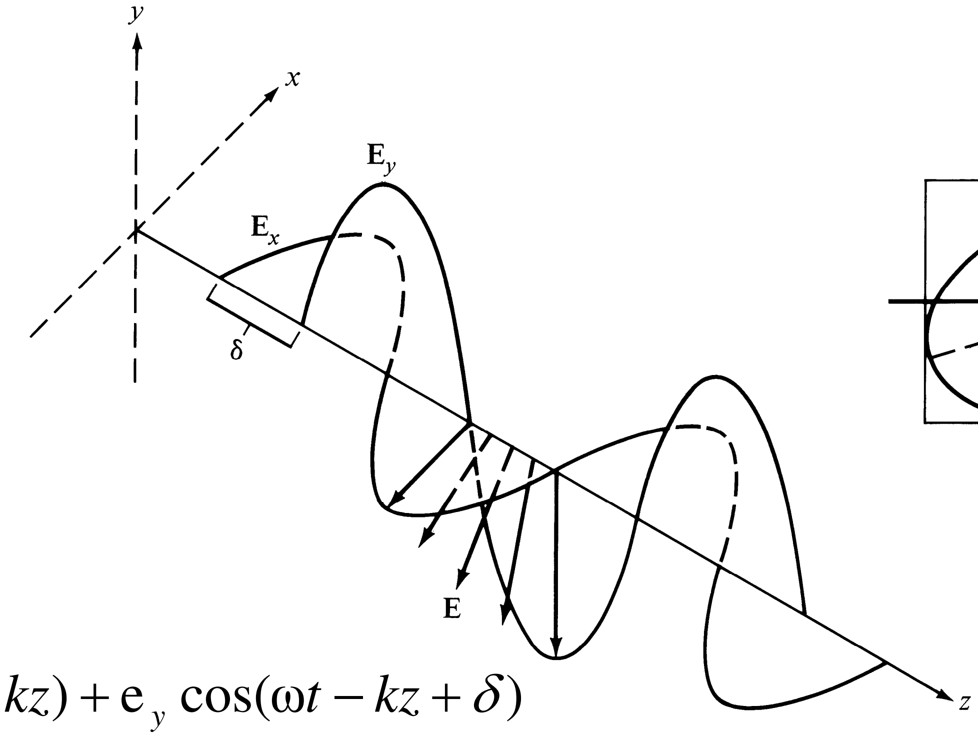
$$\vec{E} = \hat{\mathbf{e}}_x E_{0x} \cos(\omega t - kz) + \hat{\mathbf{e}}_y E_{0y} \cos(\omega t - kz + \delta)$$

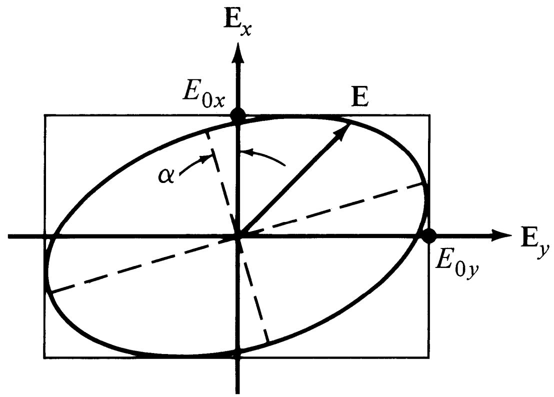
$$E = \left|\vec{E}\right| = \sqrt{{E_{0x}}^2 + {E_{0y}}^2} ; \delta = 0 \text{OR} \pi$$

$$\theta = \tan^{-1}\left(\frac{E_{0y}}{E_{0x}}\right)$$

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Elliptically Polarized plane waves





$$\vec{E} = \mathrm{e}_x E_x + \mathrm{e}_y \mathrm{E}_y$$

$$= \mathrm{e}_x E_{0x} \cos(\omega t - kz) + \mathrm{e}_y \cos(\omega t - kz + \delta)$$

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\delta = \sin^2\delta \qquad [2\text{-}16]$$

$$\tan(2\alpha) = \frac{2E_{0x}E_{0y}\cos\delta}{E_{0x}{}^2 - E_{0y}{}^2}$$

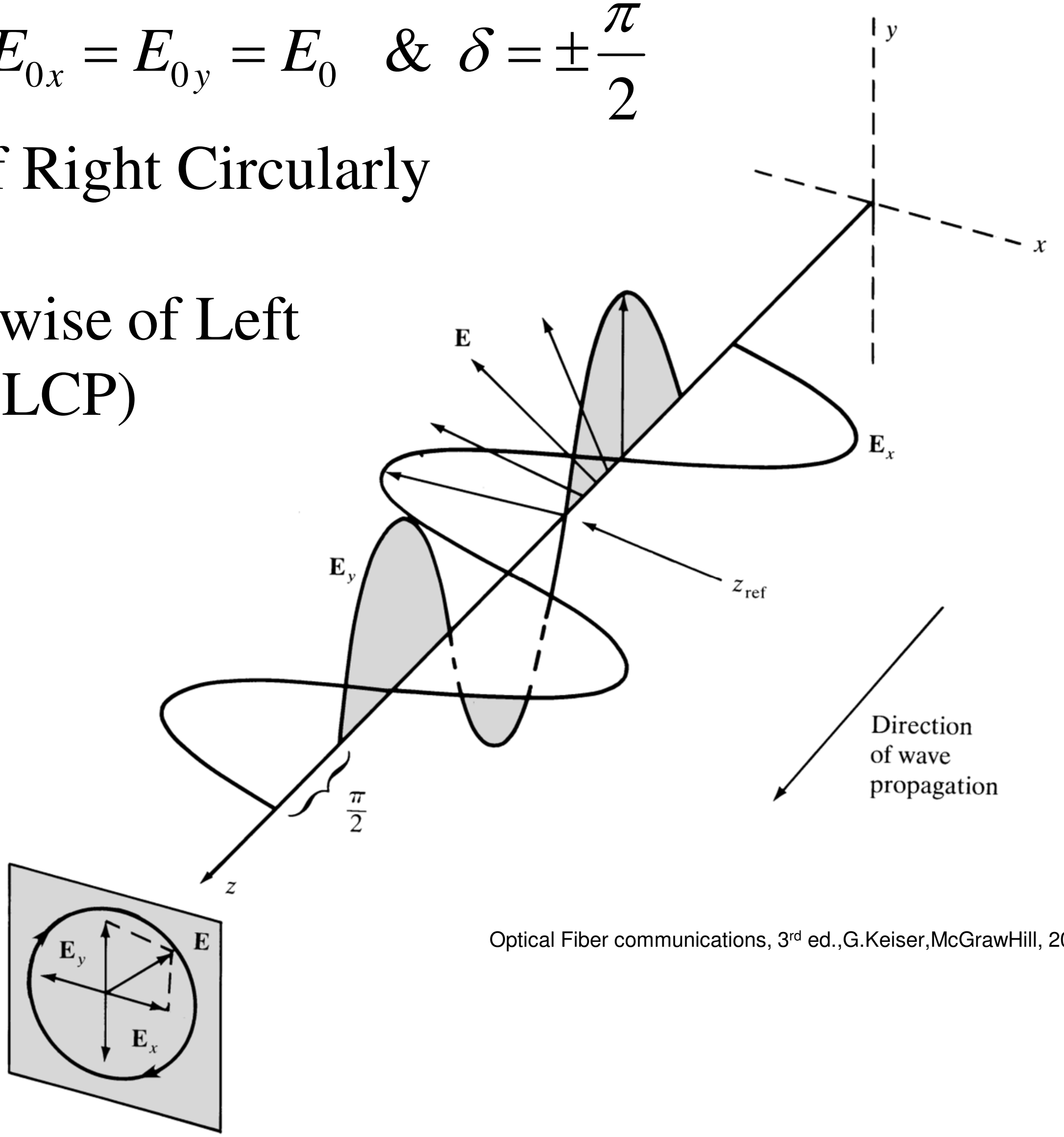
Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Circularly polarized waves

$$\text{Circular polarization}: E_{0x} = E_{0y} = E_0 \quad \& \quad \delta = \pm\frac{\pi}{2}$$

+ Sign : Clockwise of Right Circularly Polarized (RCP)

- Sign : Counterclockwise of Left Circularly Polarized (LCP)



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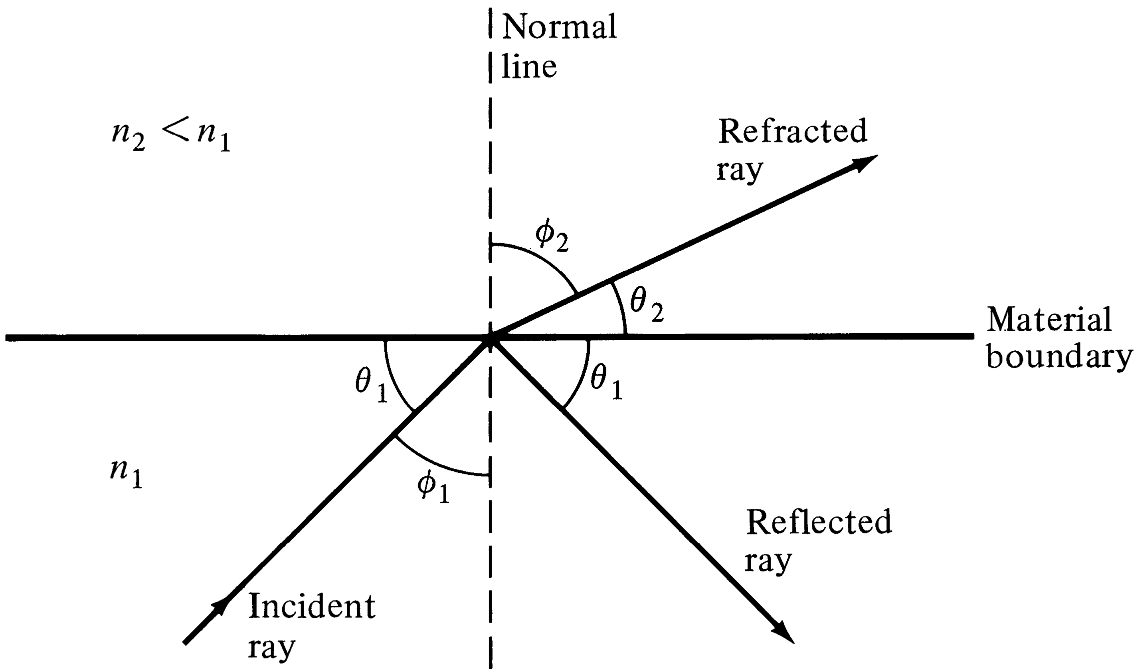
Laws of Reflection & Refraction



Reflection law: angle of incidence=angle of reflection

Snell's law of refraction:

$$n_1 \sin \phi_1 = n_2 \sin \phi_2$$

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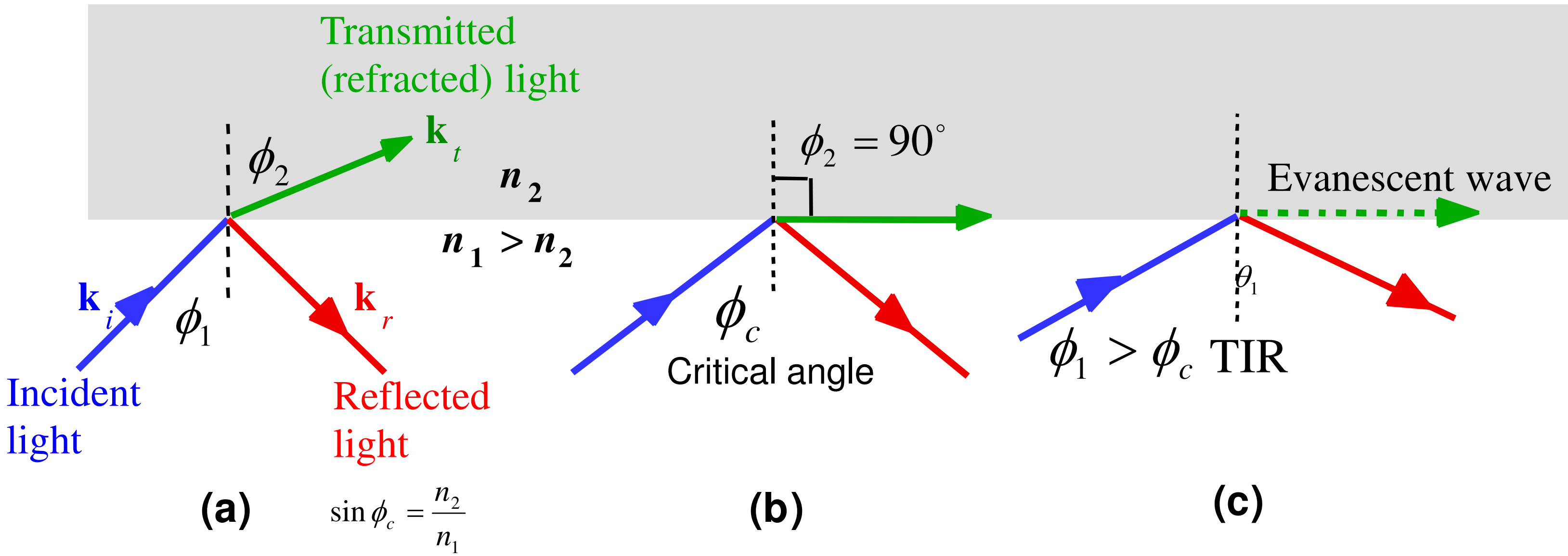
Total internal reflection, Critical angle



Light wave travelling in a more dense medium strikes a less dense medium. Depending on the incidence angle with respect to ϕ_c, which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected. (a) $\phi_1 < \phi_c$ (b) $\phi_1 = \phi_c$ (c) $\phi_1 > \phi_c$ and total internal reflection (TIR).

$$\sin\phi_c = \frac{n_2}{n_1}$$

Phase shift due to TIR

- The totally reflected wave experiences a phase shift however which is given by:

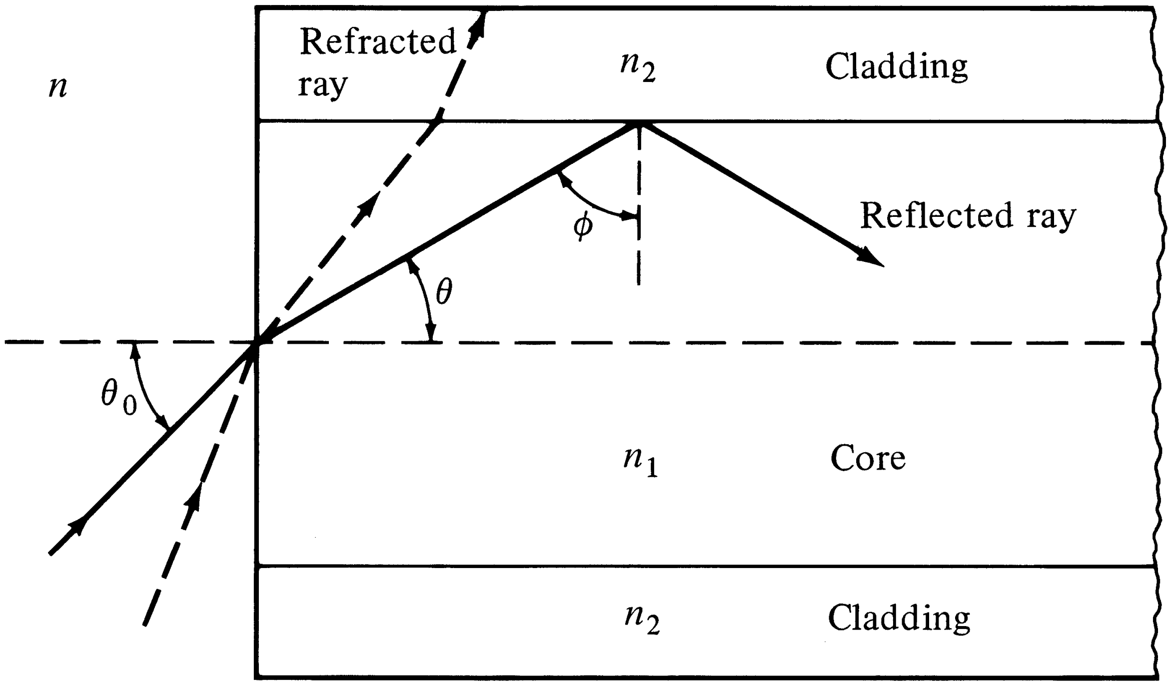
$$\tan\frac{\delta_N}{2} = -\frac{\sqrt{n^2\cos^2\theta_1 - 1}}{n\sin\theta_1}; \qquad \tan\frac{\delta_p}{2} = -\frac{n\sqrt{n^2\cos^2\theta_1 - 1}}{\sin\theta_1}$$

$$n = \frac{n_1}{n_2}$$

2.19a-b

- Where (*p*,*N*) refer to the electric field components parallel (Parallel Polarization, TM) or normal (Perpendicular Polarization, TE) to the plane of incidence respectively.

Optical waveguiding by TIR: Dielectric Slab Waveguide

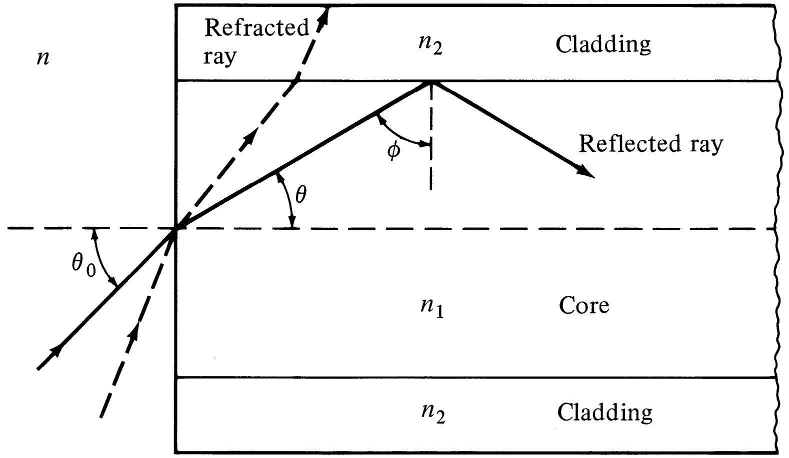


Propagation mechanism in an ideal step-index optical waveguide.

