2.2 THE INFINITE SLAB WAVEGUIDE

The simplest optical waveguide structure is the step-index planar slab waveguide. The slab waveguide, shown in Figure 2.1, consists of a high-index dielectric layer surrounded on either side by lower-index material. The slab is infinite in extent in the yz plane, but finite in the x direction. The index of refraction of the guiding slab, n_f, must be larger than that of the cover material, n_c, or the substrate material, n_s, in order for total internal reflection to occur at the interfaces. If the cover and substrate materials have the same index of refraction, the waveguide is called *symmetric*; otherwise the waveguide is called *asymmetric*. The symmetric waveguide is a special case of the asymmetric waveguide.

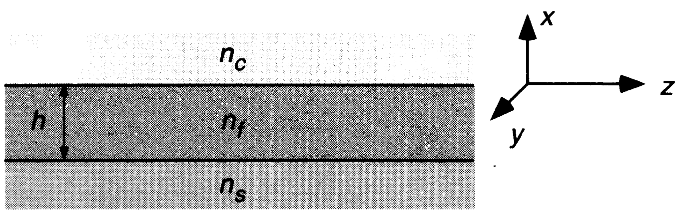

Figure 2.1 The planar slab waveguide consists of three materials, arranged such that the guiding index of refraction (n_f) is larger than the surrounding substrate (n_s) and cover (n_c) indices.

The slab waveguide is clearly an idealization of real waveguides, because real waveguides are not infinite in width. However, the one-dimensional analysis we will develop is directly applicable to many real problems, and the techniques form the foundation for further understanding. We will begin by solving the wave equation using boundary conditions for the slab waveguide structure. We will always choose the direction of propagation to be along the z axis. This will lead naturally to the concept of *modes*. We will then develop formal mode concepts such as orthogonality, completeness, and modal expansion. We will see that a waveguide structure can support only a discrete number of guided modes. The mode picture is very powerful, and will be used extensively as we delve deeper into the subject of wave propagation in structures.

2.3 ELECTROMAGNETIC ANALYSIS OF THE PLANAR WAVEGUIDE

Consider the waveguide structure shown in Figure 2.1. The three indices are chosen such that $n_f > n_s > n_c$, and the guiding layer has a thickness h. The choice of the coordinate system is critical in making the problem as simple as possible [1]. The appropriate coordinate system for this planar problem is a rectilinear cartesian system, because the three components of the field, E_x, E_y, and E_z are not coupled by reflections. For example, an electric field polarized in the y direction, E_y, will still be an E_y field upon reflection at either interface; the reflection does not couple any of the vector field into the x or z directions. Because this is an asymmetric waveguide structure, we place the $x = 0$ coordinate at one of the interfaces, choosing arbitrarily the top interface (between n_f and n_c).

We must consider two possible electric field polarizations, *transverse electric* or *transverse magnetic* [2]. The axis of the waveguide is oriented in the z direction. The k vector of the guided wave will zigzag down the z axis, striking the interfaces at angles greater than the critical angle. The field can be transverse electric (TE) or transverse magnetic (TM), depending on the orientation of the electric field. The TE case has no longitudinal component along the z axis; the electric field is transverse to the plane of incidence established by the normal to the interface, and by the k vector. Because of the different boundary conditions that control both fields, the TE and TM cases are distinguished in their mode characteristics as well as their polarization. We will consider the TE case, leaving derivation of the TM case to problems at the end of the chapter.

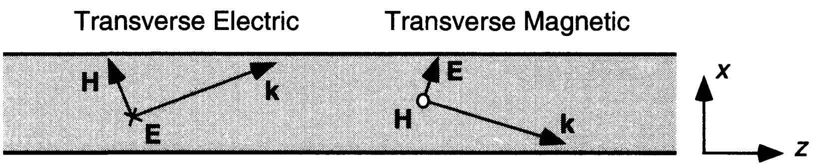

Figure 2.2 Transverse electric (TE) and transverse magnetic (TM) configurations. A cross indicates the field entering the page, and "0" indicates the field coming out of the page.

In the TE case, the E field is polarized along the y axis (into the page) of Figure 2.2. We assume the waveguide is excited by a source with frequency ω_0 and a vacuum wavevector of magnitude k_0, where $|k_0| = \omega_0/c$. To find the allowed modes of the waveguide, we must first solve the wave equation in each dielectric region, and then use the boundary conditions to connect these solutions. For a sinusoidal wave with angular frequency ω_0, the wave equation (Equation 1.26) for the electric field components in each region can be put in the scalar form:

$$\nabla^2 E_y + k_0^2 n_i^2 E_y = 0 \tag{2.1}$$

where $n_i = n_f$, n_s, or n_c, depending on the location. $E_y(x, z)$ is a function of both x and z, but because the slab is infinite in extent in the y direction, E_y is independent of y. Due to the translational invariance of the structure in the z direction, we do not expect the amplitude to vary along the z axis, but we do expect that the phase varies. We write a trial solution to Equation 2.1 in the form:

$$E_y(x, z) = E_y(x)e^{-j\beta_i z} \tag{2.2}$$

β is a propagation coefficient along the z direction, but we do not know its magnitude yet. Plugging this trial solution into Equation 2.1, and noting that $d^2E_y/dy^2 = 0$,

$$\frac{\partial^2 E_y}{\partial x^2} + (k_0^2 n_i^2 - \beta^2)E_y = 0 \tag{2.3}$$

