Quantum wells have extremely large gain compared to heterojunctions. From the dimensions, it should be obvious that the confinement factor Γ is relatively small in these lasers. A typical quantum well will be 50Å in width, while a typical mode might be 2μm in width. The confinement factor is less than 1 percent for such a system. This means that only a small fraction of the mode is experiencing gain at any moment. The gain is very high to support the entire mode. However, like the double heterojunction laser, the wings of the mode ideally do not experience any interband absorption, so losses are minimal.

16.11 MODULATION RATES IN SEMICONDUCTOR LASERS

One of the biggest applications of semiconductor lasers is in optical communication links. Of interest to the link designer is the maximum modulation speed of the laser. For slow modulation, the output of a semiconductor laser is essentially linear to the input current. If the current increases, so does the output intensity. As the modulation speed increases, complications arise due to a slight time delay between the creation of gain (injection) and the saturation of that gain by an optical field. The time domain picture is needed to determine laser modulation frequency characteristics. In this analysis, we will focus on the total inversion and the total number of photons in the cavity. We represent the total inversion by n, and the total number of photons in a given mode by Φ.

The total population is governed by three processes: injection, spontaneous recombination, and stimulated emission. If we consider a pn junction with a carrier confinement region of depth d, the total inversion is described by the rate equation

$$\begin{aligned}\frac{dn}{dt} = \frac{J}{qd} & \quad \text{(Injection)} \\ - \frac{n}{\tau_s} & \quad \text{(Spontaneous Recombination)} \\ -\sigma_s(n - n_{nom})\Phi & \quad \text{(Stimulated Emission)}\end{aligned} \tag{16.60}$$

where n_{nom} is the threshold inversion, σ_s is a collection of constants describing the strength of the optical interaction, and τ_s is the spontaneous recombination lifetime of the carriers. (The formula for gain cross section, $\sigma = A_{21}\lambda^2 g(\nu)/8\pi n^2$, that we developed for gas and ion lasers does not apply to semiconductors.) Putting all the rates together, we can write a general expression for the inversion in the semiconductor.

$$\frac{dn}{dt} = \frac{J}{qd} + \frac{-n}{\tau_s} - \sigma_s(n - n_{nom})\Phi \tag{16.61}$$

To solve this equation, we need an expression for the number of photons in the cavity, Φ. The photons are governed by their own equation. They increase in number primarily due to stimulated emission:

$$\frac{d\Phi}{dt} = +\sigma_s(n - n_{nom})\Phi \quad \text{Stimulated Emission} \tag{16.62}$$

There is also some increase in the photon count due to spontaneous emission into the mode of interest. As with the systems we described in Chapter 15, spontaneous emission couples to all the modes of the cavity, of which most are not the desired TEM_{00} modes of the cavity. Nevertheless, with the very small dimensions of typical semiconductor cavities, the number of spontaneous photons which couple into the TEM_{00} modes is not always insignificant. We can define a fraction β of the spontaneous emission to couple into the mode:

$$\frac{d\Phi}{dt} = \beta\frac{N}{\tau_s} \tag{16.63}$$

Finally, there is a loss of photons due to partial transmission at the mirrors. We define a photon lifetime, τ_p, which describes the average time a photon stays in the cavity. The rate equation for photon loss is

$$\frac{d\Phi}{dt} = -\frac{\Phi}{\tau_p} \tag{16.64}$$

Putting all these terms together,

$$\frac{d\Phi}{dt} = +\sigma_s(n - n_{nom})\Phi + \beta\frac{n}{\tau_s} - \frac{\Phi}{\tau_p} \tag{16.65}$$

Equations 16.61 and 16.65 are coupled differential equations which describe the dynamics of the gain and intensity inside the laser cavity. We can solve them first for steady state to get an expression for the output power. Assume that the junction is pumped with a DC current density $J = J_0$, with $n = n_0 > n_{nom}$, and $\Phi = \Phi_0$. At steady state, $d/dt = 0$, so from the photon equation

$$\sigma_s(n_0 - n_{nom})\Phi + \beta\frac{n_0}{\tau_s} = \frac{\Phi}{\tau_p} \tag{16.66}$$

If we solve this for $\sigma_s(n_0 - n_{nom})\Phi$ and plug this into the inversion equation, we can solve for the number of photons

$$\begin{aligned} \Phi &= \frac{\tau_p}{qd}\left(J_0 - \frac{n_0 qd}{\tau_s}\right) + \frac{\beta n_0 \tau_p}{\tau_s} \\ &= K[J_0 - J_{th}] + P_{spon} \end{aligned} \tag{16.67}$$

In the second equation, constants have been combined into single terms to simplify the appearance of the expression. The stimulated power (the first term in the equations) is generally concentrated in one or a few modes. The spontaneous power is not mode selective, but is in fact spread out over all the possible modes (on the order of 10^8) of the volume. A plot of the output power from a semiconductor laser is shown in Figure 16.28.

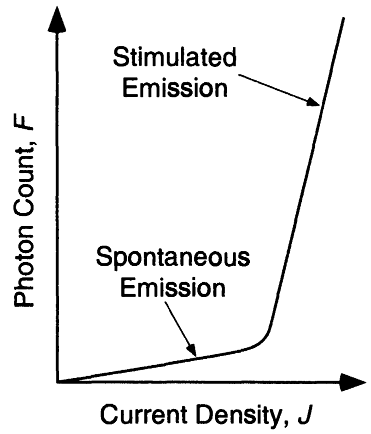

Figure 16.28 The output response from a diode laser shows that, below threshold, there is significant spontaneous emission. Once threshold is reached, stimulated emission dominates the output.