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Launching optical rays to slab waveguide

$\sin\phi_{\min} = \frac{n_2}{n_1}$; minimum angle that supports TIR



Maximum entrance angle, $\theta_{0\,\max}$ is found from the Snell's relation written at the fiber end face.

$$n\sin\theta_{0\,\max} = n_1\sin\theta_c = \sqrt{n_1^{\,2} - n_2^{\,2}}$$

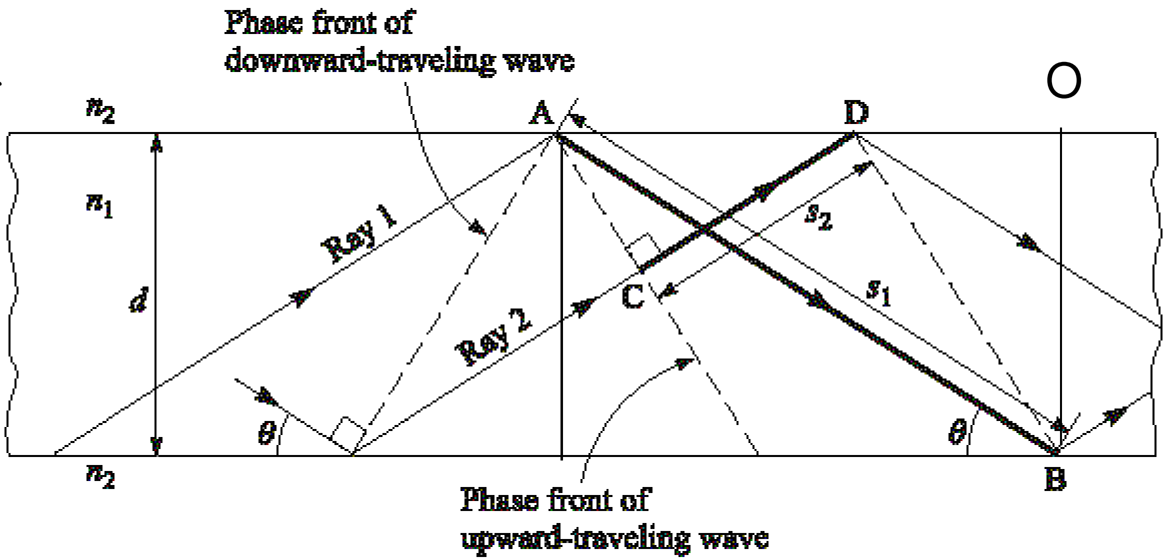
Numerical aperture:

$$\mathrm{NA} = n\sin\theta_{0\,\max} = \sqrt{n_1^{\,2} - n_2^{\,2}} \approx n_1\sqrt{2\Delta}$$

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Optical rays transmission through dielectric slab waveguide

$$n_1 > n_2; \theta < \theta_c = \frac{\pi}{2} - \phi_c$$



For TE-case, when electric waves are normal to the plane of incidence
θ must be satisfied with following relationship:

$$\tan\left(\frac{\pi n_1 d \sin\theta}{\lambda} - \frac{m\pi}{2}\right) = \left[\frac{\sqrt{n_1^{\,2}\cos^2\theta - n_2^{\,2}}}{n_1 \sin\theta}\right]$$

EM analysis of Slab waveguide

- For each particular angle, in which light ray can be faithfully transmitted along slab waveguide, we can obtain one possible propagating wave solution from a Maxwell's equations or **mode.**
- The modes with electric field perpendicular to the plane of incidence (page) are called **TE** (Transverse Electric) and numbered as: $\mathrm{TE}_0, \mathrm{TE}_1, \mathrm{TE}_2, \ldots$

  Electric field distribution of these modes for 2D slab waveguide can be expressed as:

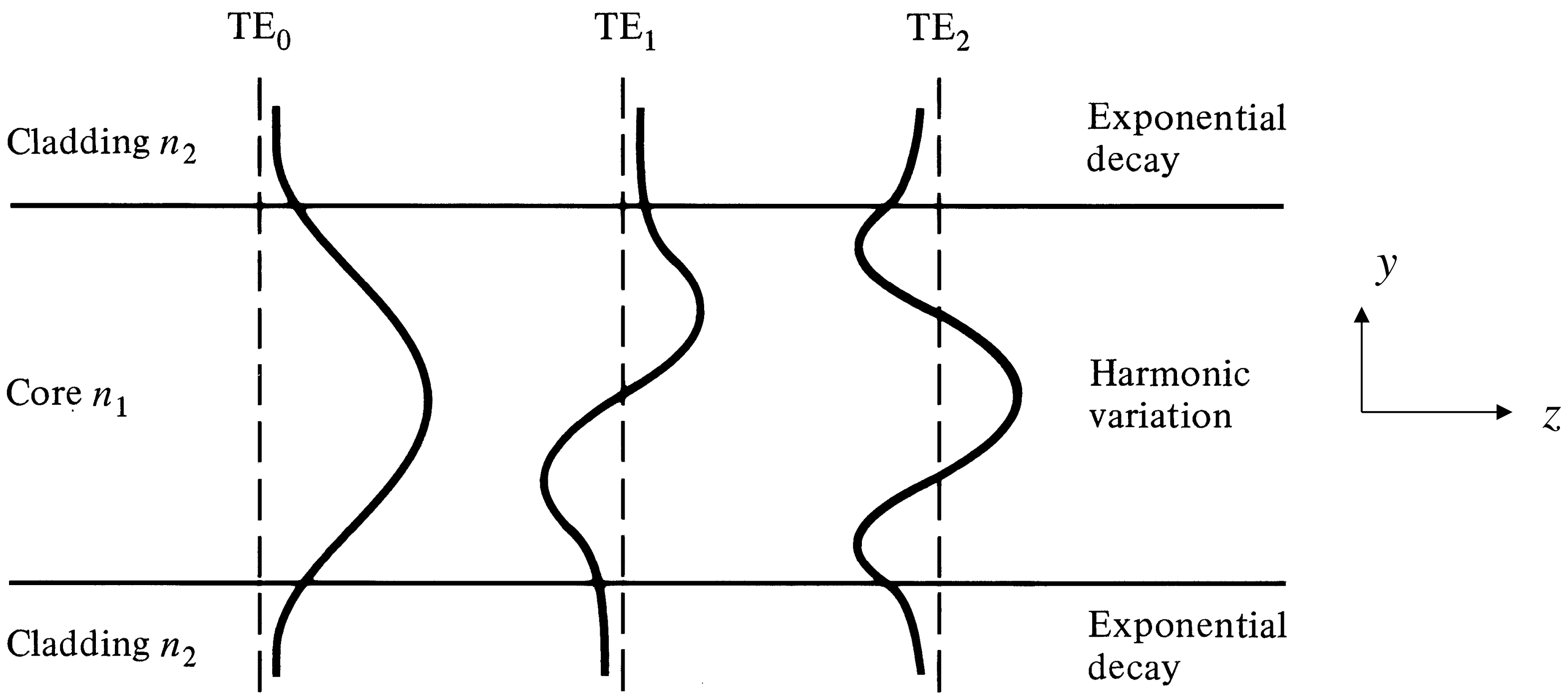
$$\vec{E}_m(x, y, z, t) = \hat{\mathrm{e}}_x f_m(y) \cos(\omega t - \beta_m z)$$

$$m = 0,1,2,3 \text{ (mode number)}$$

wave transmission along slab waveguides, fibers & other type of optical waveguides can be fully described by time & z dependency of the mode:

$$\cos(\omega t - \beta_m z) \qquad \text{or} \qquad e^{j(\omega t - \beta_m z)}$$

TE modes in slab waveguide



$$\vec{E}_m(x, y, z, t) = \hat{e}_x f_m(y)\cos(\omega t - \beta_m z)$$

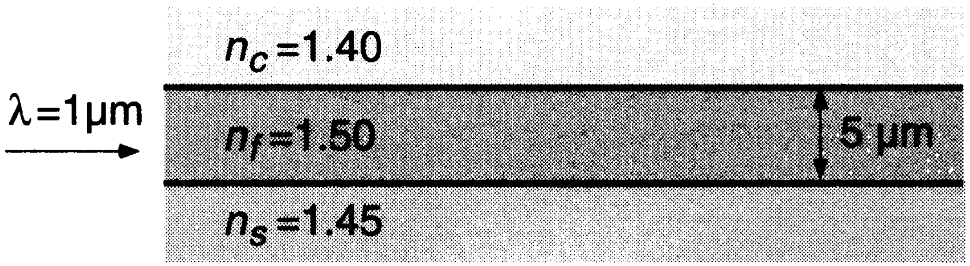
$$m = 0,1,2,3 \text{ (mode number)}$$

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Modes in slab waveguide

- The order of the mode is equal to the # of field zeros across the guide. The order of the mode is also related to the angle in which the ray congruence corresponding to this mode makes with the plane of the waveguide (or axis of the fiber). **The steeper the angle, the higher the order of the mode**.
- For higher order modes the fields are distributed more toward the edges of the guide and penetrate further into the cladding region.
- **Radiation modes** in fibers are not trapped in the core & guided by the fiber but they are still solutions of the Maxwell' eqs. with the same boundary conditions. These infinite continuum of the modes results from the optical power that is outside the fiber acceptance angle being refracted out of the core.
- In addition to bound & refracted (radiation) modes, there are **leaky modes** in optical fiber. They are partially confined to the core & attenuated by continuously radiating this power out of the core as they traverse along the fiber (results from Tunneling effect which is quantum mechanical phenomenon.) A mode remains guided as long as $n_2 k < \beta < n_1 k$

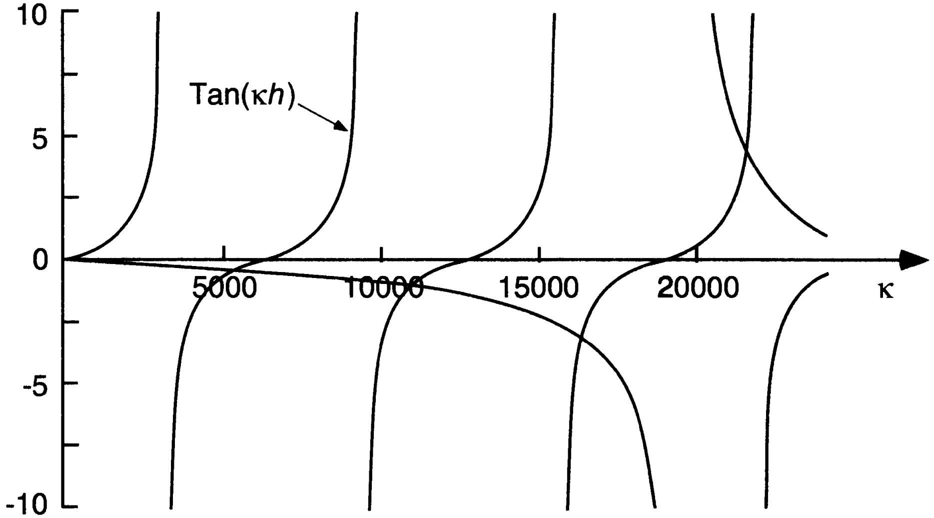
Example

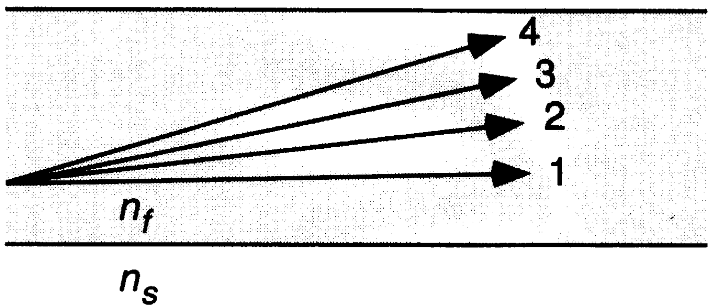


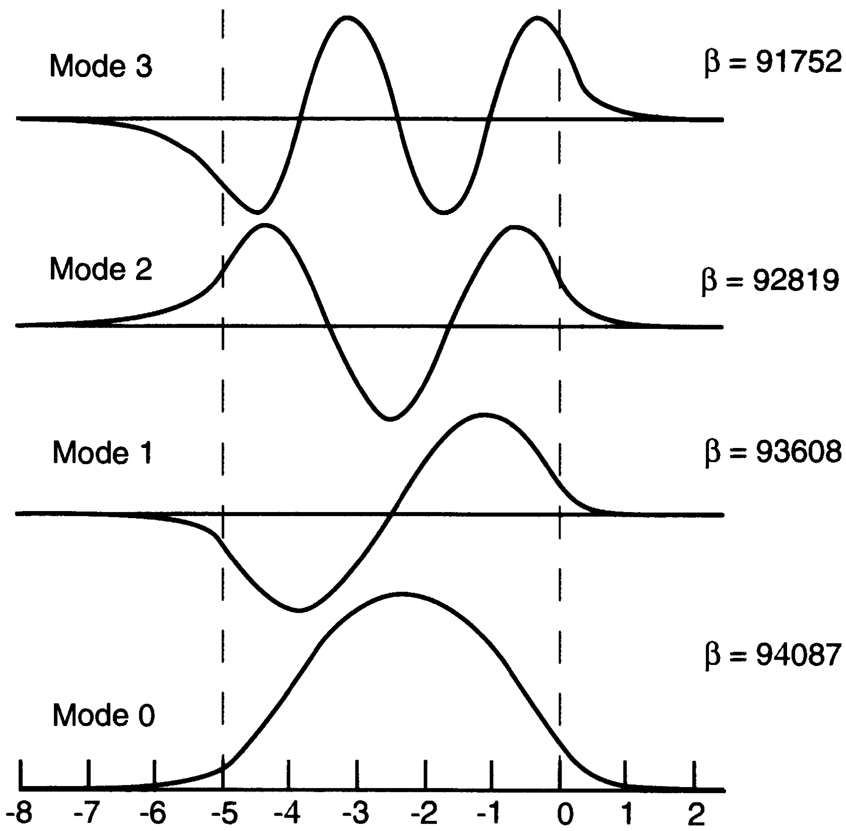
$$\gamma_c = \sqrt{\beta^2 - k_0^2(1.4)^2}$$

$$\gamma_s = \sqrt{\beta^2 - k_0^2(1.45)^2}$$

$$\beta = \sqrt{k_0^2(1.5)^2 - \kappa_f^2}$$

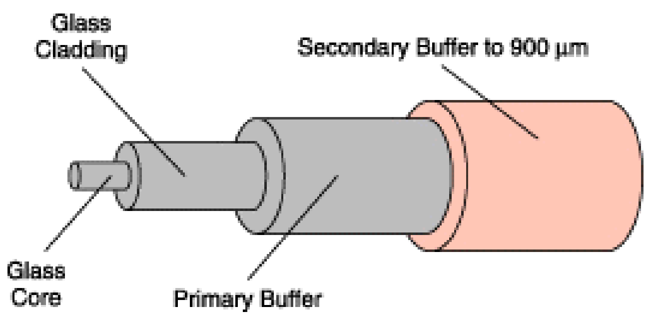






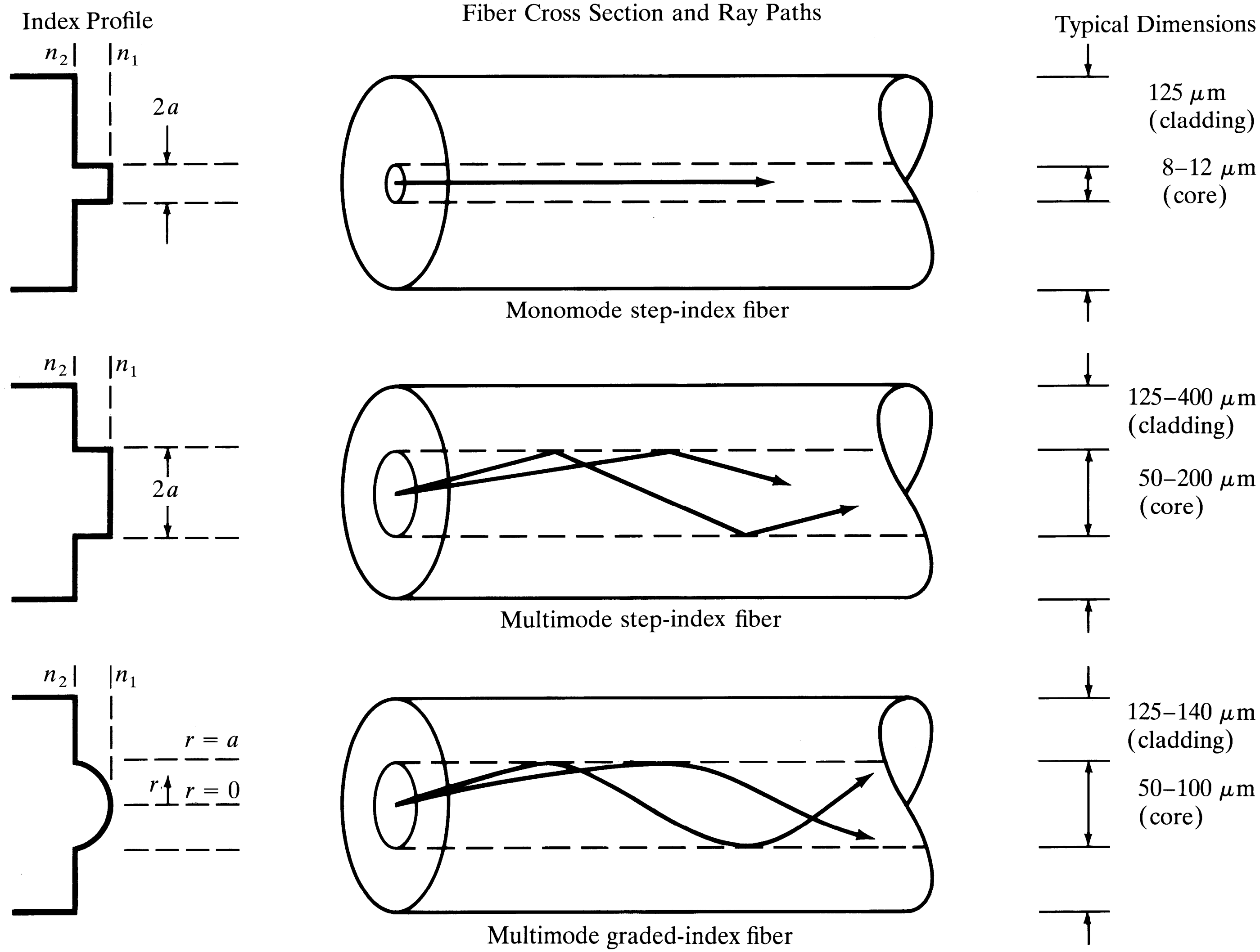
Basic structure of all optical fiber

- *Core*—carries most of light
- *Cladding*—confines light to core
- In some fibers, *substrate* glass layer to add strength
- *Inner jacket* or *primary buffer coating*—mechanical protection
- *Outer jacket* or *secondary buffer coating*—mechanical protection



Source: Optical Cable Corporation

Three Types of Optical Fibers



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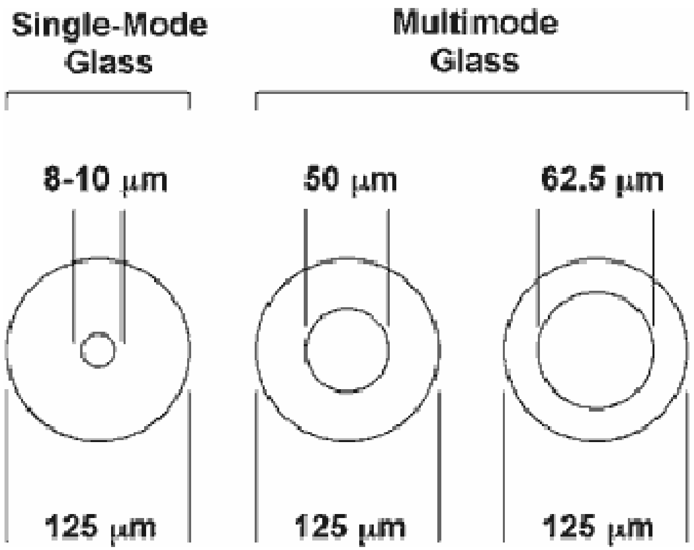
Optical fiber standard dimensions

- Core, cladding, jacketing standardized



- Jacket: 245 µm

Source: Corning

Modal Theory of Step Index fiber

- General expression of EM-wave in the circular fiber can be written as:

$$\vec{E}(r,\phi,z,t) = \sum_m A_m \vec{E}_m(r,\phi,z,t) = \sum_m A_m \vec{U}_m(r,\phi) e^{j(\omega t - \beta_m z)}$$

$$\vec{H}(r,\phi,z,t) = \sum_m A_m \vec{H}_m(r,\phi,z,t) = \sum_m A_m \vec{V}_m(r,\phi) e^{j(\omega t - \beta_m z)}$$

- Each of the characteristic solutions $\vec{E}_m(r,\phi,z,t)$ & $\vec{H}_m(r,\phi,z,t)$ is called $\boldsymbol{m}^{\text{th}}$ **mode** of the optical fiber.
- It is often sufficient to give the E-field of the mode.

$$\vec{U}_m(r,\phi) e^{j(\omega t - \beta_m z)} \qquad \mathrm{m} = 1,2,3\ldots$$

- The modal field distribution, $\vec{U}_m(r,\phi)$, and the mode propagation constant, β_m are obtained from solving the Maxwell's equations subject to the boundary conditions given by the cross sectional dimensions and the dielectric constants of the fiber.