The choice of n_i depends on the position x. For $x > 0$, we would use n_c, while for $0 > x > -h$, we would use n_f, etc. The general solution to Equation 2.3 will depend on the relative magnitude of β with respect to $k_0 n_i$. Consider the case where

$\beta > k_0 n_i$. The transverse wave equation (Equation 2.3) will have a general solution with a *real exponential* form:

$$E_y(x) = E_0 e^{\pm\sqrt{\beta^2 - k_0^2 n_i^2}\,x} \qquad \text{for } \beta > k_0 n_i \tag{2.4}$$

where E_0 is the field amplitude at $x = 0$. To be physically reasonable, we always choose the negatively decaying branch of Equation 2.4. This solution should remind you of the evanescent field of a total internally reflected (TIR) wave at an interface.

In the case where $\beta < k_0 n_i$, the solution has an oscillatory form:

$$E_y(x) = E_0 e^{\pm j\sqrt{k_0^2 n_i^2 - \beta^2}\,x} \qquad \text{for } \beta < k_0 n_i \tag{2.5}$$

So depending on the value of β, the solution can be either oscillatory or exponentially decaying. If $\beta > k_0 n_i$ we define an *attenuation coefficient*, γ, as:

$$\gamma = \sqrt{\beta^2 - k_0^2 n_i^2} \tag{2.6}$$

and describe the field as $E_y(x) = E_0 e^{-\gamma x}$. Compare this to Equation 1.114 for the evanescent field of a TIR wave. If $\beta < k_0 n_i$, then we define a *transverse wavevector*, κ, as:

$$\kappa = \sqrt{k_0^2 n_i^2 - \beta^2} \tag{2.7}$$

so $E_y(x) = E_0 e^{\pm j\kappa x}$. Using Equation 2.7, we see that β and κ can be geometrically related to the total wavevector, $k = k_0 n_f$, in the guiding film.

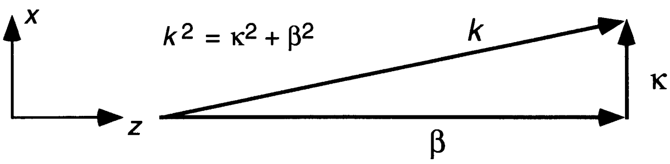

Figure 2.3 β and κ are the longitudinal and transverse components, respectively, of the wavevector k.

β and κ are called the longitudinal and transverse wavevectors, respectively, inside the guiding film. These terms will be used extensively to characterize many types of waveguide mode, so become familiar with the relation shown in Figure 2.3.

2.4 THE LONGITUDINAL WAVEVECTOR: β

The longitudinal wavevector β is used to identify individual modes. While any number of parameters could be chosen to fill this role, β is defined as the eigenvalue of the mode. Figure 2.4 plots the transverse electric field distribution in a slab waveguide for various values of β [3]. In this plot, we allow the angle between k and z to vary from 90° to 0°. This in effect varies the value of β, which is simply the z component of the wavevector, from 0 to $\beta_{max} = k$. The value of β is plotted along the horizontal axis.

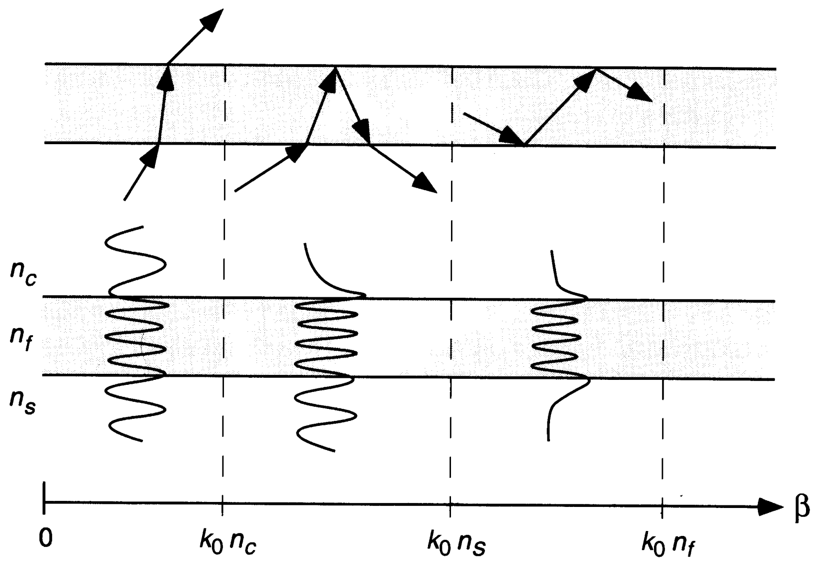

Figure 2.4 This ray and wave picture shows the electromagnetic fields as a function of β.

