LE 426 Homework

Please show all details of your solutions.

1-1. (a)What are the energies in electron volts (eV) of light at wavelengths 850, 1310, 1490 and 1550 nm?

(b) Consider a 1-ns pulse with a 100-nW amplitude at each of these wavelengths. How many photons are in such a pulse at each wavelength?

1-2. A signal travels from point A to point B. (a) If the signal power is 1 mW at point A and 0.125 mW at point B, what is the attenuation in dB? (b) If the attenuation per unit length is given by 1 dB/km, what is the distance between point A and point B? (c) What is the signal power at point B if the attenuation is 15 dB?

1-3. A transmission line has a bandwidth of 2 MHz. If the signal-to-noise ratio at the receiving end is 20 dB, what is the maximum data rate that this line can support?

1-4. Derive the wave equation from the Maxwell's equations.

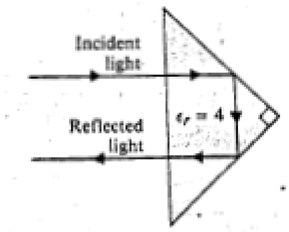
Please show all details of your solutions.

2-1. Derive Snell's laws of reflection and refraction based on the boundary conditions (i.e., continuity of tangential components of electric and magnetic fields).

2-2. Derive the reflection coefficients for TE (normal, perpendicular) and TM (parallel) cases.

2-3. The wave $\mathbf{E} = [3\hat{\mathbf{x}} + 2\hat{\mathbf{y}} - 4\hat{\mathbf{z}}]E_0 e^{j(4x+3z)}$ in free space (z ≤ 0 region) impinges upon the dielectric medium (z ≥ 0 region, $\mu_r = 1$, $\varepsilon_r = 2.5$).
(a) Find the wavenumber vector, plane of incidence, and angle of incidence.
(b) Determine the reflected wave and the transmitted wave.
(HINT: decompose **E** into perpendicular and parallel polarizations.)

2-4. Glass isosceles triangular prisms shown in the figure below are used in optical instruments. Assuming $\varepsilon_r = 4$ for glass, calculate the percentage of the incident light power reflected back by the prism.



2-5. Show that the phase shift upon total internal reflection for the parallel polarization (TM polarization) is given by

$$\tan\frac{\delta_p}{2} = -\frac{n\sqrt{n^2\cos^2\theta_1 - 1}}{\sin\theta_1}; n = \frac{n_1}{n_2}.$$

where $\theta_1 = \pi/2 - \phi_1$ and ϕ_1 denotes the incident angle.

Please show all details of your solutions.

3-1. Consider an infinite slab waveguide with n_s, n_f, n_c = 1.45, 1.46, 1.0, respectively, and h = 5 μm. Assume that $\lambda = 1$ μm,

(a) Find the number of TE modes

(b) Find β, u, w_c, w_s of each mode.

(c) Plot the field patterns of the fundamental and the highest TE modes.

3-2. Repeat problem 1 when n_f is changed to 1.5.

3-3. Repeat problem 1 when h is changed to 10.

3-4. Derive the characteristic equation for the TM modes of a slab waveguide given by

$$\tan(u_f h) = \frac{u_f \left[\frac{n_f^2}{n_s^2} w_s + \frac{n_f^2}{n_c^2} w_s \right]}{u_f^2 - \frac{n_f^4}{n_c^2 n_s^2} w_c w_s}.$$

3-5. Derive the characteristic equations for the TE and TM modes of a *symmetric waveguide* given by equations (2.30) and (2.31), respectively.

Please show all details of your solutions.

4-1. Derive the characteristic equation for the step-index fiber given by equation (5.25) in the note.

4-2. A manufacturer wishes to make a step-index fiber with $V = 75$ and NA = 0.30 to be used at 820 nm. Given $n_1 = 1.458$, find the core size and n_2.

4-3. The HE_{11} mode field distribution can be approximated by a Gaussian shape,

$$E(r) = E_0 \exp\left[-\left(\frac{r}{W_0}\right)^2\right],$$

show that the mode field diameter (MFD) can be calculated from

$$\text{MFD} = 2W_0 = 2\left[\frac{2\int_0^\infty E^2(r) r^3 dr}{\int_0^\infty E^2(r) r dr}\right]^{1/2}.$$

Please show all details of your solutions.

5-1. A certain optical fiber has an attenuation of 0.6 dB/km at 1310 nm and 0.3 dB/km at 1550 nm. Suppose the following two optical signals are launched simultaneously into the fiber: an optical power of 150 μW at 1310 nm and an optical power of 100 μW at 1550 nm. What are the power level in μW of these two signals at (a) 8 km and (b) 20 km?

5-2. An LED with a spectral bandwidth of 20 nm and a central frequency of 850 nm is to be used to transmit digital pulses on an optical fiber with n_1,n_2 = 1.5,1.48, respectively. The material dispersion of this fiber is approximately 300 ps/km·nm.

a) What is the maximum bit rate for a transmission length of 2 km for a single mode fiber?

b) What are the modal dispersion, its corresponding pulse spread and the maximum bit rate for a multimode fiber of 2 km length?

c) What is the maximum core radius for a single mode fiber?

5-3. Consider an optical fiber with n_1,n_2 = 1.5,1.48, respectively, and the core radius of 10 μm. Suppose the fiber has the property shown in figure in slide 27, find the wavelength at which the maximum waveguide dispersion occurs and the corresponding waveguide dispersion.