- Most important characteristics of the EM transmission along the fiber are determined by the mode propagation constant, $\beta_m(\omega)$, which depends on the mode & in general varies with frequency or wavelength. <u>This quantity is always between the plane propagation constant (wave number) of the core & the cladding media</u> .

$$n_2 k < \beta_m(\omega) < n_1 k$$

- At each frequency or wavelength, there exists only a finite number of guided or propagating modes that can carry light energy over a long distance along the fiber. Each of these modes can propagate in the fiber only if the frequency is above the **cut-off frequency**, ω_c , (or the source wavelength is shorter than the cut-off wavelength) obtained from cut-off condition that is:

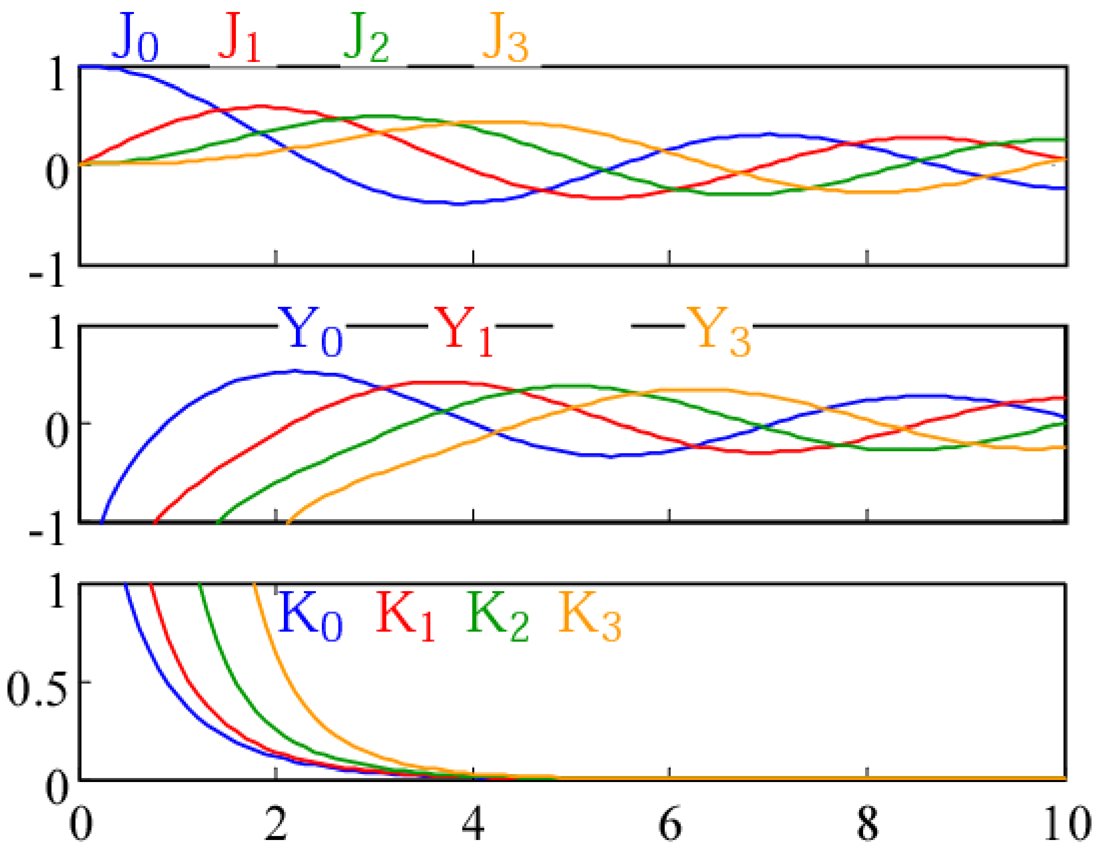
$$\beta_m(\omega_c) = n_2 k$$

- To minimize the signal distortion, the fiber is often operated in a **single mode** regime. In this regime only the lowest order mode (fundamental mode) can propagate in the fiber and all higher order modes are under cut-off condition (non-propagating).
- **Multi-mode** fibers are also extensively used for many applications. In these fibers many modes carry the optical signal collectively & simultaneously.

Optical Fiber Modes

- The optical fiber has a circular waveguide instead of planar
- The solutions to Maxwell's equations
  - Fields in core are non-decaying
    - J, Y Bessel functions of first and second kind
  - Fields in cladding are decaying
    - K modified Bessel functions of second kind
- Solutions vary with radius r and angle θ
- There are two mode number to specify the mode
  - m is the radial mode number
  - ν is the angular mode number

Bessel Functions



Bessel Function Relationships

- Bessel function recursive relationships

$$J_{-n}(x) = (-1)^n J_n(x)$$

$$K_{-n}(x) = K_n(x)$$

$$J_{n-1}(x) = \frac{2n}{x} J_n(x) - J_{n+1}(x)$$

$$K_{n-1}(x) = -\frac{2n}{x} K_n(x) + K_{n+1}(x)$$

$$J_0(x) = \frac{2}{x} J_1(x) - J_2(x)$$

$$K_0(x) = -\frac{2}{x} K_1(x) + K_2(x)$$

- Small argument approximations

$$K_n(x) \to \begin{cases} -\ln\left(\frac{x}{2}\right) - 0.5772 & n = 0 \\ \frac{(n-1)!}{2}\left(\frac{2}{x}\right)^n & n > 0 \end{cases}$$

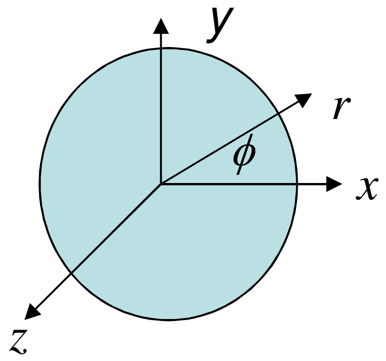
Mode designation in circular cylindrical waveguide (Optical Fiber)

$\mathrm{TE}_{\nu m}$ modes : The electric field vector lies in transverse plane.

$\mathrm{TM}_{\nu m}$ modes : The magnetic field vector lies in transverse plane.

Hybrid $\mathrm{HE}_{\nu m}$ modes : E_z component is larger than H_z component.

Hybrid $\mathrm{EH}_{\nu m}$ modes : H_z component is larger than E_z component.



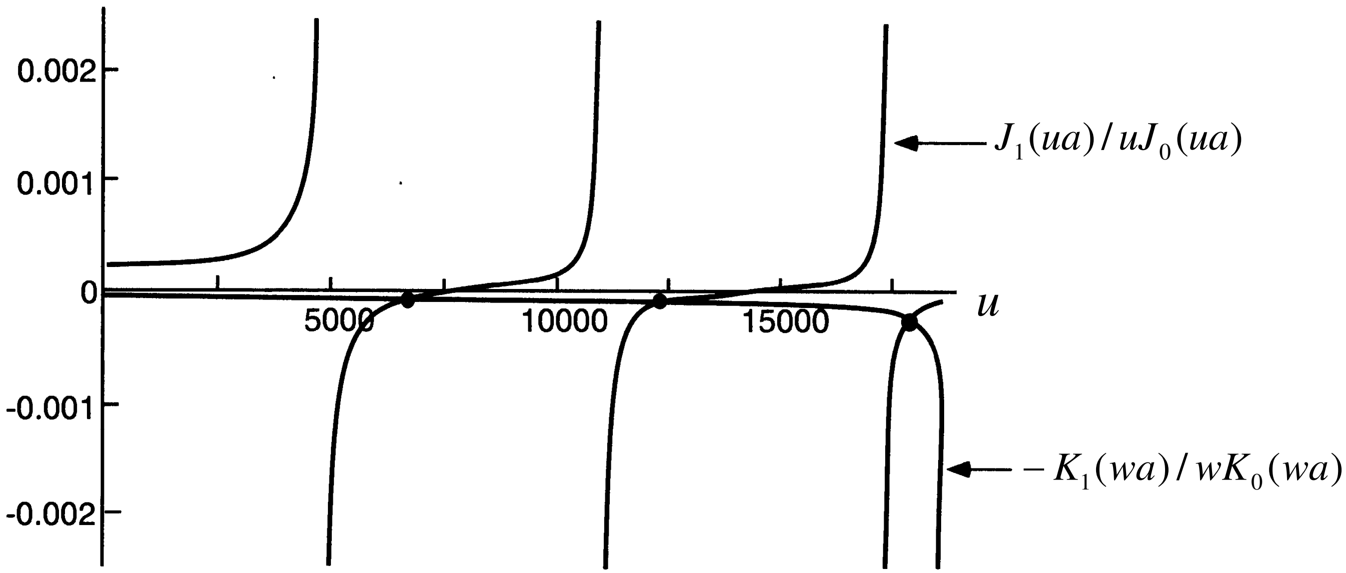
ν= # of variation cycles or zeros in ϕ direction.

m= # of variation cycles or zeros in r direction.

Example of TE Modes

Given $n_1 = 1.5$, $n_2 = 1.45$, $a = 5$ μm. $\lambda = 1.3$ μm. Find TE modes. The characteristic equation is given by:

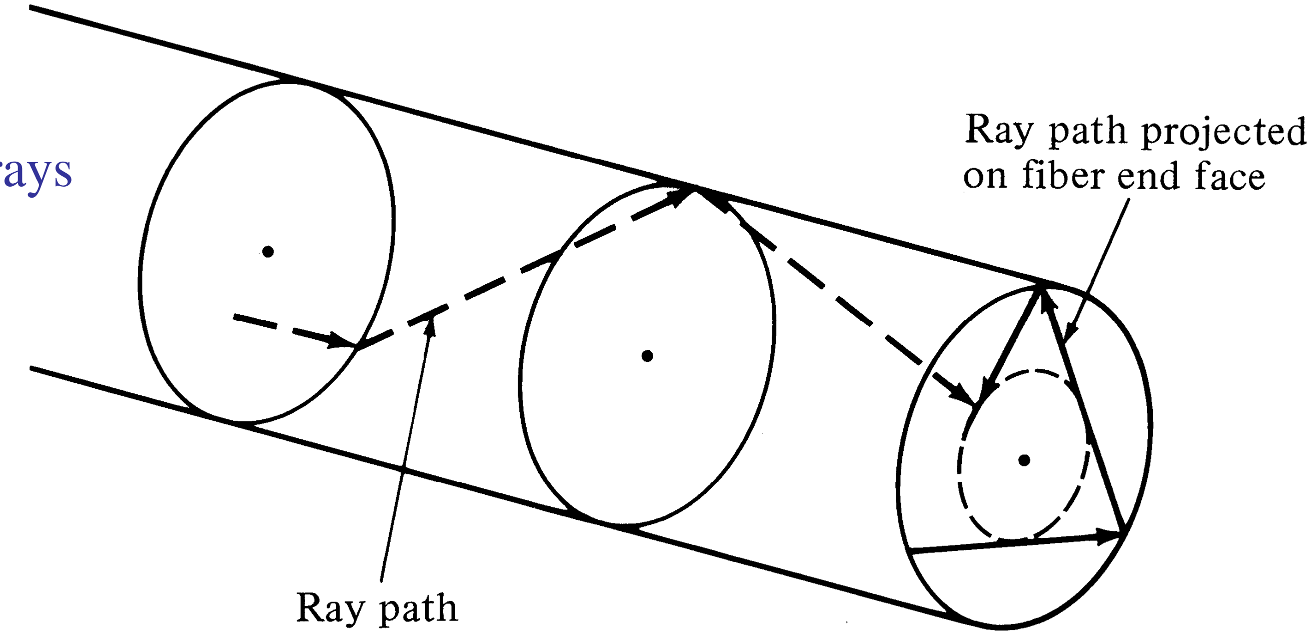
$$\frac{J_1(ua)}{uJ_0(ua)} + \frac{K_1(wa)}{wK_0(wa)} = 0$$



Characteristic equation plot

Ray Optics Theory (Step-Index Fiber)

Skew rays



When ν=0 (no ϕ-variation), rays lie in a plane that intersects the fiber axis, which is called "meridional" rays. Hybrid modes has ϕ-variation, resulting in "skew" rays.

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Characteristic Equation

- Under the weakly guiding approximation $(n_1 - n_2) << 1$
  - Valid for standard telecommunications fibers

$$\boxed{ua\frac{J_\nu'(ua)}{J_\nu(ua)} = wa\frac{K_\nu'(wa)}{K_\nu(wa)}}$$

$$u^2 = k_1^2 - \beta^2 = n_1^2\, k_o^2 - \beta^2 \qquad w^2 = \beta^2 - k_2^2 = \beta^2 - n_2^2\, k_o^2$$

$$J_l'(x) = \pm J_{l\pm1}(x) \mp l\,\frac{J_l(x)}{x} \qquad K_l'(x) = -K_{l\pm1}(x) \mp l\,\frac{K_l(x)}{x}$$

- Substitute to eliminate the derivatives

$$ua\frac{J_{\nu-1}(ua)}{J_\nu(ua)} = -wa\frac{K_{\nu-1}(wa)}{K_\nu(wa)} \quad or \quad ua\frac{J_{\nu+1}(ua)}{J_\nu(ua)} = wa\frac{K_{\nu+1}(wa)}{K_\nu(wa)}$$

HE Modes EH Modes

Lowest Order Modes

- Look at the ν=*-1, 0, 1* modes

$$x = ua \qquad y = wa \qquad V = \sqrt{x^2 + y^2} = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2}$$

- Use Bessel function properties to get positive order and highest order on top

□ **ν=-1**

$$x\frac{J_{-2}(x)}{J_{-1}(x)} = -y\frac{K_{-2}(y)}{K_{-1}(y)} \qquad x\frac{J_0(x)}{J_1(x)} = -y\frac{K_0(y)}{K_1(y)}$$

$$\boxed{x\frac{J_2(x)}{J_1(x)} = y\frac{K_2(y)}{K_1(y)}}$$

□ **ν=0**

$$x\frac{J_{-1}(x)}{J_0(x)} = -y\frac{K_{-1}(y)}{K_0(y)}$$

$$x\frac{J_1(x)}{J_0(x)} = y\frac{K_1(y)}{K_0(y)} \qquad \boxed{x\frac{J_1(x)}{J_0(x)} = y\frac{K_1(y)}{K_0(y)}}$$

Lowest Order Modes cont.

- ν **=+1**

$$x\frac{J_0(x)}{J_1(x)} = -y\frac{K_0(y)}{K_1(y)}$$

$$x\frac{J_2(x)}{J_1(x)} = y\frac{K_2(y)}{K_1(y)}$$

$$x\frac{J_2(x)}{J_1(x)} = y\frac{K_2(y)}{K_1(y)}$$

- So the 6 equations collapse down to 2 equations

$$\boxed{x\frac{J_2(x)}{J_1(x)} = y\frac{K_2(y)}{K_1(y)}}$$

$$\boxed{x\frac{J_1(x)}{J_0(x)} = y\frac{K_1(y)}{K_0(y)}}$$

lowest modes

First Mode Cut-Off

- First mode
  - What is the smallest allowable V?
  - Let $y \rightarrow 0$ and the corresponding $x \rightarrow V$

$$V\frac{J_1(V)}{J_0(V)} = \lim_{y\to 0} y\frac{K_1(y)}{K_0(y)} = \lim_{y\to 0}\frac{y\left(\frac{1}{2}\right)\left(\frac{2}{y}\right)}{-\ln\left(\frac{y}{2}\right)-0.5772} = 0$$

$$J_1(V) = 0$$

  - So V=0, no cut-off for lowest order mode
  - Same as a symmetric slab waveguide

Second Mode Cut-Off

- Second mode

$$V\frac{J_2(V)}{J_1(V)}=\lim_{y\to 0} y\frac{K_2(y)}{K_1(y)}=\lim_{y\to 0}\frac{y\left(\frac{1}{2}\right)\left(\frac{2}{y}\right)^2}{\left(\frac{1}{2}\right)\left(\frac{2}{y}\right)}=2$$

$$J_2(V)=\frac{2}{V}J_1(V)$$

$$J_{n+1}(x)=\frac{2n}{x}J_n(x)-J_{n-1}(x)$$

$$J_2(x)=\frac{2}{x}J_1(x)-J_0(x)$$

$$\frac{2}{V}J_1(V)-J_o(V)=\frac{2}{V}J_1(V)$$

$$J_o(V)=0$$

$$\boxed{V=2.405}$$

Cut-off Condition

ν	Mode	Cutoff condition
0	TE_{0m}, TM_{0m}	$J_0(ua) = 0$
1	HE_{1m}, EH_{1m}	$J_1(ua) = 0$
≥ 2	$EH_{\nu m}$	$J_\nu(ua) = 0$
	$HE_{\nu m}$	$\left(\frac{n_1^2}{n_2^2} + 1\right) J_{\nu-1}(ua) = \frac{ua}{\nu - 1} J_\nu(ua)$