The top sketch of Figure 2.4 shows the ray picture of the field, while the lower sketch shows the wave picture (solutions to Equations 2.4 and 2.5). There are three special points on the β axis, and the first one is at $\beta = k_0 n_c$. For $\beta < k_0 n_c$, solutions to the wave equation in all regions of space are oscillatory (Equation 2.5). The ray picture shows that when $\beta \approx 0$, the wave travels nearly perpendicular to the z axis of the waveguide. Like light going through a sheet of glass, the ray refracts at the dielectric interfaces, but is not trapped. An oscillatory wave is present in the three distinct dielectric regions.

The second special point occurs at $k_0 n_s$. For $k_0 n_c < \beta < k_0 n_s$, the ray picture shows a ray total-internal-reflecting at the film-cover interface, but refracting at the lower substrate-film interface. In the wave picture, the field becomes evanescent in the cover region. The field will still be oscillatory in the film and substrate regions. This condition is called a substrate mode.

As β increases beyond $k_0 n_s$, the evanescent conditions are satisfied in both the cover and substrate region, while oscillatory solutions are found in the film itself. Such solutions are, in fact, the guided modes of the film. The ray picture depicts a ray trapped between the two interfaces.

If β continues to increase beyond $k_0 n_f$ (although physically it is not clear how this could ever be done, since β is simply the z component of $k_0 n_f$), then Equation 2.4 is satisfied everywhere, so the three regions must have exponential solutions. The only way to satisfy boundary conditions is to choose exponentially increasing fields in the surrounding dielectric regions, causing the field to explode toward infinity as the distance from the film increases. Satisfying this solution would require infinite energy, so the entire premise of the solution is unphysical.

A guided wave must satisfy the condition that

$$k_0 n_s < \beta < k_0 n_f \tag{2.8}$$

where it is assumed that $n_c \leq n_s$. This is a universal condition for any dielectric waveguide, regardless of geometry.

2.5 EIGENVALUES FOR THE SLAB WAVEGUIDE

To find the values of β that lead to allowed solutions to the wave equation, we must apply the boundary conditions to the general solutions developed in Equations 2.4 and 2.5. Assume that β satisfies Equation 2.8. Then the transverse portions of the electric field amplitudes in the three regions are

$$\begin{aligned} E_y(x) &= Ae^{-\gamma_c x} \qquad 0 < x \\ E_y(x) &= B\cos(\kappa_f x) + C\sin(\kappa_f x) \qquad -h < x < 0 \\ E_y(x) &= D\,e^{\gamma_s (x+h)} \qquad x < -h \end{aligned} \tag{2.9}$$

where A, B, C, and D are amplitude coefficients to be determined from the boundary conditions, γ_c and γ_s refer to the attenuation coefficients in the cover and substrate, respectively (Equation 2.8), and κ_f is the transverse component of k in the guiding film (from Equation 2.6). The boundary conditions that connect the solutions at the interfaces are:

1. Tangential E is continuous.
2. Tangential H is continuous.

We rarely worry about continuity of the normal components of D and B, because these conditions are almost always satisfied when we satisfy the transverse conditions. Since E_y, as defined in Equation 2.9, is transverse to the interface, boundary condition 1 is straightforward to apply. What about the condition for continuity of magnetic field H? Should we write down a set of equations similar to Equations 2.9 that describe the magnetic field as a function of position? Indeed, we could do that, but there is usually a simpler way to derive expressions for the magnetic field. If we assume that the fields are harmonic, then we can describe the magnetic intensity in terms of the electric intensity, and derive a simple boundary condition for the magnetic terms. Recall that

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.10}$$

For a sinusoidal field,

$$\mathbf{B}(t) = \mu \mathbf{H}(t) = \mu \mathbf{H}_0 e^{j\omega t} \tag{2.11}$$

so

$$\nabla \times \mathbf{E}(t) = -\mu j\omega \mathbf{H}(t) \tag{2.12}$$

We need an expression for the tangential component (the z component in this case) of **H**. Expanding the $\nabla \times$ term of Equation 2.10 into its individual components, and taking the z component, we get

$$\hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -\mu j\omega H_z \tag{2.13}$$

Since there is no E_x component to the field (it would not vary with y even if it did exist due to the infinite planar structure), we get an explicit equation for the tangential component of the magnetic field, H_z,

$$H_z = \frac{j}{\mu\omega}\frac{\partial E_y}{\partial x} \tag{2.14}$$

The tangential component of **H**, H_z, is defined in terms of the electric field quantities. Since μ and ω are identical in all the media, the continuity of the tangential magnetic field is guaranteed if $\partial E_y/\partial x$ is made continuous across the interface. We can now find the amplitude coefficients, A, B, C, and D.