Next, we can determine how the laser will respond to a *small-signal* modulation of the drive current. We replace J_0 with $J_0 + \Delta J(t)$. Similarly, we will look for variations in the photon count and inversion in terms of small-signal variations, $n = n_0 + \Delta n(t)$ and $\Phi = \Phi_0 + \Delta\Phi(t)$. Plug these values into the rate equations to get

$$\begin{aligned}\frac{dn_0}{dt} + \frac{d\Delta n(t)}{dt} = \frac{J_0}{qd} + \frac{\Delta J}{qd} - \frac{n_0}{\tau_s} - \frac{\Delta n(t)}{\tau_s}\\ -\sigma_s[n_0 - \Delta n(t) - n_0][\Phi_0 + \Delta\Phi(t)]\end{aligned} \tag{16.68}$$

$$\begin{aligned}\frac{dP_0}{dt} + \frac{d\Delta P(t)}{dt} = \sigma_s[n_0 - \Delta n(t) - n_0][\Phi_0 + \Delta\Phi(t)]\\ + \beta\left[\frac{n_0 + \Delta n(t)}{\tau_s}\right] - \frac{P_0 + \Delta P(t)}{\tau_p}\end{aligned} \tag{16.69}$$

After doing a little algebra to expand terms, throwing out second-order terms such as $\Delta P(t)\Delta N(t)$, and canceling common terms, we end up with two coupled equations

$$\begin{aligned}\frac{d\Delta n(t)}{dt} &= \frac{\Delta J}{qd} - \left(\frac{1}{\tau_s} + \sigma_s P_0\right)\Delta n(t) - \frac{\Delta P(t)}{\tau_p}\\ \frac{d\Delta P(t)}{dt} &= \left[\sigma_s P_0 + \frac{\beta}{\tau_s}\right]\Delta n\end{aligned} \tag{16.70}$$

The classic method to solve these coupled equations is to convert them into one second-order differential equation. The photon equation becomes

$$\frac{d^2\Delta P(t)}{dt^2} + \left[\frac{1}{\tau_s} + \sigma_s P_0\right]\frac{d\Delta P(t)}{dt} + \frac{1}{\tau_p}\left[P_0 + \frac{\beta}{\tau_s}\right]\Delta P(t) = \frac{\Delta J}{qd}\left[\sigma_s P_0 + \frac{\beta}{\tau_s}\right] \tag{16.71}$$

This is not very enlightening, but we can use it to determine the response to certain driving functions. We assume the current has a harmonic driving term $J(t) = \Delta J e^{j\omega t}$. Let's look for a solution to Φ in terms of a similar harmonic function, $\Phi(t) = \Delta\Phi e^{j\omega t}$. Plugging this trial solution into the equation, we find it yields a solution so long as we choose the ratio of the amplitudes to be

$$\frac{\Delta\Phi}{\Delta J} = \frac{P_0 + \beta/\tau_s}{\left[\frac{1}{\tau_p}(\sigma_s P_0 + \beta/\tau_s) - \omega^2\right] + j\omega\left[\frac{1}{\tau_s} + \sigma_s P_0\right]} \tag{16.72}$$

This equation takes a little inspection to appreciate. The denominator has a resonance term in it. For very low frequencies, the constants in the equation dominate, yielding a constant modulation index. But at very large frequencies, the ω^2 term in the denominator dominates, causing the modulation to roll off rapidly with frequency above a critical value. The critical frequency for modulation is when the denominator is minimized, or when

$$\omega^2 = \frac{1}{\tau_p}\left(\sigma_s P_0 + \frac{\beta}{\tau_s}\right) \approx \frac{\sigma_s P_0}{\tau_p} \tag{16.73}$$

A plot of the transfer function is shown in Figure 16.29.

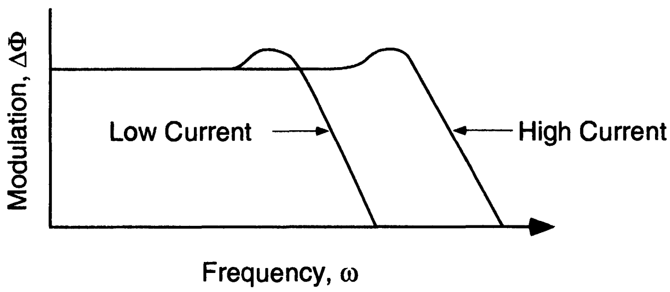

Figure 16.29 The modulation of the output power by current modulation is a fairly flat function at low frequency, but shows a resonance at higher frequencies. The resonant frequency depends on the current and photon lifetime.

We see from Equation 16.73 that the maximum modulation frequency can be increased by operating the laser at higher power. This increases the saturating fields, so the optical output is more responsive to changes in the gain. One can also decrease the photon lifetime of the laser. This can be done by reducing the length of the laser, or by reducing the reflectivity of the laser mirrors. However, the maximum frequency only increases as the square root of changes in power of photon lifetime, so it is not easy to make dramatic strides in the frequency response. Current laboratory records indicate that modulation speeds up to 30GHz are feasible, although commercial devices are still operating in the 5–10GHz range. At these frequencies, packaging issues such as lead inductance, junction capacitance, and series resistance become as serious as the fundamental laser physics.

16.12 SUMMARY

In this chapter, we have made a cursory review of semiconductor laser technology, starting with the fundamental equations that govern the carrier distributions in semiconductors, and applying these laws to semiconductor laser design.

The field of semiconductor lasers is large and dynamic. We have done little justice to the scope of the field, nor to the excitement that it currently is experiencing as it pushes open new applications and frontiers. The interested reader is encouraged to follow through by reading some of the suggested readings at the end of this chapter to find more complete descriptions of the semiconductor laser.

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Supplementary Reading

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