Number of Modes

- The number of modes can be characterized by the normalized frequency

$$V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2}$$

- Most standard optical fibers are characterized by their numerical aperture

$$\mathrm{NA} = \sqrt{n_1^2 - n_2^2}$$

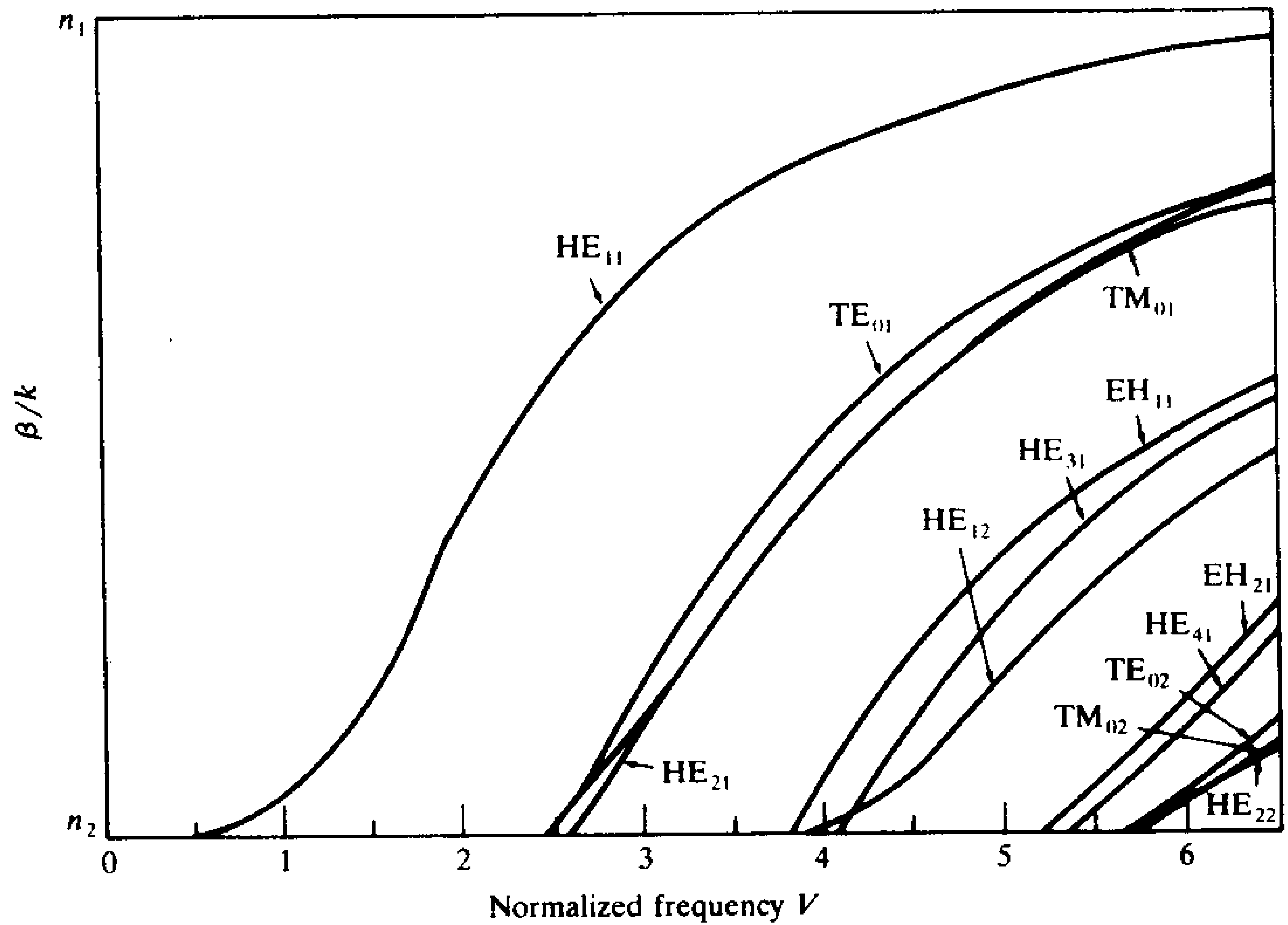
- Normalized frequency is related to numerical aperture

$$V = \frac{2\pi}{\lambda} a \ \mathrm{N}A$$

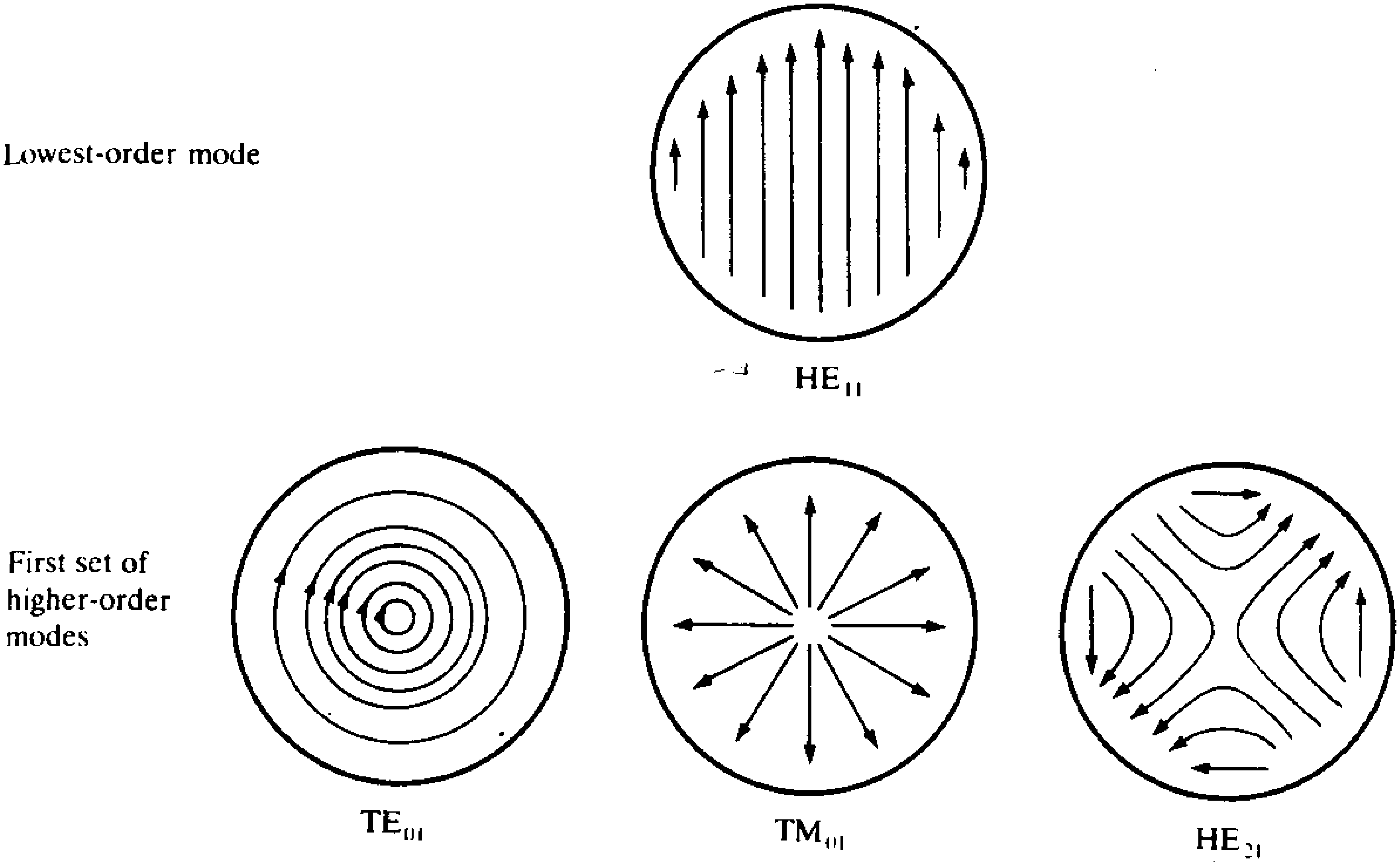
- The optical fiber is single mode if **V<2.405**
- For large normalized frequency the number of modes is approximately

$$\#\mathrm{Modes} \approx \frac{4}{\pi^2} V^2 \qquad V >> 1$$

Fiber Modes


n_1
n_2
β/k
HE$_{11}$
TE$_{01}$
TM$_{01}$
EH$_{11}$
HE$_{31}$
HE$_{12}$
EH$_{21}$
HE$_{41}$
TE$_{02}$
TM$_{02}$
HE$_{21}$
HE$_{22}$
0
1
2
3
4
5
6
Normalized frequency V

Electric Field Profiles



Weakly Guided Modes

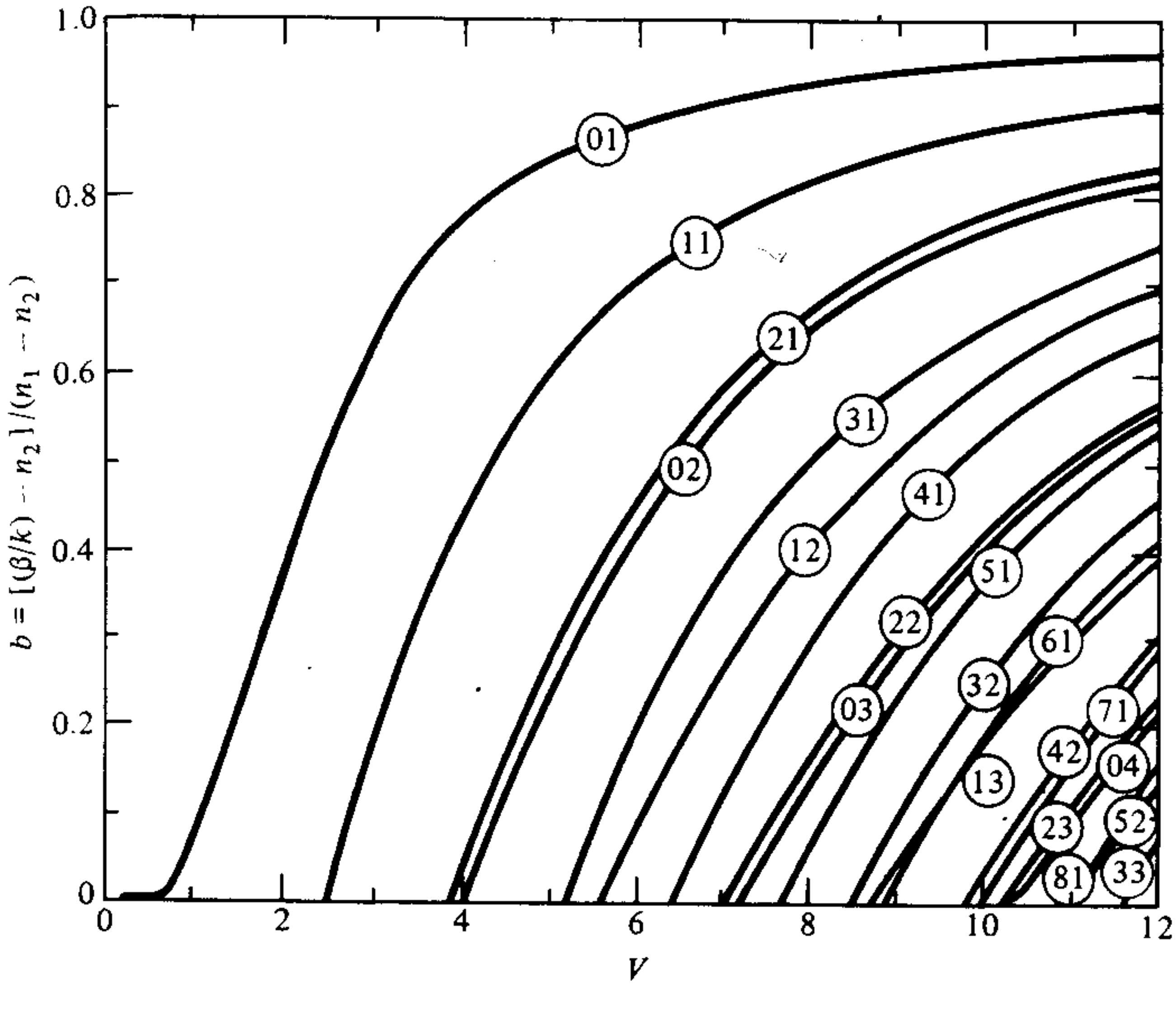
- The refractive index difference between the core and cladding is very small, i.e., $n_1 - n_2 << 1$
- There is degeneracy between modes
  - Groups of modes travel with the same velocity (β equal)
- Modes are approximated with nearly linearly polarized modes called LP modes
  - LP_{01} from HE_{11} (Fundamental Mode)
  - LP_{0m} from HE_{1m}
  - LP_{1m} sum of TE_{0m}, TM_{0m}, and HE_{2m}
  - $LP_{\nu m}$ sum of $HE_{\nu+1,m}$ and $EH_{\nu-1,m}$

Lower-order LP modes

TABLE 2-2
Composition of the lower-order linearly polarized modes

LP-mode designation	Traditional-mode designation and number of modes	Number of degenerate modes
LP_{01}	$HE_{11} \times 2$	2
LP_{11}	$TE_{01}, TM_{01}, HE_{21} \times 2$	4
LP_{21}	$EH_{11} \times 2, HE_{31} \times 2$	4
LP_{02}	$HE_{12} \times 2$	2
LP_{31}	$EH_{21} \times 2, HE_{41} \times 2$	4
LP_{12}	$TE_{02}, TM_{02}, HE_{22} \times 2$	4
LP_{41}	$EH_{31} \times 2, HE_{51} \times 2$	4
LP_{22}	$EH_{12} \times 2, HE_{32} \times 2$	4
LP_{03}	$HE_{13} \times 2$	2
LP_{51}	$EH_{41} \times 2, HE_{61} \times 2$	4

Plot of *b*-*V*

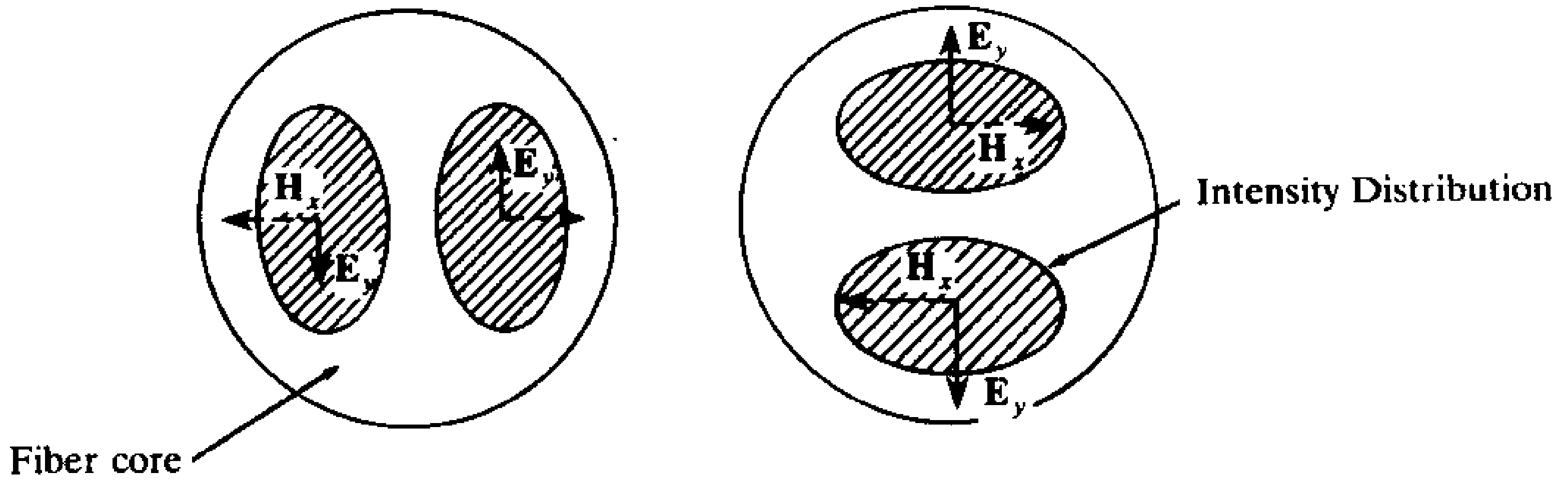


1.0
0.8
0.6
0.4
0.2
0
b = [(β/k) − n2]/(n1 − n2)
0
2
4
6
8
10
12
V
01
11
21
02
31
41
12
51
22
61
32
03
71
42
04
13
23
52
81
33

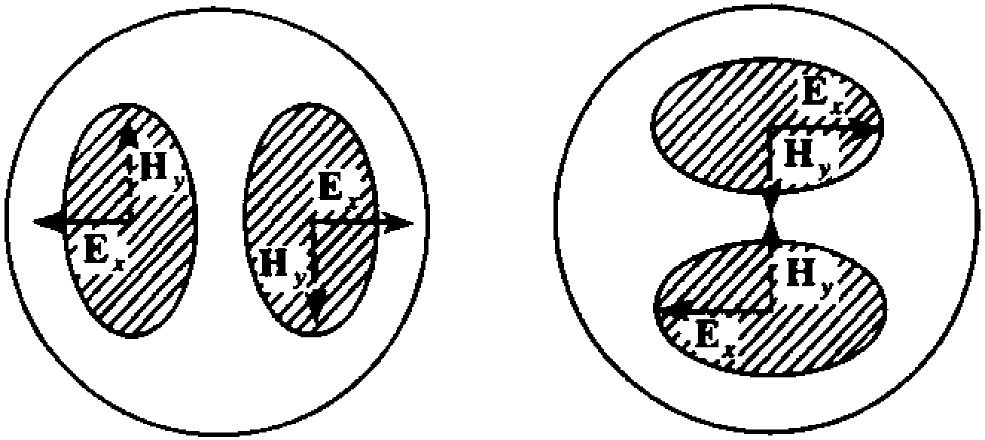
Cut-off V-parameter for low-order LP_{lm} modes