At the $x = 0$ interface, the condition that E_y be continuous requires that

$$Ae^{-\gamma_c 0} = B\cos(\kappa_f 0) + C\sin(\kappa_f 0) \tag{2.15}$$

which is satisfied only if $A = B$. Making the magnetic field continuous at $x = 0$ requires that the first derivative of E_y, $\partial E_y/\partial x$ be continuous at $x = 0$,

$$\begin{aligned} -A\gamma_c e^{-\gamma_c 0} &= -B\kappa_f\sin(\kappa_f 0) + C\kappa_f\cos(\kappa_f 0) \\ -A\gamma_c &= +C\kappa_f \end{aligned} \tag{2.16}$$

yielding

$$C = -A\frac{\gamma_c}{\kappa_f} \tag{2.17}$$

All coefficients are written in terms of A. Using these coefficients, and applying the condition that E_y be continuous at $x = -h$ (h is a positive number) yields

$$A\left[\cos(-\kappa_f h) - \frac{\gamma_c}{\kappa_f}\sin(-\kappa_f h)\right] = De^{\gamma_s(h-h)} \tag{2.18}$$

This can be solved for D (noting $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$)

$$D = A\left[\cos(\kappa_f h) + \frac{\gamma_c}{\kappa_f}\sin(\kappa_f h)\right] \tag{2.19}$$

Putting all the terms together,

$$\begin{aligned} E_y &= Ae^{-\gamma_c x} && x > 0 \\ E_y &= A\left[\cos(\kappa_f x) - \frac{\gamma_c}{\kappa_f}\sin(\kappa_f x)\right] && -h < x < 0 \\ E_y &= A\left[\cos(\kappa_f h) + \frac{\gamma_c}{\kappa_f}\sin(\kappa_f h)\right]e^{\gamma_s(x+h)} && x < -h \end{aligned} \tag{2.20}$$

where A is the amplitude at the $x = 0$ interface. Equation 2.20 describes the amplitude of the electric field in all regions of the problem. Note that negative values of x must be used in the guiding and substrate layers—otherwise the formula will give nonsensical values. This equation is very handy for plotting out the mode profiles of guided modes. We will apply it to many problems in this chapter and text.

Having found the amplitude coefficients in Equation 2.20, is this description of the transverse electric field complete? No! The propagation and decay constants, γ_c, γ_s, and κ_f all depend on β, which is still undefined. The fourth and final boundary condition, namely the continuity of $\partial E_y/\partial x$ at $x = -h$, gives an equation for β.

$$\left.\frac{\partial E_y}{\partial x}\right|_{x=-h} = A[\kappa_f \sin(\kappa_f h) - \gamma_c \cos(\kappa_f h)] \qquad \text{(film term)}$$
$$= A\left[\cos(\kappa_f h) + \frac{\gamma_c}{\kappa_f}\sin(\kappa_f h)\right]\gamma_s \qquad \text{(substrate term)} \tag{2.21}$$

Divide both sides of the equation by $\cos(\kappa_f h)$ to get the *eigenvalue equation* for β.

$$\tan(h\kappa_f) = \frac{\gamma_c + \gamma_s}{\kappa_f\left[1 - \dfrac{\gamma_c\gamma_s}{\kappa_f^2}\right]} \tag{2.22}$$

This is a transcendental equation that must be solved numerically or graphically. All terms depend on the value of β. It is called the *characteristic equation* for the TE modes of a slab waveguide. Solution of this equation will yield the eigenvalues, β_{TE} that correspond to allowed TE modes in the waveguide.

Had we set up our initial problem with transverse magnetic fields, as opposed to transverse electric fields, we would have arrived at a different characteristic equation for the eigenvalues, β_{TM}. We leave it as an exercise (see Problem 2.1) to confirm that, for the TM case, the eigenvalue equation for β is

$$\tan(h\kappa_f) = \frac{\kappa_f\left[\dfrac{n_f^2}{n_s^2}\gamma_s + \dfrac{n_f^2}{n_c^2}\gamma_c\right]}{\kappa_f^2 - \dfrac{n_f^4}{n_c^2 n_s^2}\gamma_c\gamma_s} \tag{2.23}$$

Each waveguide structure that we explore in this text will have a characteristic equation that must be solved to find the eigenvalues of the modes.