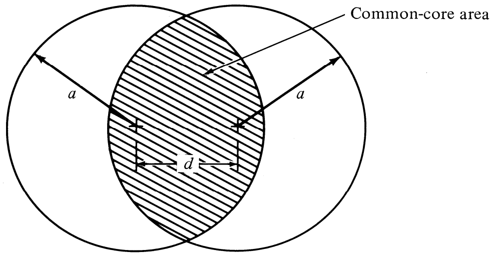
Please show all details of your solutions.

6-1. A laser diode has lateral ($\phi = 0°$) and transverse ($\phi = 90°$) half-power beamwidths of $2\theta =$ 60° and 30°, respectively. What are the transverse and lateral power distribution coefficients for this device?

6-2. A GaAs optical source that has a refractive index of 3.6 is closely coupled to a step-index fiber which has a core refractive index of 1.465. If the source size is smaller than the fiber core, and the small gap between the source and the fiber is filled with a gel that has a refractive index of 1.305, what is the power loss in decibels from the source into the fiber?

6-3. Show that the common area of two misaligned fiber cores shown in the figure below is given by

$$A_{comm} = 2a^2 \arccos \frac{d}{2a} - d\left(a^2 - \frac{d^2}{4}\right)^{1/2}$$



Please show all details of your solutions.

7-1. If the absorption coefficient of silicon is 0.05 μm^{-1} at 860 nm, find the penetration depth at which $P(x)/P_{in} = 1/e = 0.368$.

7-2. An InGaAs *pin* photodiode has the following parameters at 1550 nm: I_D = 1.0 nA, η = 0.95, R_L=500 Ω, and the surface leakage current is negligible. The incident optical power is 500 nW and the receiver bandwidth is 150 MHz. Compare the noise currents due to bulk dark current, surface dark current, and the total noise current.

7-3. Derive equation (6.19), i.e.,

$$M_{\text{opt}}^{x+2} = \frac{2qI_L + 4k_B T / R_L}{xq(I_P + I_D)}.$$

Please show all details of your solutions.

8-1. A transmission system sends out information at 200,000 b/s. During the transmission process, fluctuation noise is added to the signal so that at the decoder output the signal pulses are 1 V in amplitude and the rms noise voltage is 0.2 V.
(a) Assuming that ones and zeros are equally likely to be transmitted, what is the average time in which an error occurs?
(b) How is this time changed if the voltage amplitude is doubled with the rms noise voltage remaining the same?

8-2. An LED operating at 1300 nm injects 25 μW of optical power into a fiber. If the attenuation between the LED and the photodetector is 40 dB and the photodetector quantum efficiency is 0.65, what is the probability that fewer than 5 electron-hole pairs will be generated at the detector in a 1-ns interval?

8-3. Derive the bit error rate for synchronous heterodyne ASK receiver given by

$$\text{BER} = \frac{1}{2}\text{erfc}\left(\sqrt{\eta \bar{N}_p / 2}\right)$$

Please show all details of your solutions.

9-1. A 1550-nm single-mode digital fiber optic link needs to operate at 565 Mbps over 50 km without repeaters. A single-mode InGaAsP laser launches an average optical power of -13 dBm into the fiber. The fiber has a loss of 0.35 dB/km, and there is a splice with a loss of 0.1 dB every kilometer. The coupling loss at the receiver is 0.5 dB, and the receiver uses an InGaAs APD with a sensitivity of -39 dBm. Excess-noise penalties are predicted to be 1.5 dB. Set up an optical power budget for this link and find the system margin.

9-2. A 90-Mbps NRZ data transmission system uses a GaAlAs laser diode having 1-nm spectral width. The rise time of the laser transmitter output is 2 ns. The transmission distance is 7 km over a graded-index fiber having an 800-MHz·km bandwidth-distance product and material dispersion 70 ps/(nm·km).
(a) If the receiver bandwidth is 90 MHz and the mode-mixing factor $q = 0.7$, what is the system rise time? Does this rise time meet the NRZ data requirement of being less than 70 percent of a pulse width?
(b) What is the system rise time if there is no mode mixing in the 7-km link, that is $q = 1.0$?

9-3. Consider a SCM system having 120 channels, each modulated at 2.3 percent. The link consists of 12 km of single-mode fiber having a loss of 1 dB/km, plus a connector having a 0.5 dB loss on each end. The laser source couples 2 mW of optical power into the fiber and has RIN = -135 dB/Hz. The *pin* photodiode receiver has a responsivity of 0.6 A/W, $B_e = 5$ GHz, $I_D = 10$ nA, $R_{eq} = 50\ \Omega$, and $F_t = 3$ dB. Find the carrier-to-noise ratio for this system.

Please show all details of your solutions.

10-1. A DWDM optical transmission system is designed to have 100-GHz channel spacings, how many wavelength channels can be utilized in the 1536-to-1556-nm spectral band?

10-2. A 2×2 waveguide coupler has $\kappa = 0.4\ \text{mm}^{-1}$, $\alpha = 0.06\ \text{mm}^{-1}$, and $\Delta\beta = 0$. How long should the coupler be to make a 3-dB power divider? If that length is doubled, what fraction of the input power emerges from the second channel?

10-3. Consider an N-node star network in which 0 dBm of optical power is coupled from any given transmitter into the star. Let the fiber loss be 0.3 dB/km. Assume the stations are located 2 km from the star, the receiver sensitivity is -38 dBm, each connector has a 1-dB loss, the excess loss in the star coupler is 3 dB, and the link margin is 3 dB,
(a) Determine the maximum number of stations N that can be incorporated on this network.
(b) How many stations can be attached if the receiver sensitivity is -32 dBm?