	m=1	**m=2**	**m=3**
l=0	0	3.832	7.016
l=1	2.405	5.520	8.654

Transverse Electric Field of LP_{11} mode

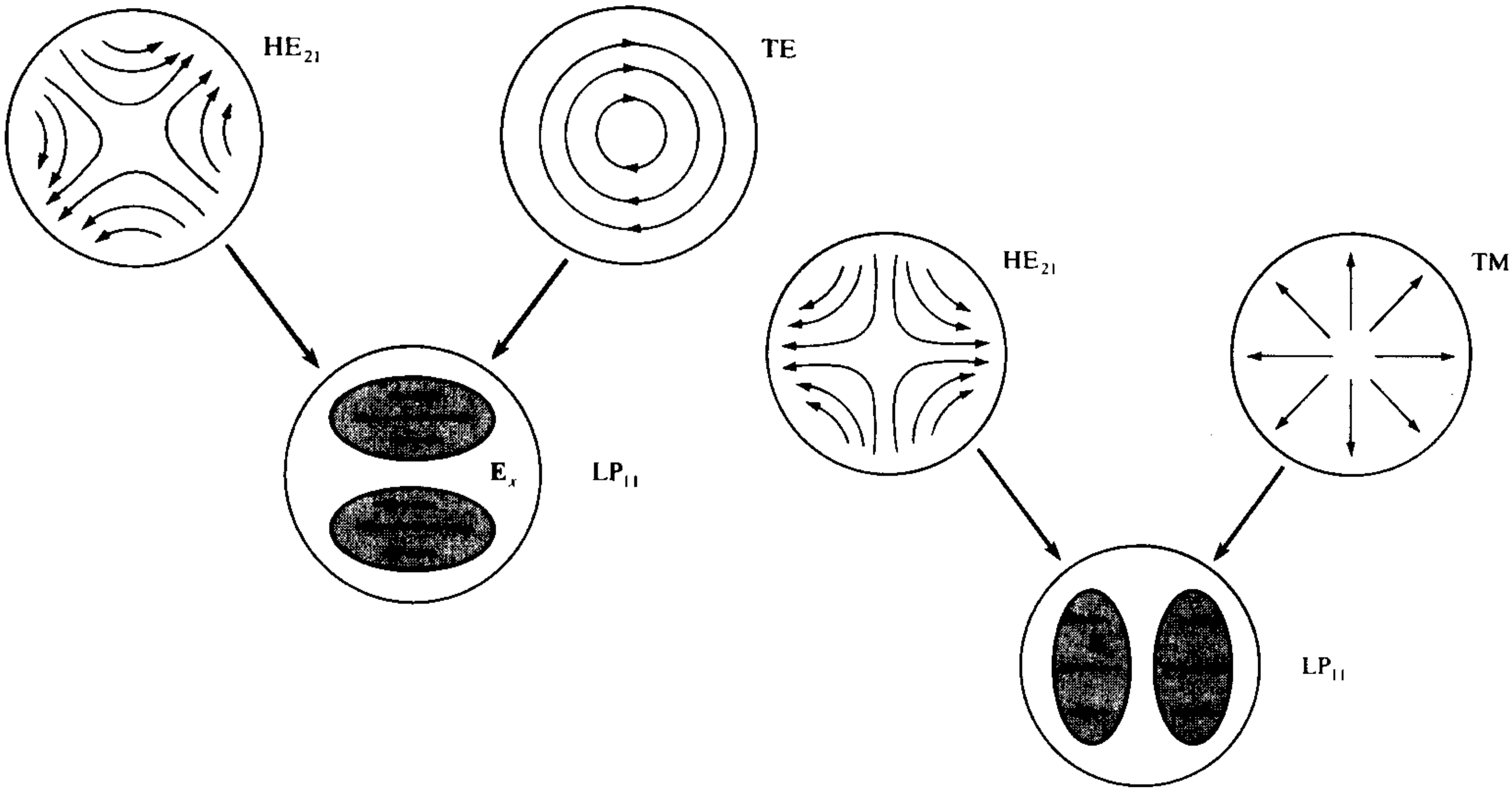


E vertically polarized

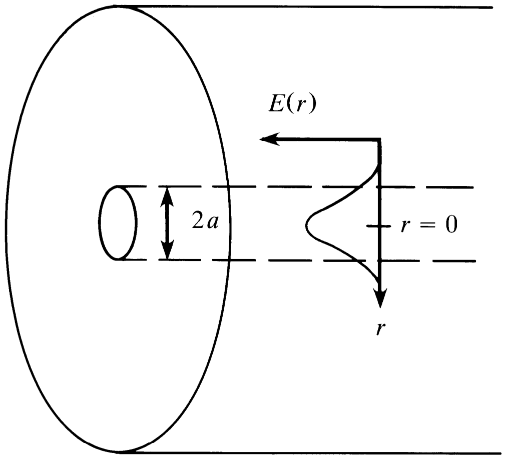


E horizontally polarized

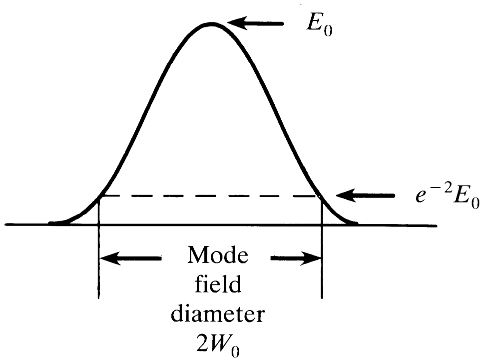
LP_{11} mode

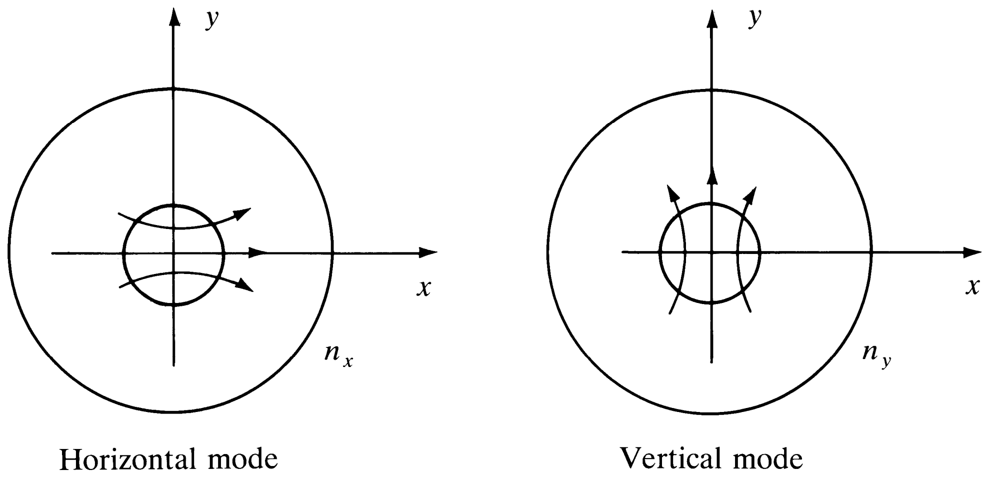


Fundamental Mode (HE_{11}) Field Distribution



Mode field diameter (MFD)





Polarizations of fundamental mode

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Single mode Operation

- The cut-off wavelength or frequency for each mode is obtained from:

$$\beta_{lm}(\omega_c) = n_2 k = \frac{2\pi n_2}{\lambda_c} = \frac{\omega_c n_2}{c}$$

- **Single mode operation** is possible (Single mode fiber) when:

$$V \leq 2.405$$

Only HE_{11} can propagate faithfully along optical fiber

Single-Mode Fibers

$$\Delta = 0.1\% \text{ to } 1\% \quad ; \qquad a = 6 \text{ to } 12\ \mu\text{m};$$

$$\text{V} = 2.3 \text{ to } 2.4 \quad @ \text{ max frequency or min } \lambda$$

- **Example**: A fiber with a radius of 4 micrometer and $n_1 = 1.500$ & $n_2 = 1.498$ has a normalized frequency of *V=2.38* at a wavelength 1 micrometer. The fiber is single-mode for all wavelengths greater and equal to 1 micrometer.

MFD (Mode Field Diameter): The electric field of the first fundamental mode can be written as:

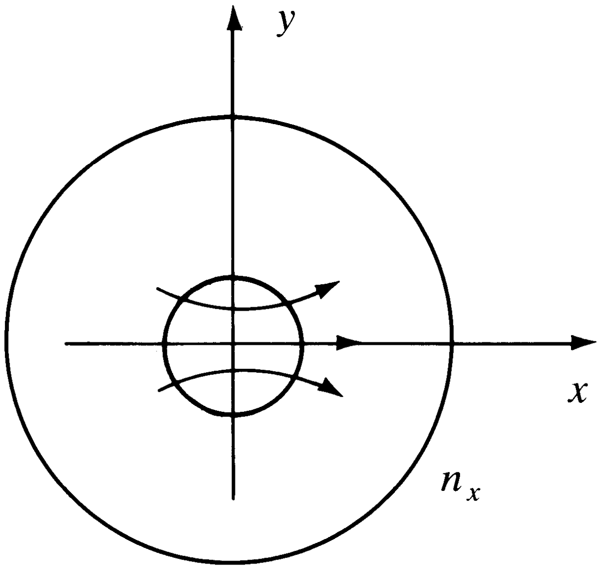
$$E(r) = E_0 \exp(-\frac{r^2}{W_0{}^2});$$

$$\text{MFD} = 2W_0 = 2\left[\frac{2\int_0^\infty E^2(r)r^3dr}{\int_0^\infty E^2(r)rdr}\right]^{1/2}$$

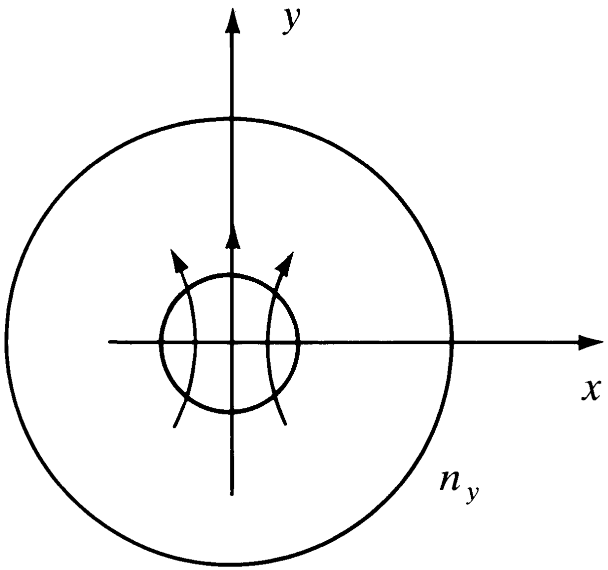
Birefringence in single-mode fibers

- Because of asymmetries the refractive indices for the two degenerate modes (vertical & horizontal polarizations) are different. This difference is referred to as **birefringence**, B_f :

$$B_f = n_y - n_x$$



Horizontal mode



Vertical mode

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Fiber Beat Length

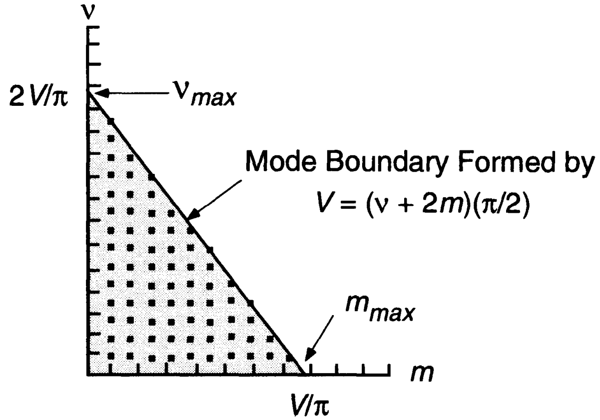
- In general, a linearly polarized mode is a combination of both of the degenerate modes. As the modal wave travels along the fiber, the difference in the refractive indices would change the phase difference between these two components & thereby the state of the polarization of the mode. However after certain length referred to as **fiber beat length,** the modal wave will produce its original state of polarization. This length is simply given by:

$$L_p = \frac{2\pi}{kB_f}$$

Multi-Mode Operation

- Total number of modes, M, supported by a multi-mode fiber is approximately (When V is large) given by:

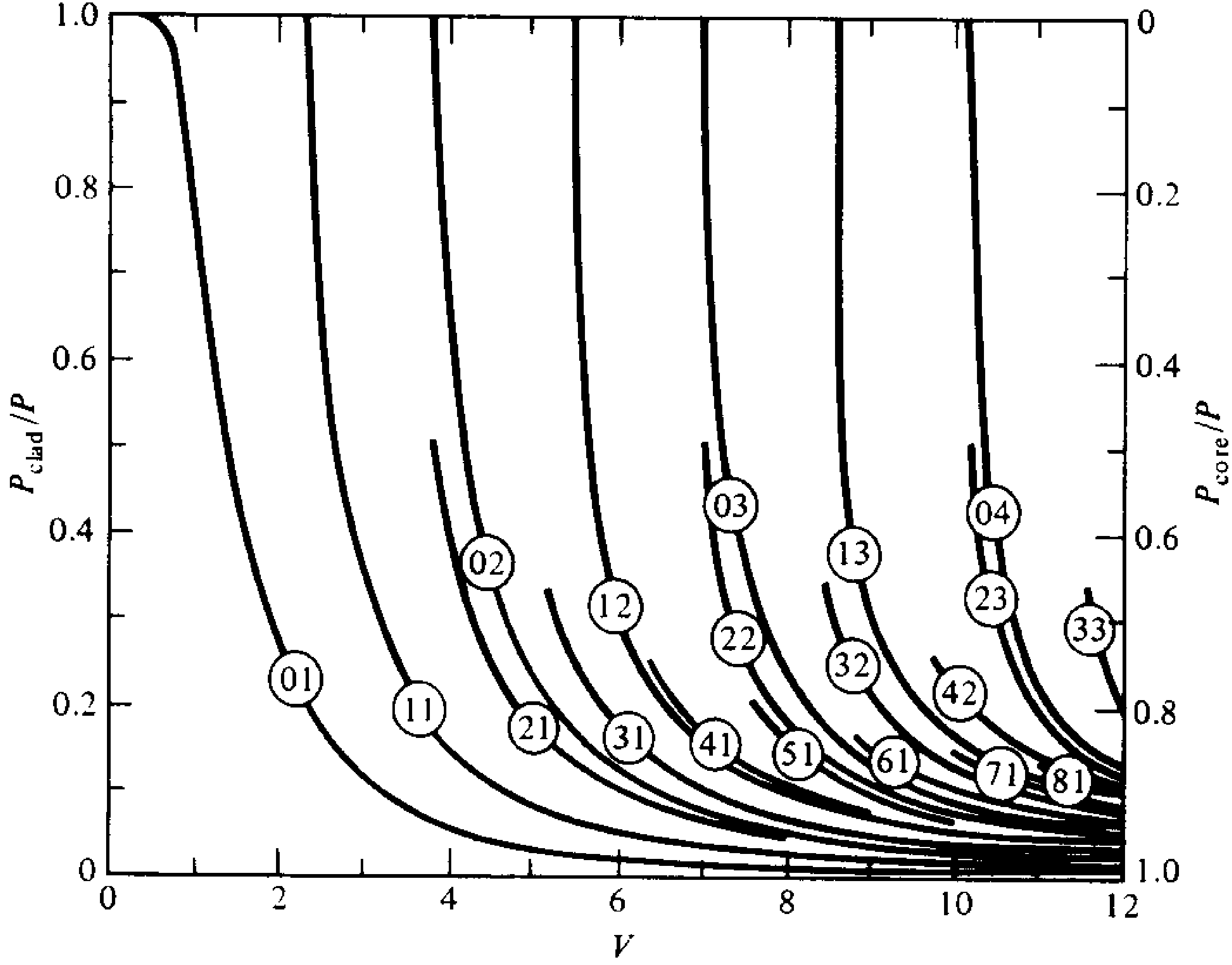
$$M = \frac{4V^2}{\pi^2} \approx \frac{V^2}{2}$$



- **Power distribution in the core & the cladding:** Another quantity of interest is the ratio of the mode power in the cladding, P_{clad} to the total optical power in the fiber, P, which at the wavelengths (or frequencies) far from the cut-off is given by:

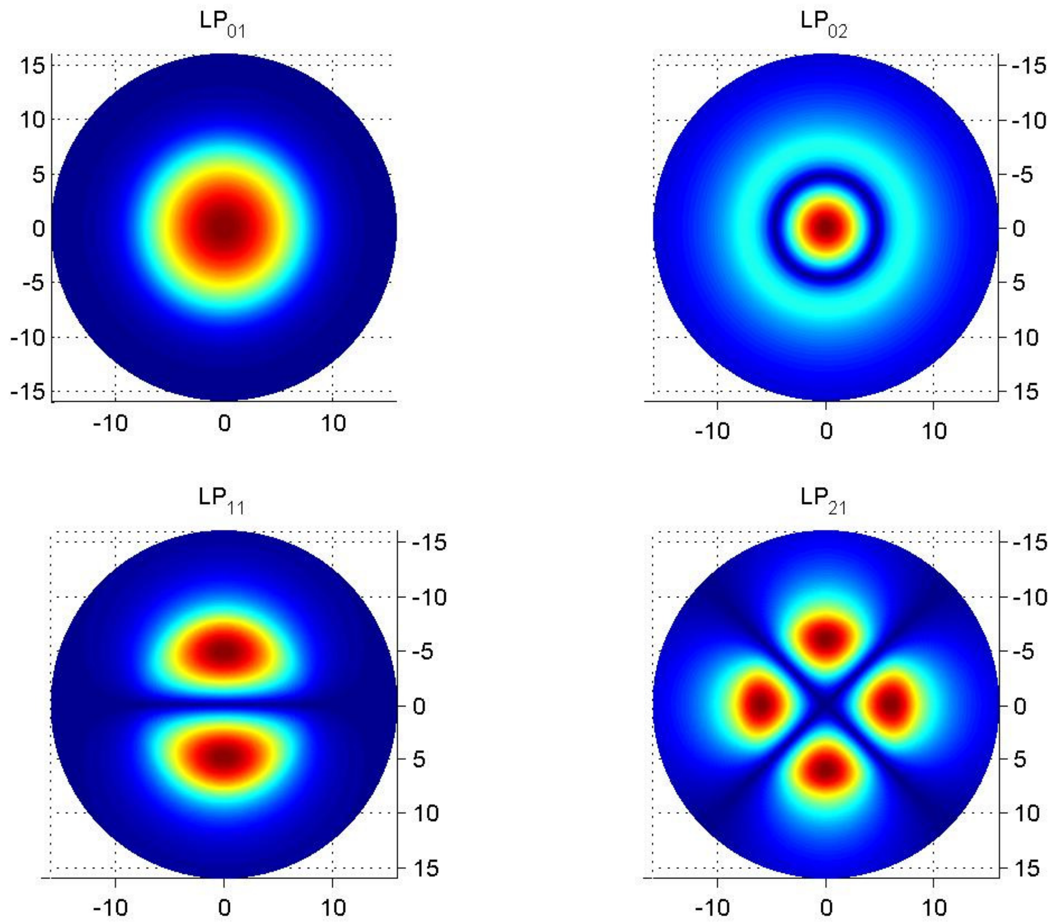
$$\frac{P_{clad}}{P} \approx \frac{4}{3\sqrt{M}}$$

Power Flow



1.0
0.8
0.6
0.4
0.2
0
P_{clad}/P
0
0.2
0.4
0.6
0.8
1.0
P_{core}/P
0
2
4
6
8
10
12
V
01
11
21
31
41
51
61
71
81
02
12
22
32
42
03
13
23
33
04

Intensity Profiles



Standard Single Mode Optical Fibers

- Most common single mode optical fiber: SMF28 from Corning
  - Core diameter d_{core}=8.2 μm
  - Outer cladding diameter: d_{clad}=125μm
  - Step index
  - Numerical Aperture NA=0.14
    - NA=sin(θ)
    - $\Delta\theta=8°$
    - λ_{cutoff}= 1260nm (single mode for $\lambda>\lambda_{cutoff}$)
    - Single mode for both λ=1300nm and λ=1550nm standard telecommunications wavelengths

Standard Multimode Optical Fibers

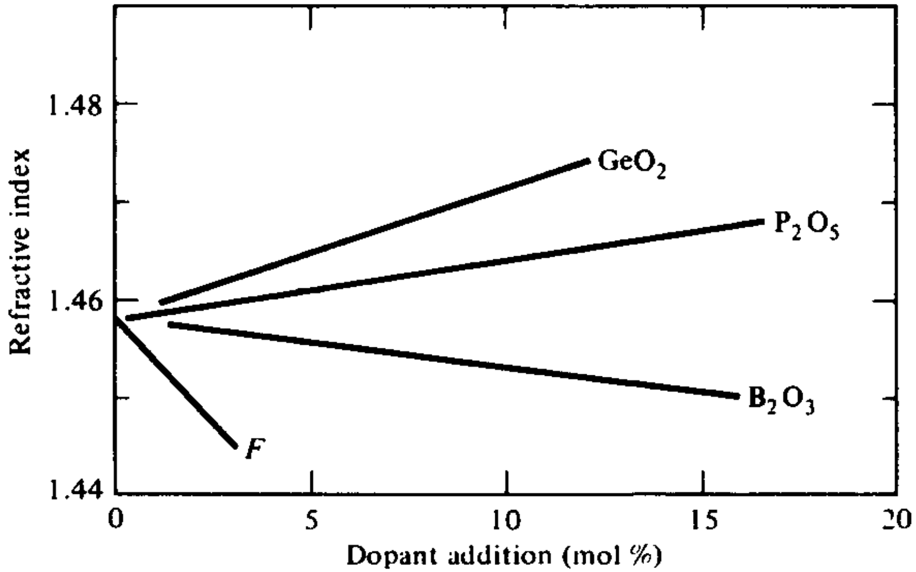
- Most common multimode optical fiber: 62.5/125 from Corning
  - Core diameter d_{core}= 62.5 μm
  - Outer cladding diameter: d_{clad}=125μm
  - Graded index
  - Numerical Aperture NA=0.275
    - NA=sin(θ)
    - Δθ=16°
    - Many modes

Fiber Materials

- Two major types: Glass optical fiber and Plastic (or Polymer) optical fiber (POF)
- Glass fiber features:
  - Advantages: low attenuation, cheap and abundant raw material (mostly sand)
  - Disadvantages: high installation cost, complexity, requires skilled technicians, inflexible, easy-to-break

Glass Materials

- Oxide glass
  - Silica (SiO_2) is most common; good over wide range, especially around 1.5 μm
  - Use *dopants* to change refractive index.