The transcendental equation can be solved routinely with a numerical package on a personal computer, or it can be solved graphically. The amplitude distribution (Equation 2.20) and the transcendental equations for finding the eigenvalue (Equations 2.22 and 2.23) are three of the equations that you will use repeatedly, so they should be programmed into a numerical routine which can be called and modified as desired for different situations. To provide insight into the eigenequation, Example 2.1 below shows the graphical solution.

Example 2.1 Graphical and Numerical Solution to the β Eigenvalue Equation

Consider the planar dielectric structure shown in Figure 2.5. The guiding index has value 1.50, the substrate index is 1.45, and the cover index is 1.40. This is an asymmetric waveguide. The thickness of the guiding layer is 5μm. We want to determine the allowed values of β using Equation 2.22 for this structure. Assume that light with wavelength of 1μm is used to excite the waveguide.

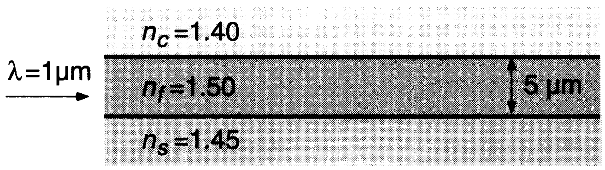

Figure 2.5 The planar slab waveguide configuration used in Example 2.1.

Solution:

We will use κ_f as the variable for plotting all the terms of the equation. This choice is arbitrary (we could have chosen β), but it makes the argument of the $\tan(\kappa_f h)$ term linear. All variables must be defined in terms of κ_f:

$$\begin{aligned} \gamma_c &= \sqrt{\beta^2 - k_0^2(1.4)^2} \\ \gamma_s &= \sqrt{\beta^2 - k_0^2(1.45)^2} \\ \beta &= \sqrt{k_0^2(1.5)^2 - \kappa_f^2} \end{aligned} \tag{2.24}$$

Using these values, both sides of the TE characteristic equation (Equation 2.22) are plotted as a function of κ_f on the graph in Figure 2.6. The variable κ_f ranges from a value of 0 (when $\beta = k_0 n_f$), to $\kappa_{max} = \sqrt{k_0^2 n_f^2 - k_0^2 n_s^2}$. The $\tan(\kappa_f h)$ term generates the typical pattern

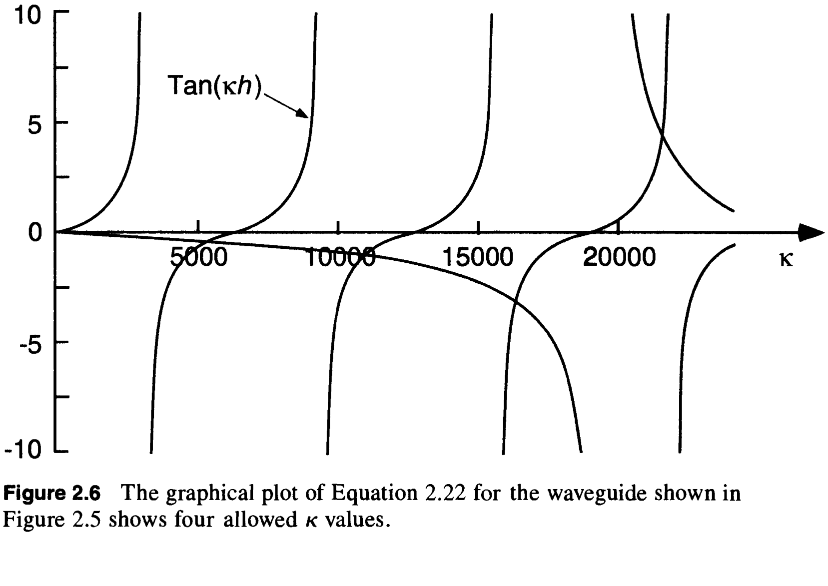

Figure 2.6 The graphical plot of Equation 2.22 for the waveguide shown in Figure 2.5 shows four allowed κ values.

of a repeating function extending from $-\infty$ to $+\infty$. The right-hand side of Equation 2.22 yields a slower function that diverges toward $-\infty$ around $\kappa = 20{,}000\ \mathrm{cm}^{-1}$ and then comes in from $+\infty$. At the points where the two curves cross, Equation 2.22 is satisfied. These points represent allowed values of κ for this waveguide. From the plot we see that the allowed κ values are approximately 5,500, 12,000, 16,500, and 21,500 cm^{-1}.