- Metal Halide (Metal+Halogen)
  - Fluoride glass : very low loss for mid-infrared (2-5 μm) range)
- Active glass, like active electronic devices : can be used to amplify or attenuate; erbium and neodymium are most common.

Plastic Optical Fiber

- Advantages:
  - Simpler and less expensive components
  - Lighter weight
  - greater flexibility and ease in handling and connecting (POF diameters are 1 mm compared with 8-100 mm for glass)
  - Lower installation cost
- Disadvantages:
  - High attenuation
  - Limited production

POF Material

- Typically,
  - Core : PMMA (poly(methylmethacryclate)) (acrylic), PFP (perfluorinated polymers), polysterene
  - Cladding : Silicone resin, Fluorinated polymer
- Features: High refractive index difference (core : 1.5-1.6, cladding : 1.46) ; Large NA
- Examples:
  - Polysterene core (n=1.6), methy methacrylate cladding (n=1.49) -> NA = 0.6

Plastic fibers

TABLE 2-4
Sample characteristics of PMMA and PFP polymer optical fibers

	PMMA POF	PFP POF
Core diameter	0.4 mm	0.125–0.30 mm
Cladding diameter	1.0 mm	0.25–0.60 mm
Numerical aperture	0.25	0.20
Attenuation	150 dB/km at 650 nm	60–80 dB/km at 650–1300 nm
Bandwidth	2.5 Gb/s over 100 m	2.5 Gb/s over 300 m

[*Optical Fiber Communications*, 3rd Edition, by Gerd Keiser, Mc Graw-Hill, 2000]

Comparison between glass fiber, POF, copper wire

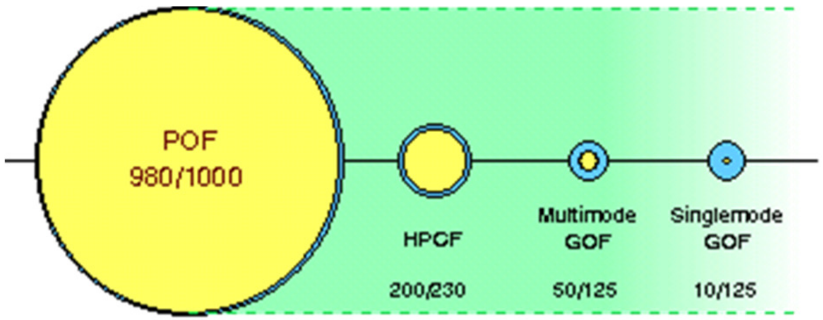
	PLASTIC	GLASS	COPPER
Component costs	Potentially low	More expensive	Low
Loss	High–medium (short distance)	Medium–low (long distance)	High
Connections	Easy to connect, requires little training or special tools	Takes longer, requires special tools and training	High
Handling	Easy	Requires training and care	Easy
Flexibility	Flexible	Brittle	Flexible
Wavelength operating range	Visible	Infrared	NA
Numerical aperture	High (0.4)	Low (0.1–0.2)	NA
Bandwidth	High (11 Gbps over 100 m)	Large (40 Gbps)	Limited to 100 m at 100 Mbps
Test equipment	Low cost	Expensive	High
System costs	Low overall	High	Medium

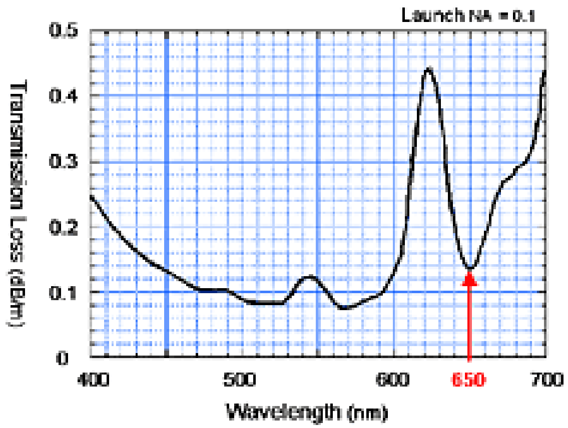
Types of optical fibers & Applications

- Single mode glass—long distance communications
- Multimode glass—short distance communications
- Plastic—consumer short distance, electronics & cars
- Hybrid or polymer clad (glass core, plastic cladding)—lighting, consumer applications

Plastic optical fiber (POF)

- 1000 µm diameter, 980 µm core
- Strong
- Uses LEDs in visible range, 650 nm
- Not suitable for long-distance uses
- Does not transmit infrared





Source: Pofeska/Mitsubishi Rayon Co.

Requirements for fabricating useful optical fiber

- Materials must be extremely pure
  - Impurity < 1 part per billion for metals
  - Impurity < 1 part per 10 million for water
- About 1000 times more pure than traditional chemical purification techniques allow
- Dimensions must be controlled to extremely high degree
  - Core size, position, cladding size tolerances ~ 1 micron or less
  - Roughly 1 wavelength of light
  - Refractive indices must also be very precisely controlled
- Must be made in long lengths
- Must have tensile strength

Fiber Fabrication Procedure

- Direct-melt : like traditional glass-making, fibers made directly from the molten state of purified components of silicate glasses
- Vapor phase oxidation : Three Steps are Involved
  - Making a Preform Glass Cylinder
  - Drawing of preform down into thin fiber
  - Jacketing and cabling

Fiber Fabrication (cont’d)

- First SiO_2 particles are formed by reaction of metal halides vapors and oxygen, then collected on a bulk glass and sintered -> *preform*
- Preform : A solid glass rod or tube which is a scale model of desired fiber.
- typically around 10 to 25 nm in diameter and 60 to 120 cm long.

Purification of silica : a two-step process

- First: use distillation
  - Heat silica to boiling point (2230^{o} C), condense gas
  - Metals are heavier and do not boil at this temperature
  - Yields impurity levels of ~ 10^{-6}
- Second stage takes place when fiber fabricated

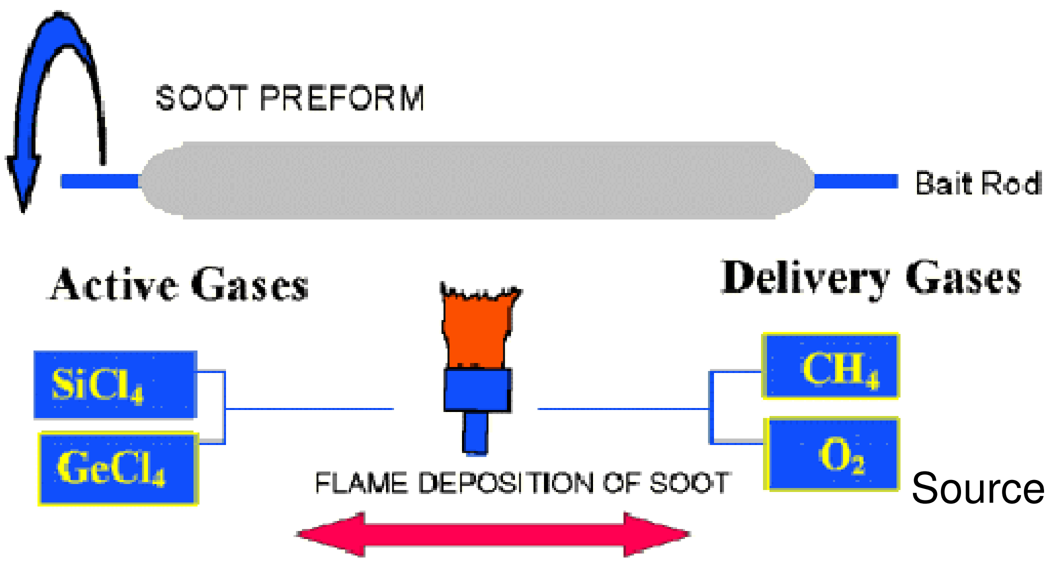
Fiber fabrication process

- Called “Outside Vapor Deposition Process” or OVD process
- Stages
  - Laydown
  - Consolidation
  - Drawing

First stage: Laydown

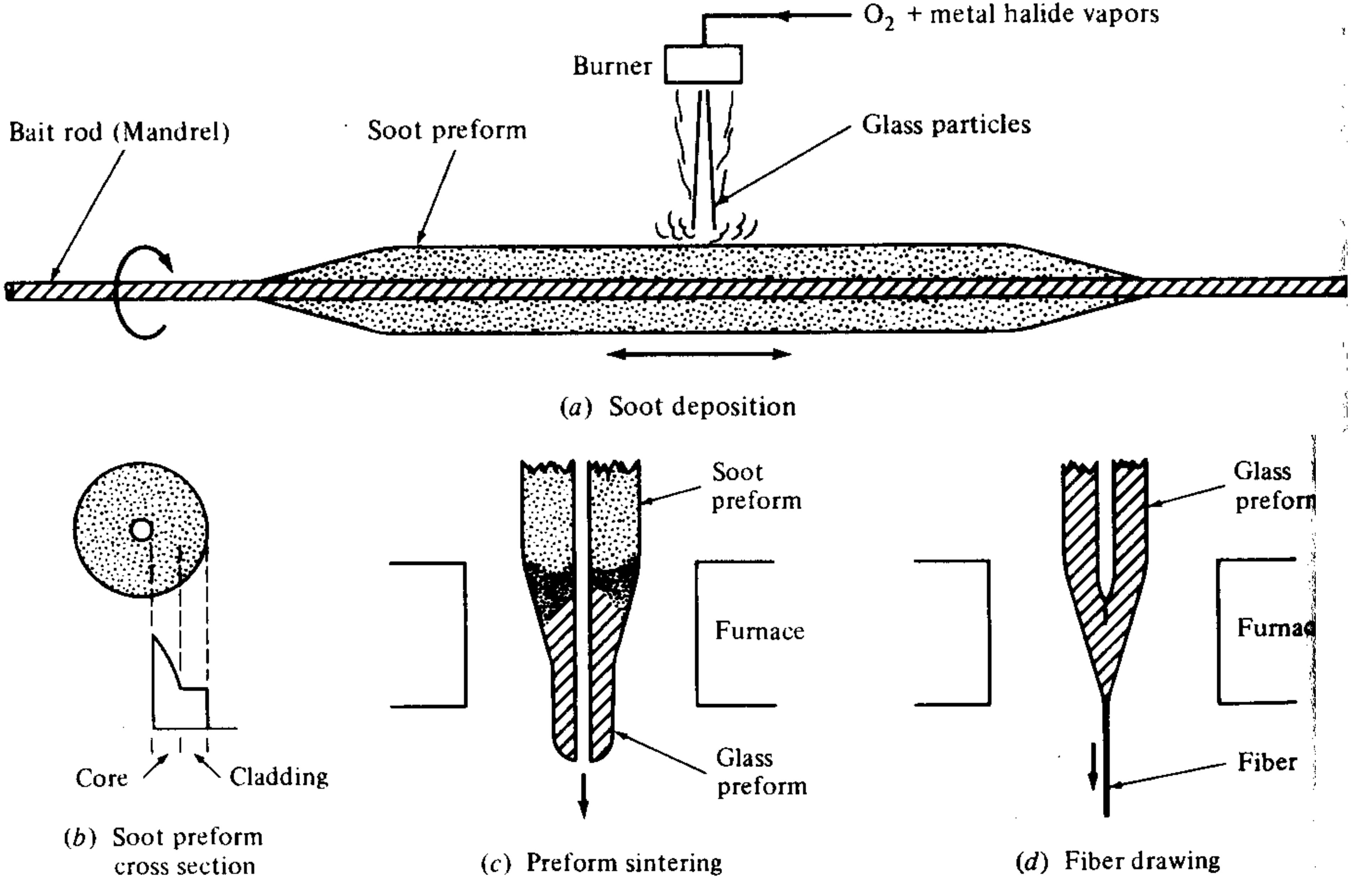
- Vapor deposition from ultrapure vapors
- Soot preform made when vapors exposed to burner and form fine soot particles of silica and germanium

(From particles of silica and germania)



Source: Corning

Outside Vapor Phase Oxidation (OVPO)



(a) Soot deposition

(b) Soot preform cross section

(c) Preform sintering

(d) Fiber drawing

Laydown (continued)

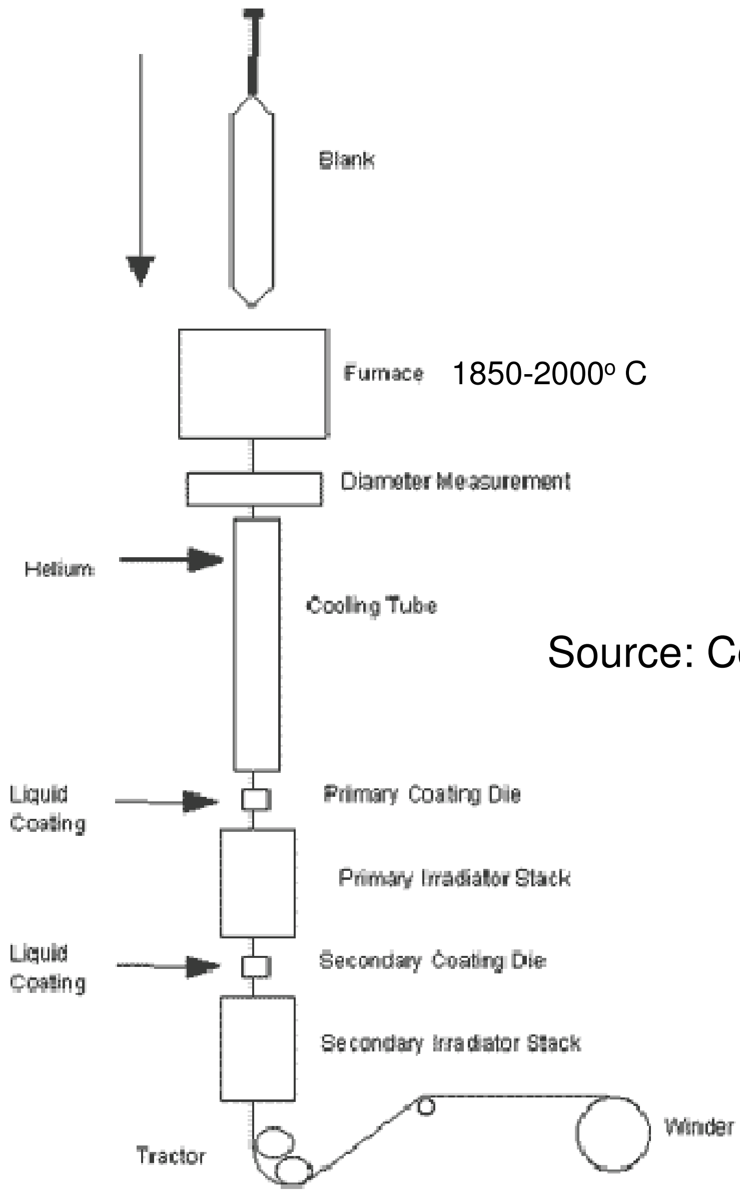
- Particles deposited on surface of rotating bait rod
  - Core first
  - Then silica cladding
- Vapor deposition process purifies fiber material as impurities do not deposit as rapidly
- Preform is somewhat porous at this stage

Second stage: consolidation

- Bait rod removed
- Placed in high-temperature consolidation furnace
  - Water vapor removed
  - Preform sintered into solid, dense, transparent glass
- Has same cross-section profile as final fiber, but is much larger (1-2.5 cm, final: 125 μm = .0125 cm)

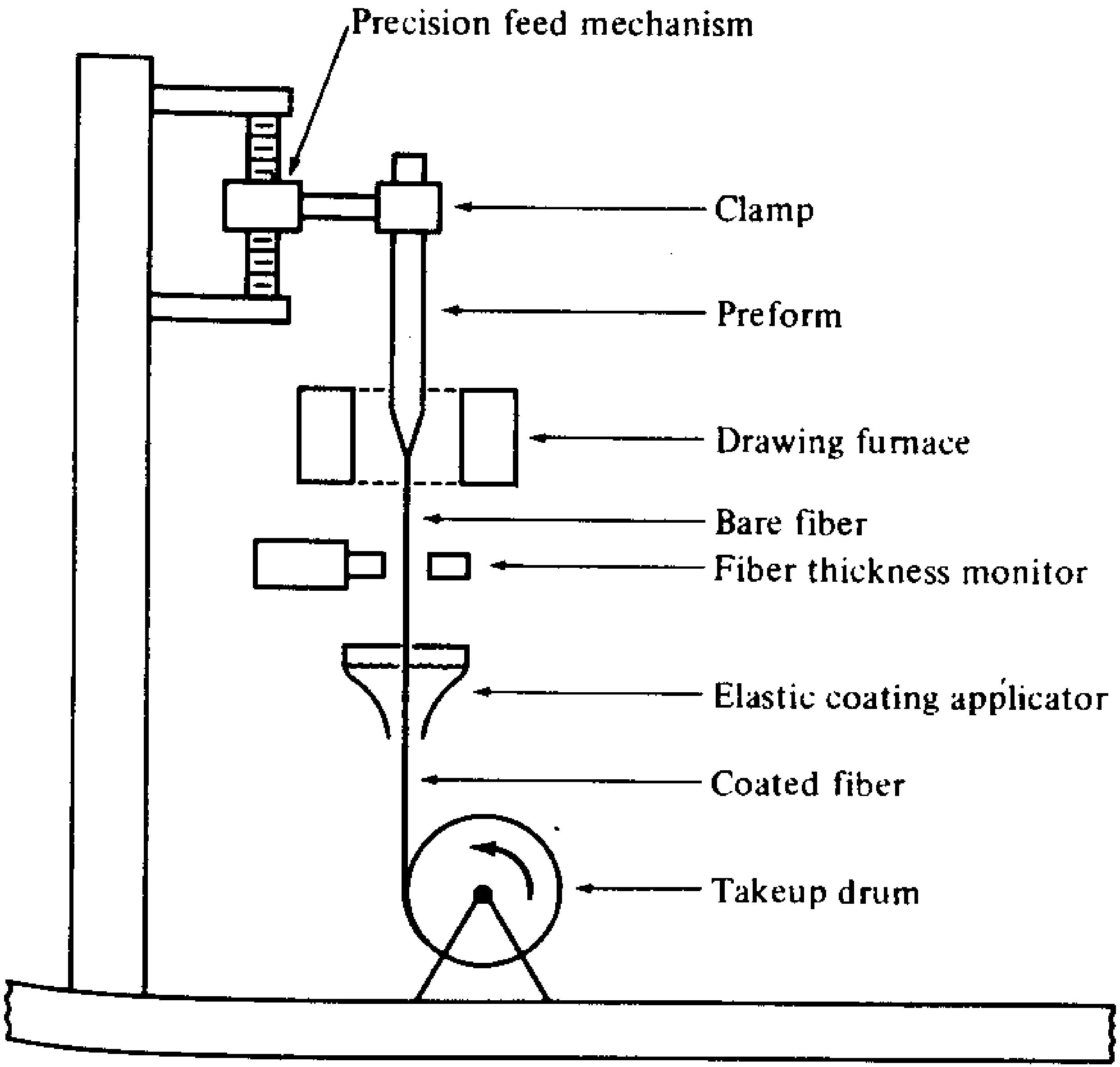
Third stage: drawing

- Done in "draw tower"
- Glass blank from consolidation stage lowered into draw furnace
- Tip heated until "gob" of glass falls
  - Pulls behind it a thin strand of glass
- Gob cut off
- Strand threaded into computer-controlled tractor assembly
- Sensors control speed of drawing to make precise diameter