This plot was generated using *Mathematica*, although there are several other suitable numeric packages that can perform these calculations and plots. To serve as a guide, the *Mathematica* code is listed below:

```
nf=1.50;
ns=1.45;
nc=1.40;
h=0.0005;
lambda= 10^(-4);
k=2 Pi/lambda;
beta=Sqrt[ k^2 nf^2- kappa^2];
kappamax=Sqrt[k^2 nf^2 - k^2 ns^2];
gammas=Sqrt[beta^2-k^2 ns^2];
gammac=Sqrt[beta^2-k^2 nc^2];
Plot[{Tan[kappa h], (gammas + gammac) /
    (kappa(1 - gammas gammac/kappa^2))}, {kappa, 1, kappamax},
    PlotRange ->{-10,10}]
```

The transcendental characteristic equation must be solved numerically, which is a relatively straightforward action for many mathematical software packages. Again using *Mathematica*, the following command was used repeatedly to find each root of the equation.

```
FindRoot[Tan[kappa h] == (gammas + gammac)/ (kappa(1- gammas
    gammac/kappa^2)), {kappa, 5000}]
```

The last bracket of the command tells *Mathematica* to begin its search around a value of $\kappa = 5000$. The program returned the first κ value of 5497.16. To find higher roots, we used values taken from the graph as starting points, and let the computer return the more accurate value. Numerically, the eigenvalues for κ were found to be 5497.16, 10963.2, 16351, and 21545 cm^{-1}. The eigenvalues in terms of β can be found directly from the individual κ values, using Equation 2.24, to be 94087, 93608, 92819, and 91752 cm^{-1}, respectively.

This example shows some typical features of optical waveguides. First, the guiding film need not be very thick. It is generally on the order of a few wavelengths. Second, the index difference required to achieve a guiding structure is small. In this case, $\Delta n = 0.05$ between the core and substrate. This is actually a huge difference compared to many practical devices which have index differences as small as 0.001. Finally, inspection of Figure 2.6 shows that if the waveguide is made too thin, so that the argument κh does not extend beyond approximately $\pi/2$, it is possible that the two sets of lines will never cross, and there will be no mode allowed in the structure. Material growth experimentalists sometimes get caught in this problem after discovering that an exotic new optical material they have designed can only be grown in layers thinner than 0.1 μm before internal strain ruins the layer.

The example yielded four solutions for β, or four allowed modes. What does this mean? Each mode has the same wavelength of light; they each just travel in a slightly different direction within the waveguide. In the ray picture, the modes would be shown as four discrete rays traveling at slightly different angles, as shown in Figure 2.7. Notice that only a few discrete rays actually propagate in the waveguide.

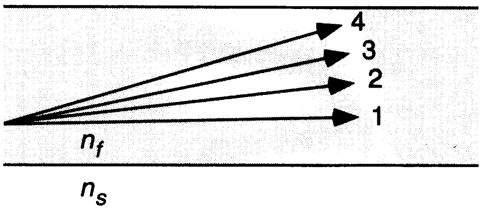

Figure 2.7 In this ray depiction of the four allowed modes in the waveguide, each ray has the same magnitude of k vector. They are simply oriented slightly differently with respect to the z axis.

To those familiar with basic quantum mechanics, the problem outlined in Example 2.1 should look very familiar. This graphical technique is often used to find the allowed energy eigenvalues of a particle in a finite potential well [4]. The analogy between the particle-in-a-box and the optical waveguide problem is very strong: both situations describe *waves* which are confined between two reflecting boundaries. In both cases, the waves partially tunnel into the surrounding potential barrier before turning around. Only certain allowed energies, in the case of the particle, or transverse propagation coefficients (κ), in the case of the optical wave, are found to create a standing wave in the one-dimensional system.

To complete the solution, the coefficient, A, should be related to a physical parameter. In practice, A is related to the power carried in the waveguide. The power is calculated by integrating the z component of the Poynting vector over the cross-sectional area of the guide:

$$S_z = \frac{1}{2} Re(\mathbf{E} \times \mathbf{H} \cdot \hat{z}) \tag{2.25}$$

Note that we are using the time-averaged power. The average power in a TE mode is

$$P_z = \frac{1}{2} \int_{-\infty}^{\infty} E_y H_x dx = \left(\frac{\beta}{2\omega\mu_0}\right) \int_{-\infty}^{\infty} |E_y|^2 dx \tag{2.26}$$

Since the integral spans only one direction, the integral has units of power per unit length (in the y direction).

It is much more enlightening to see the actual mode solutions that correspond to each value of β. Using the following *Mathematica* commands to implement

Equation 2.20, and to normalize each mode according to Equation 2.26, we plotted the total amplitude profile for each of the allowed modes in Example 2.1.

```
wave[x_]:= Exp[-gammac x] /; x>0
wave[x_]:= Cos[kappa x] -(gammac/kappa) Sin [kappa x]/; (x<=0) &&
    (x>-h)
wave[x_]:= (Cos[kappa h] + (gammac/kappa) Sin[kappa h])Exp[gammas
    (x+h)] /; x<=-h;
amplitude=1/ Sqrt[beta/(2 omega mu) * NIntegrate[(wave[x])^2, {x,
    -0.001, 0.0002}]]
Plot[amplitude * wave[x] ,{x, -h-0.0003, 0.0002}]
```

Figure 2.8 below illustrates the amplitude solutions for the four modes of Example 2.1.