Source: Corning

Fiber Drawing

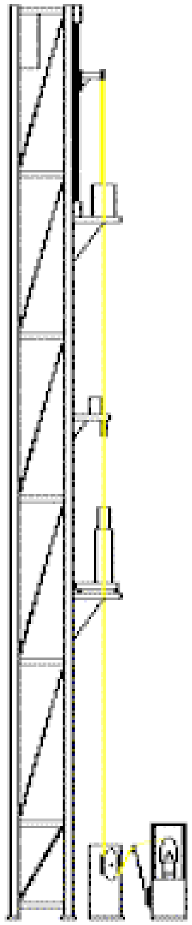


Drawing (continued)

- Diameter measured hundreds of times per second
  - Ensures precise outside dimension
- Primary and secondary coatings (jackets) applied
- At end, fiber wound onto spools for further processing

Gob forming,
Source: Corning

Draw tower



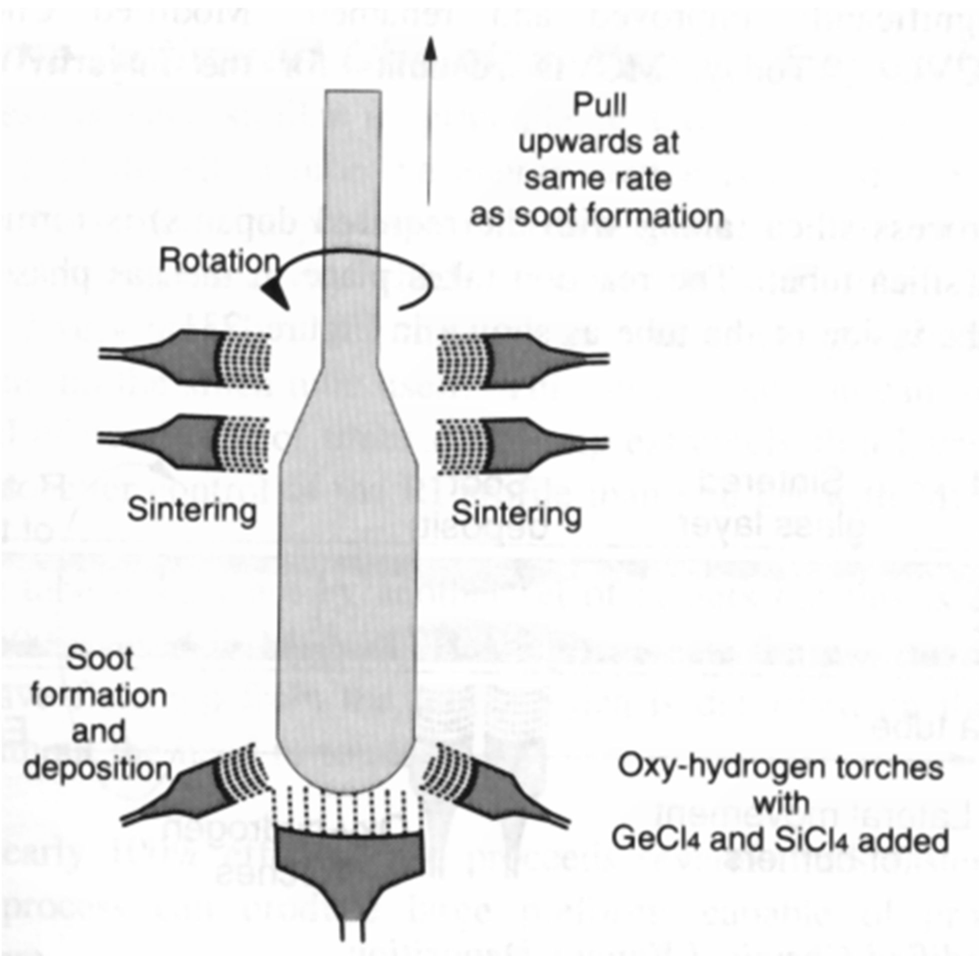
Technical Characteristics	
Drawing Speed	up to 80 feet/second
Tower Height	up to 30 feet
Max. Preform Length	50 inches
Max. No. of Preforms	1
Take-up Spool Capacity	500 miles
Max. No. of Spools	2
Fiber Diameter Accuracy	+/- 0.000025 inches
Preform Feed Speed Accuracy	+/- 0.01%
Max. Drawing Temperature	2400 deg C

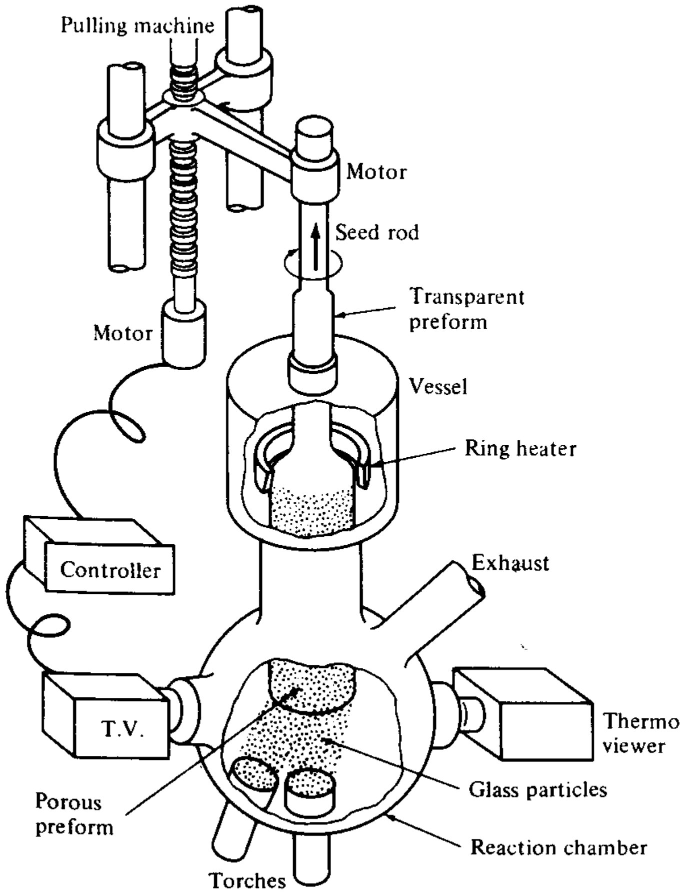
Source: Axsys

Other methods used to make fiber

- Vapor phase axial deposition (VAD)
  - Batch process
  - Preforms can be drawn up to 250 km
  - Flame hydrolysis
    - Soot formed and deposited by torches

VAD process (continued)





Source: Dutton

Other methods used to make fiber (continued)

- Modified chemical vapor deposition (MCVD)
  - Silica formed inside silica tube in gaseous phase reaction
  - Soot deposited on inside of tube
  - Burners traverse tube
    - Sinters soot
    - Produces highly controllable RI profile
  - At end, tube evacuated, sides collapse

MCVD process

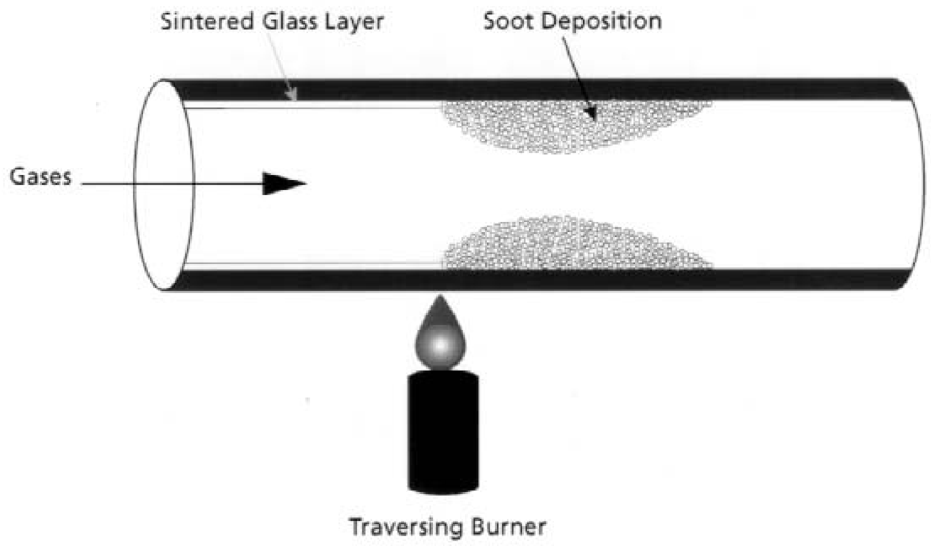
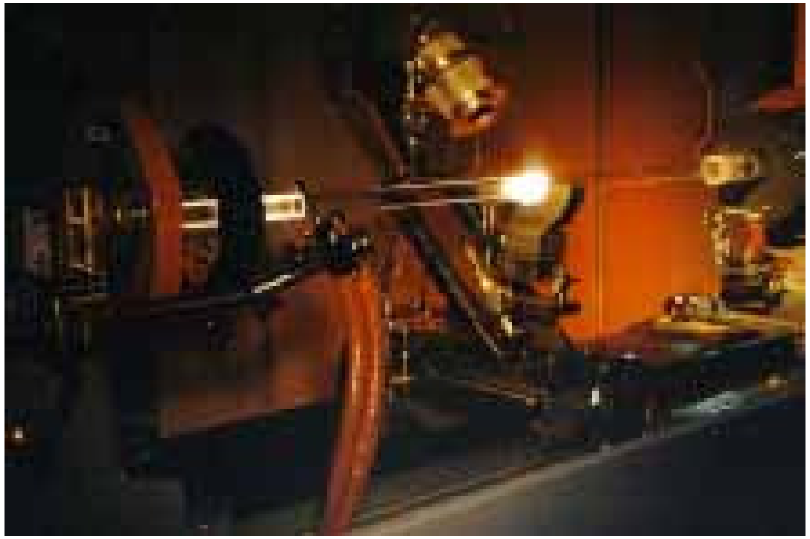
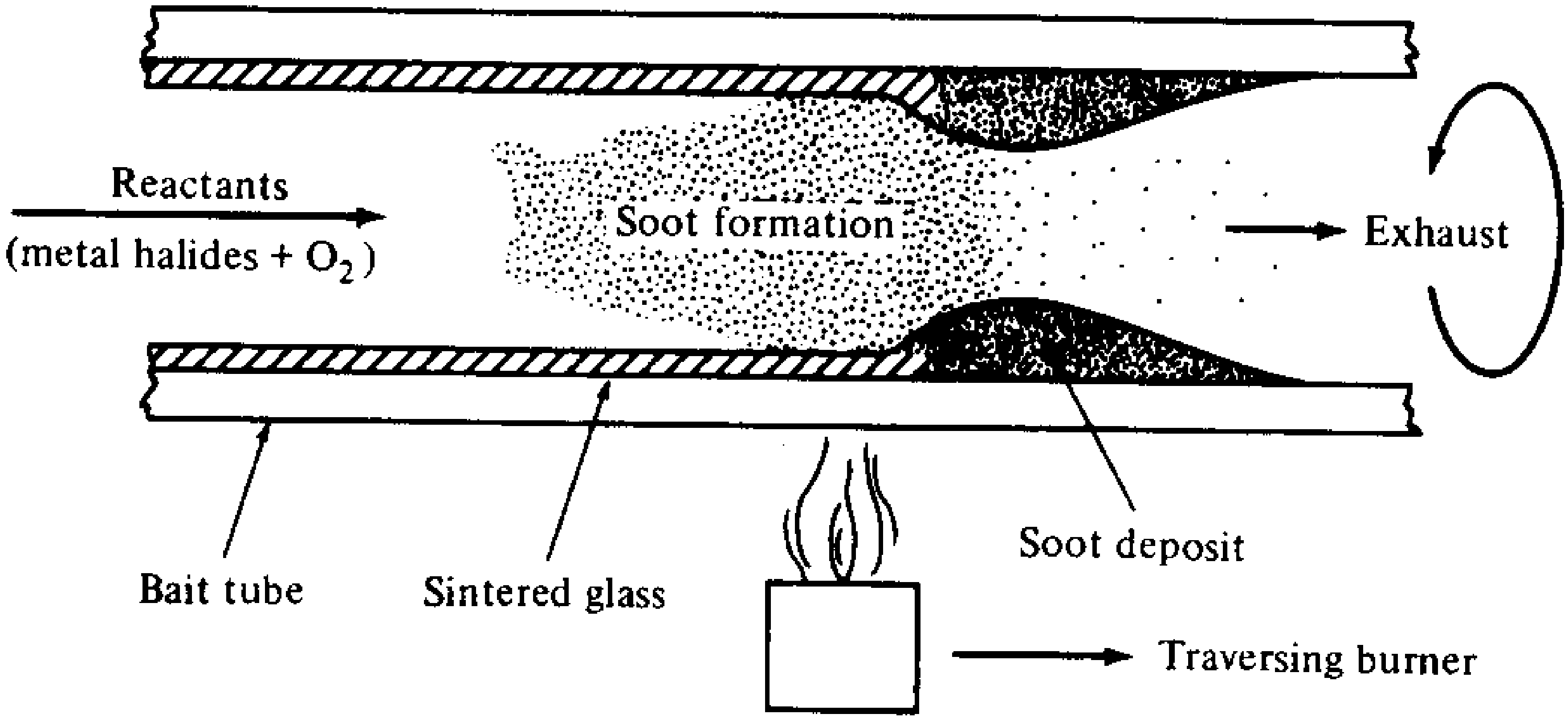


Figure 2 PREFORM FABRICATION

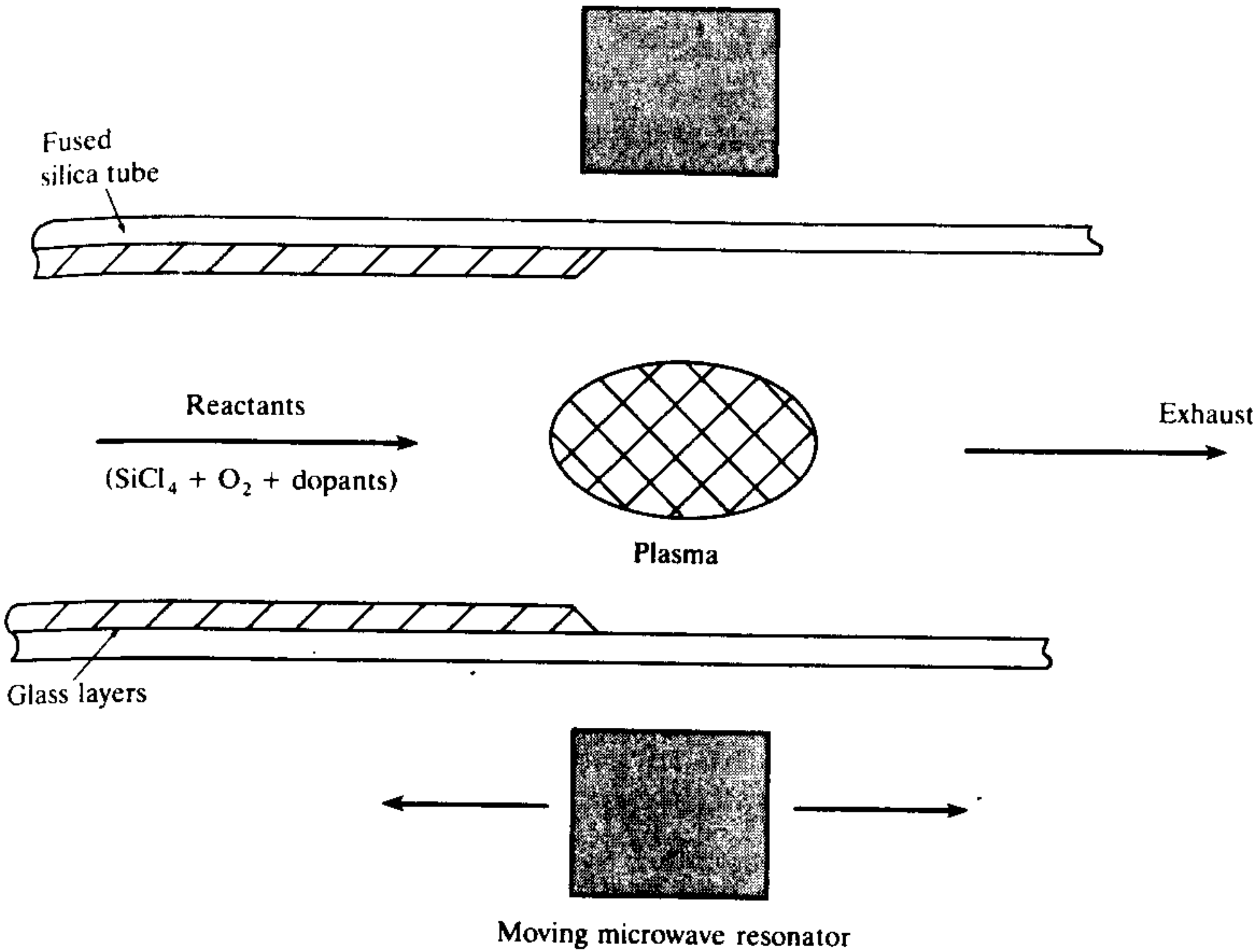


Source: Fotec

Modified Chemical Vapor Deposition (MCVD)

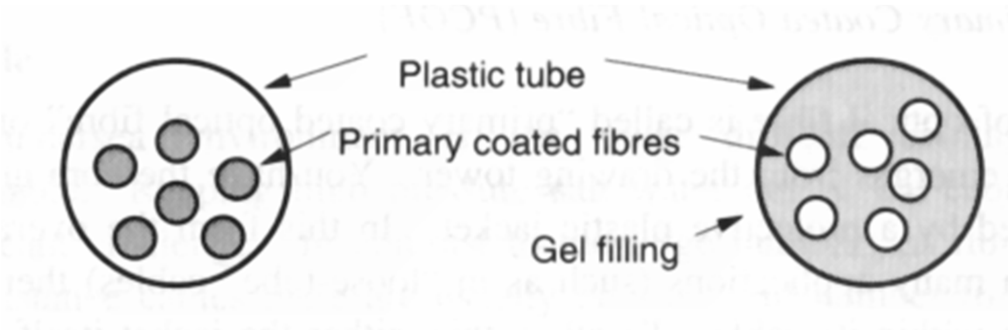


Plasma-Activated Chemical Vapor Deposition (PCVD)



Basic cable construction: types

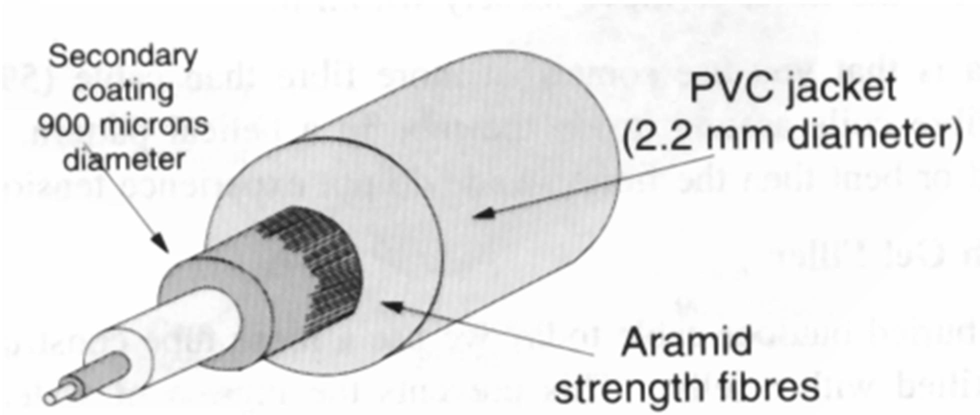
- Tight buffered
  - No room for fibers to move inside of cable
- Loose tube
  - Multiple fibers loose inside of outer plastic tube
  - Advantage is that with extra length of fiber inside tube due to curling, less likelihood of damage in sharp bends
- Loose tube with gel filler
  - Multiple fibers immersed in gel inside of plastic tube

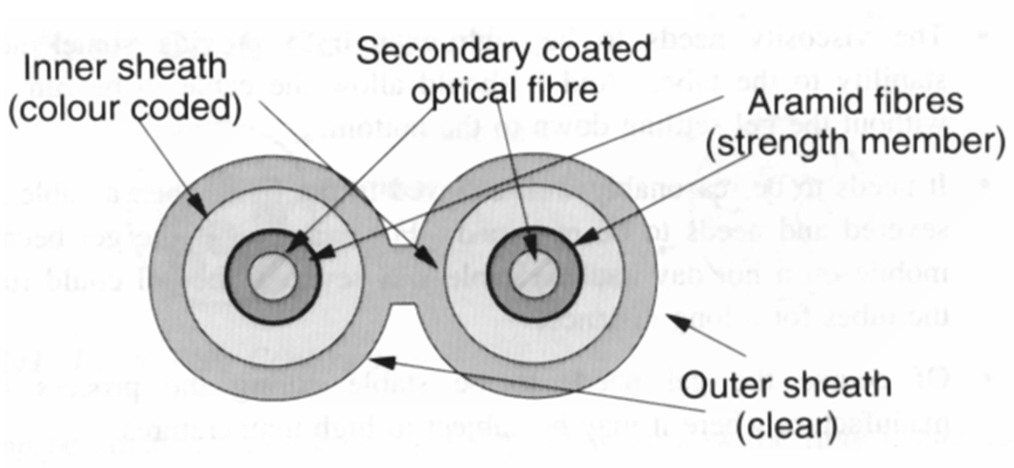


Source: Dutton

Typical indoor cable

- Single core or double core
  - Utilize substrate for additional strength (aramid or fiberglass)

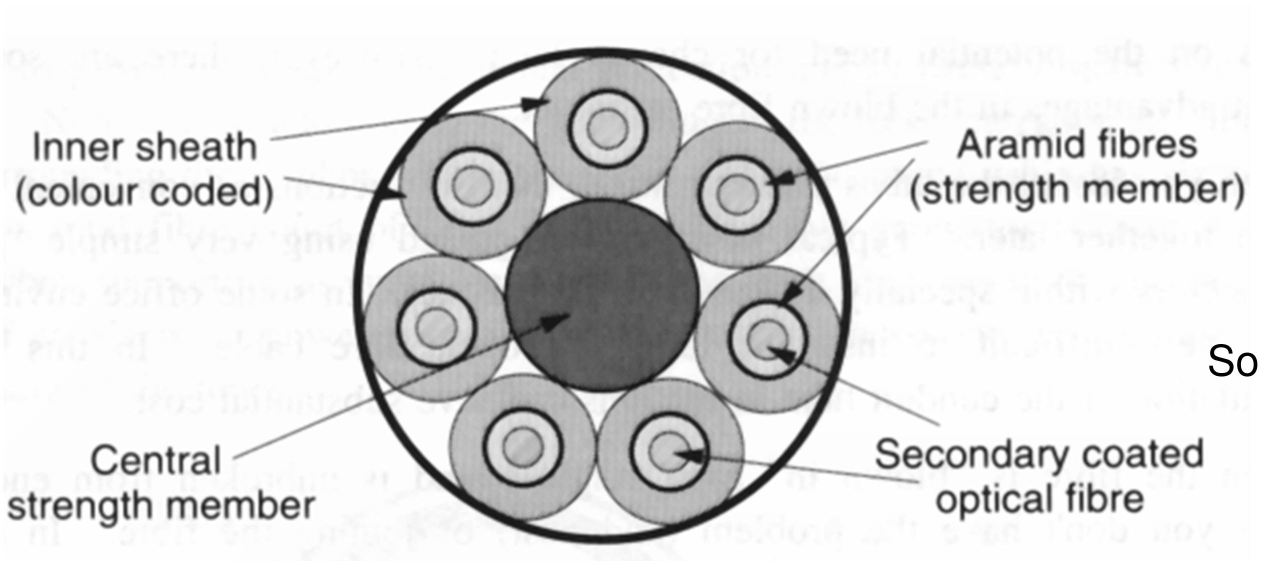




Source: Dutton

Tight buffered indoor cable

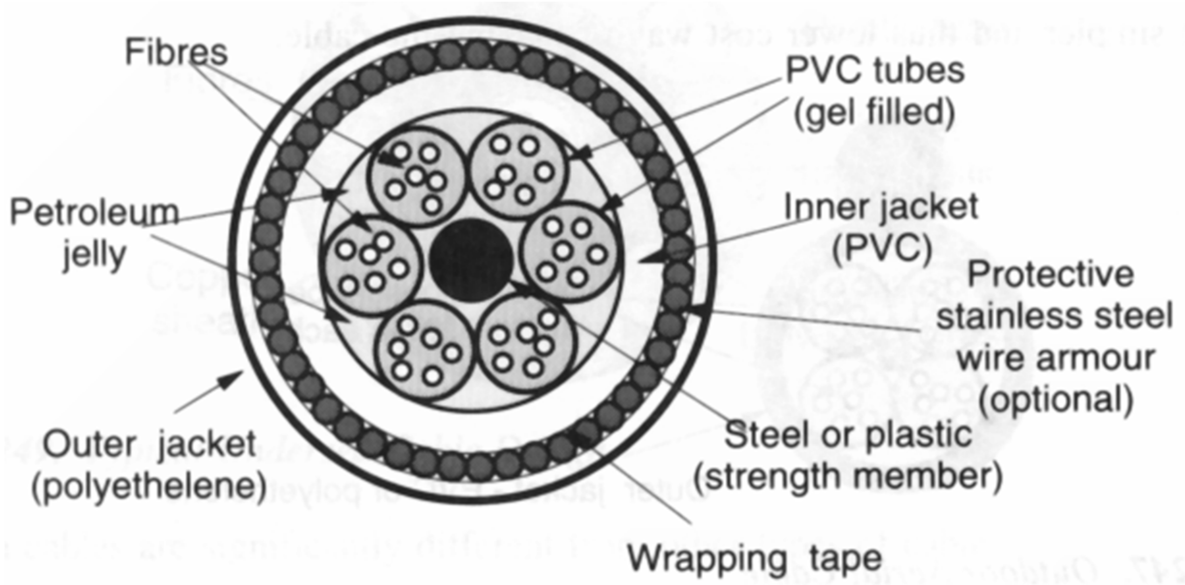
- Application: building risers
- 6 or 12 fibers typically
- Central strength member supports weight of cable
- Tight buffering means that fibers are not put under tension due to their own weight



Source: Dutton

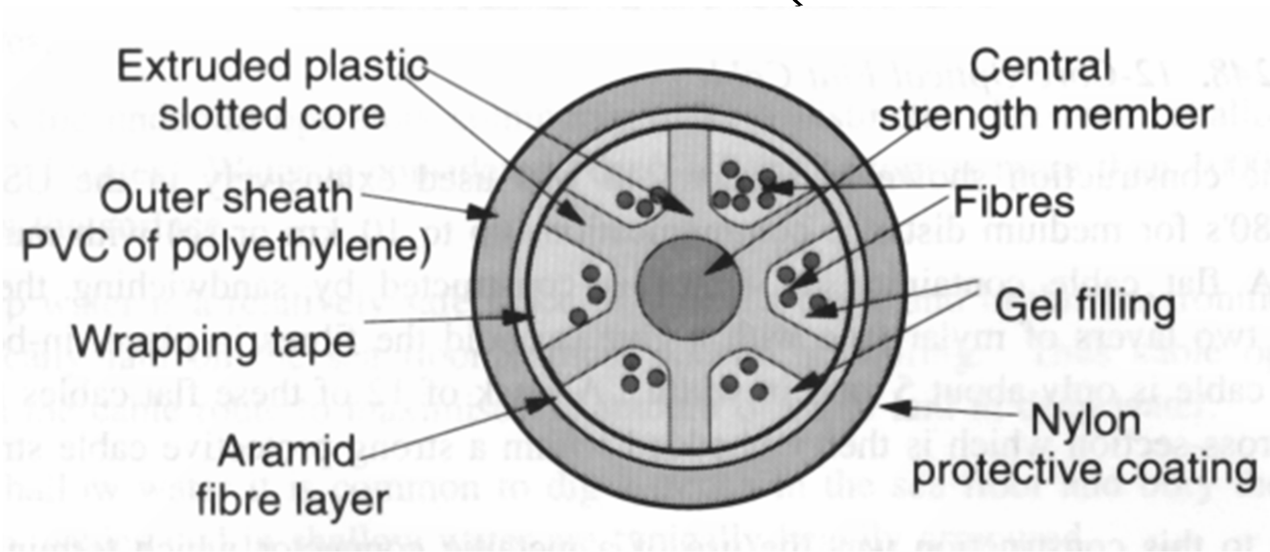
Outdoor cable

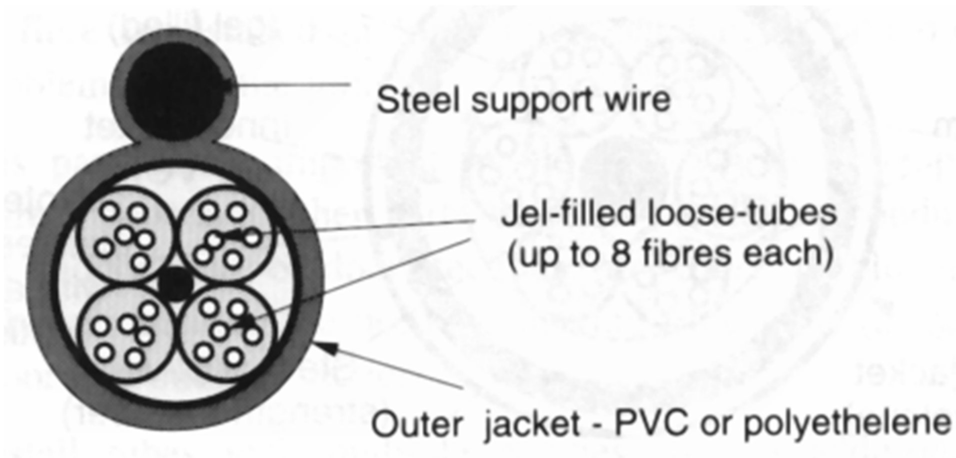
- More rugged, larger number of fibers per cable
  - 6 fibers/tube, 6 tubes = 36 fibers
  - 8 fibers/tube, 12 tubes = 96 fibers
- Steel or plastic used for strength member
- Outer nylon layer in locations where termites are a problem



Source: Dutton

Outdoor cable (continued)

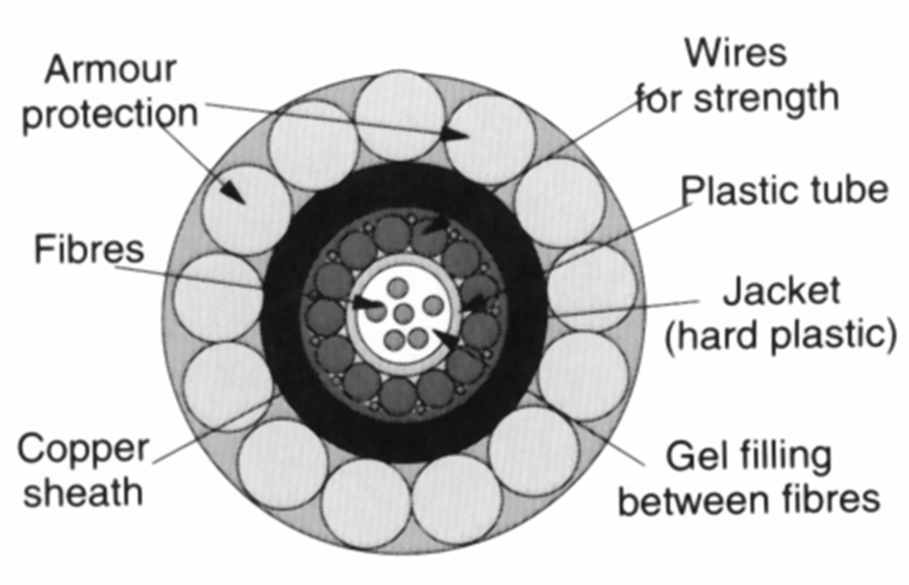




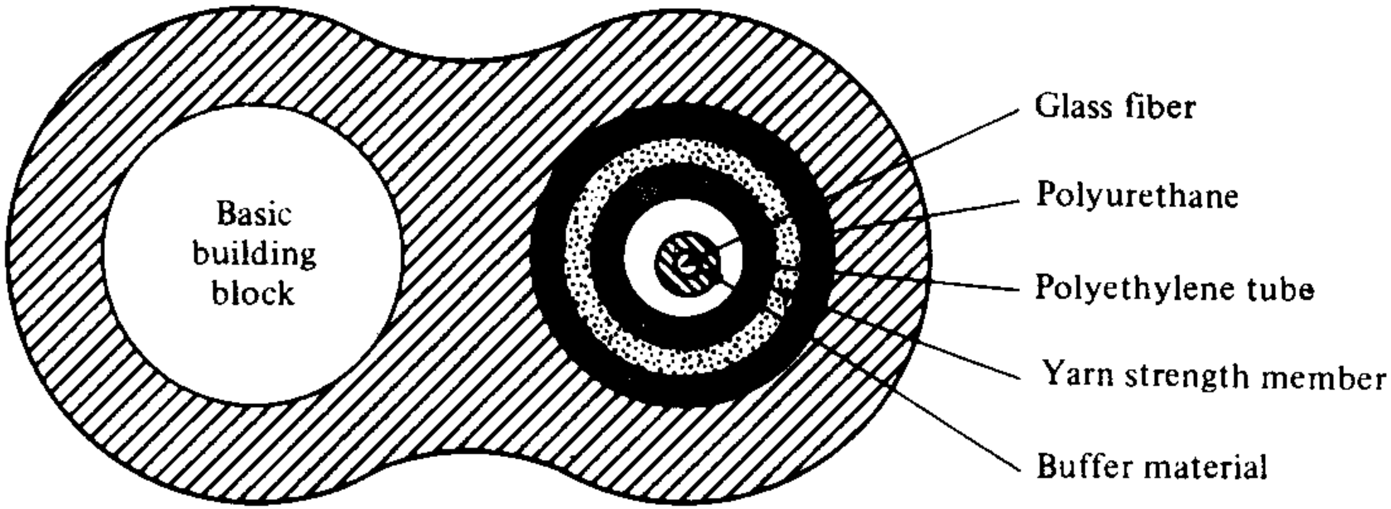
Source: Dutton

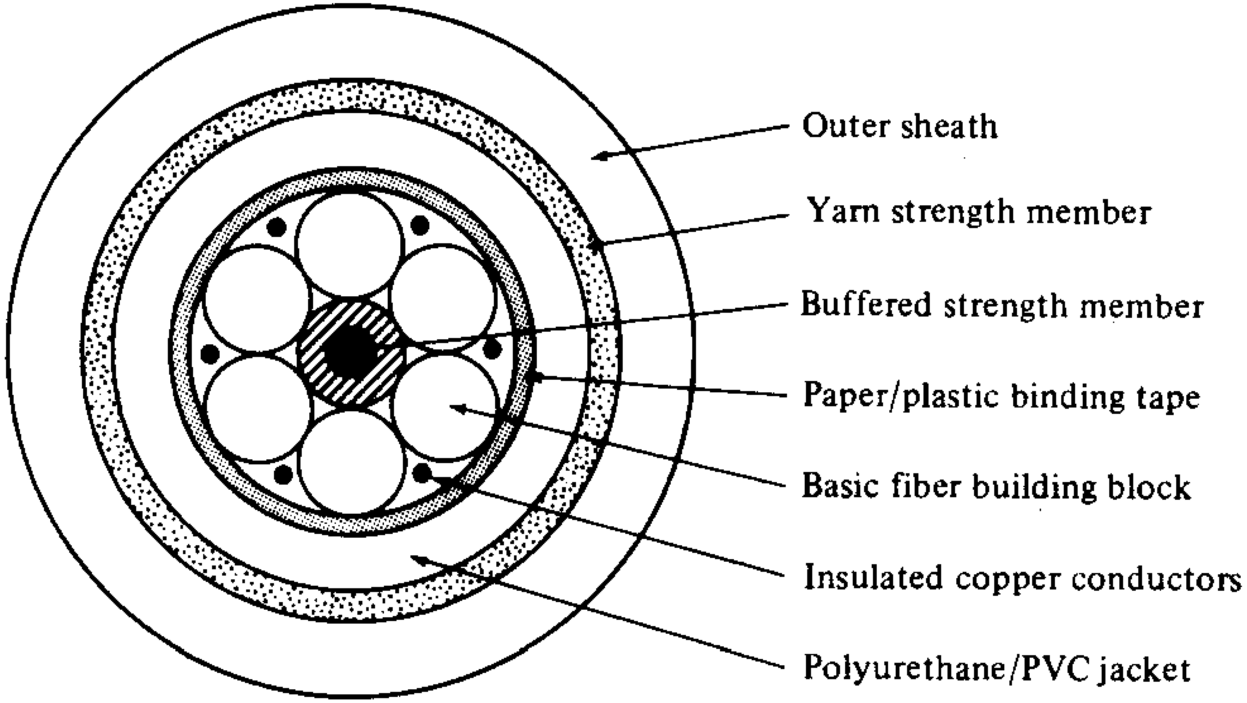
Submarine cable

- Smaller number of fibers because mechanical requirements much greater
  - 4 to 20 typically
- Must withstand high pressure, damage from anchors, trawlers, etc.
- Cables for shallow water are in greatest danger
  - Typically heavily armored



Source: Dutton


Basic
building
block
Glass fiber
Polyurethane
Polyethylene tube
Yarn strength member
Buffer material


Outer sheath
Yarn strength member
Buffered strength member
Paper/plastic binding tape
Basic fiber building block
Insulated copper conductors
Polyurethane/PVC jacket

Fiber Cross-sections