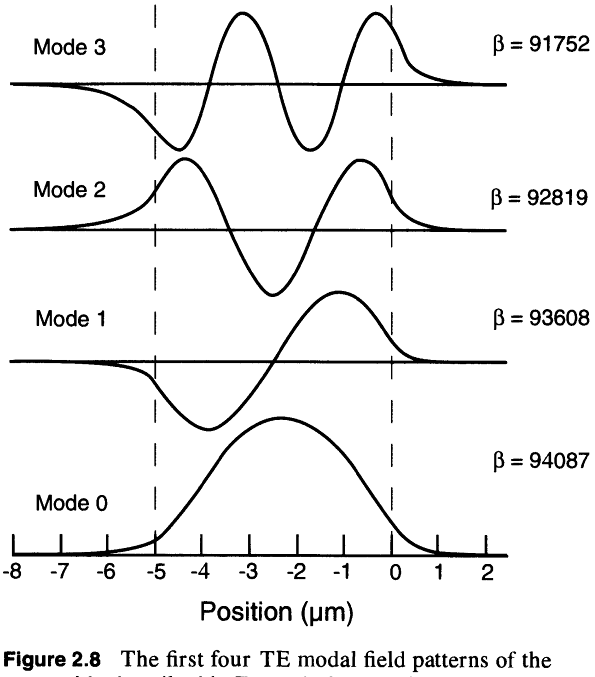

Figure 2.8 The first four TE modal field patterns of the waveguide described in Example 2.1 are shown. The vertical lines represent the location of the dielectric interfaces.

Since the waveguide is asymmetric, the modes are slightly asymmetric, although it is not obvious to the casual glance. Notice that the modes have alternating even and odd symmetry, and that the evanescent tails of the higher-order modes extend slightly further into the cladding than do the tails of the lowest-order mode. The modes are labeled by the *number of nodes* they have. The TE_0 mode is the lowest order (which means the mode with the smallest value of κ), and it has no nodes. The TE_1 mode has one zero crossing in the waveguide, the TE_2 mode has 2 nodes, etc. There will also be a set of TM modes with similar designations.

2.6 OPTICAL MODE CONFINEMENT

How much of the mode's energy resides inside the core, and how much energy is carried in the evanescent tail? Power in the guiding layer is found by integrating the Poynting vector over the area of the waveguide structure. The fraction of the power contained in the core is simply

$$\frac{P_{core}}{P_{total}} = \frac{\int_{-h}^{0} E_y(x)H_x^*(x)dx}{\int_{-\infty}^{\infty} E_y(x)H_x^*(x)dx} \tag{2.27}$$

The expression for the fraction of power contained in the cladding is:

$$\frac{P_{clad}}{P_{total}} = 1 - \frac{P_{core}}{P_{total}} \tag{2.28}$$

There is not a simple, general, closed-form expression for these integrals, although they are straightforward to evaluate numerically. In general, higher-order modes are less confined than their lower-order counterparts, and are therefore more susceptible to bending loss and evanescent coupling. This general trend can be seen in Figure 2.8, where mode 0 has a power confinement of 99.47 percent, while mode 3 has only 85.9 percent confinement. These percentages were determined by the numeric evaluation of Equation 2.27 for each mode. Problem 2.9 explores the dependence of the mode confinement on the size and relative indices of the waveguide. Mode confinement is an important property for waveguide designs. A mode that is loosely confined will be more affected by bends and neighboring structures than will a tightly bound mode.

2.7 THE SYMMETRIC WAVEGUIDE

Figure 2.9 shows a symmetric waveguide, where a guiding film with index n_f and thickness h is surrounded on both sides by an index n_s. It is convenient to place the coordinate system in the middle of this waveguide since the fields will reflect the symmetry of the structure.

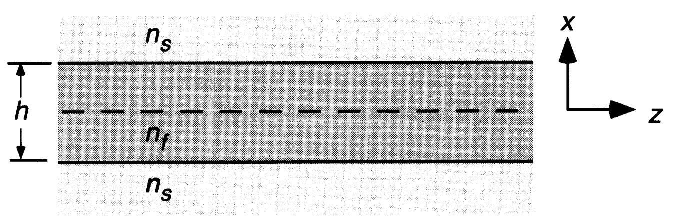

Figure 2.9 The symmetric waveguide is surrounded by material with the same index of refraction. The axis of symmetry is usually chosen to be the $x = 0$ axis.

We leave it as an exercise for the reader to show that the general field description of a TE mode within the symmetric structure (for the coordinate system shown in Figure 2.9) is:

$$E_y = Ae^{-\gamma(x-h/2)} \qquad \text{for } x \geq h/2$$

$$E_y = A\frac{\cos \kappa x}{\cos \kappa h/2} \quad \text{or} \quad A\frac{\sin \kappa x}{\sin \kappa h/2} \qquad \text{for } -h/2 < x < h/2 \tag{2.29}$$

$$E_y = \pm Ae^{\gamma(x+h/2)} \qquad \text{for } x \leq -h/2$$

The magnetic amplitude of the TM mode can be similarly described. There are two choices for the description of the field in the guiding layer, depending on whether a symmetric (cosine) or antisymmetric (sine) mode is excited. The fact that the modes can be uniquely characterized in terms of even or odd groups is a natural consequence of the *even* symmetry of the index structure. The sign of the field in the lower substrate is positive for the even modes, and negative for the odd modes. The characteristic eigenvalue equation for the TE modes in a symmetric waveguide is:

$$\begin{aligned} \tan \kappa h/2 &= \frac{\gamma}{\kappa} \quad \text{for even (cos) modes} \\ &= -\frac{\kappa}{\gamma} \quad \text{for odd (sin) modes} \end{aligned} \tag{2.30}$$

The characteristic equation for the TM modes is

$$\begin{aligned} \tan \kappa h/2 &= \left(\frac{n_f}{n_s}\right)^2 \frac{\gamma}{\kappa} \quad \text{for even (cos) modes} \\ &= -\left(\frac{n_s}{n_f}\right)^2 \frac{\kappa}{\gamma} \quad \text{for odd (sin) modes} \end{aligned} \tag{2.31}$$

A unique feature of the symmetric waveguide is that it can *always* support at least one mode. Consider the graphical solution for a symmetric waveguide described in Example 2.2 below.

Example 2.2 The Symmetric Waveguide

Suppose the waveguide shown in Figure 2.9 has a film index of $n_f = 1.49$, and cladding index equal to $n_s = 1.485$. The difference in index between the two layers is very small. Let's calculate what the allowed values of β are for this structure. Let the wavelength be 0.8μm. We will use the graphical solution, as it best demonstrates why the symmetric waveguide will always support at least one mode. Two thicknesses will be examined; $h = 3\mu$m, and $h = 15\mu$m.

Solution:

There are only two variables in this problem: γ and κ. As in the last example, we will plot functions in terms of κ.

$$\gamma_s = \sqrt{\beta^2 - k_0^2 n_s^2} = \sqrt{k_0^2(n_f^2 - n_s^2) - \kappa^2} \tag{2.32}$$
$$\beta = \sqrt{k_0^2 n_f^2 - \kappa^2}$$

Plugging numbers into these expressions, using $k_0 = 2\pi/\lambda = 7.853 \times 10^4 \text{cm}^{-1}$, yields

$$\gamma = \sqrt{9.176 \times 10^7 - \kappa^2} \tag{2.33}$$
$$\beta = \sqrt{1.3694 \times 10^{10} - \kappa^2}$$

To find the eigenvalues of the TE modes, we must solve Equation 2.30. Graphically, the functions $\tan \kappa h/2$, γ/κ, and $-\kappa/\gamma$ are plotted on the same graph as a function of κ. These are plotted against κ for the case where $h = 3\mu\text{m}$ in Figure 2.10.

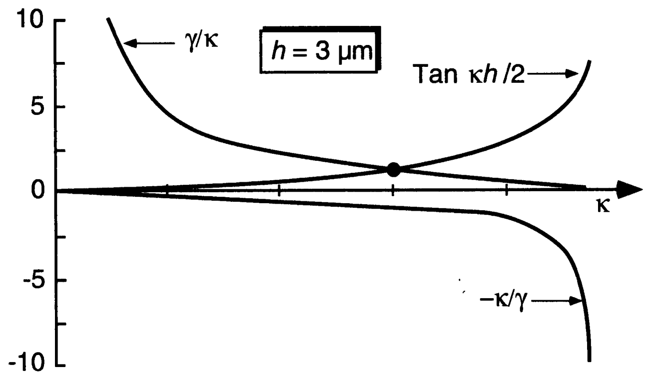

Figure 2.10 For the thin waveguide there is only one allowed mode, which occurs near $\kappa = 6000\text{cm}^{-1}$.

The top curve, which corresponds to the even mode, begins at $+\infty$, and terminates with a value of 0. Notice that the $\tan \kappa h/2$ starts at zero and increases. It is inevitable that the two curves will cross, so there *must* be at least one mode. In fact, we can generalize this statement: a symmetric waveguide will always carry at least one guided mode.

As the waveguide is made thicker, more modes are allowed. Consider the graphical plot of the equations for the case when the waveguide slab is $15\mu\text{m}$ thick, as shown in Figure 2.11.

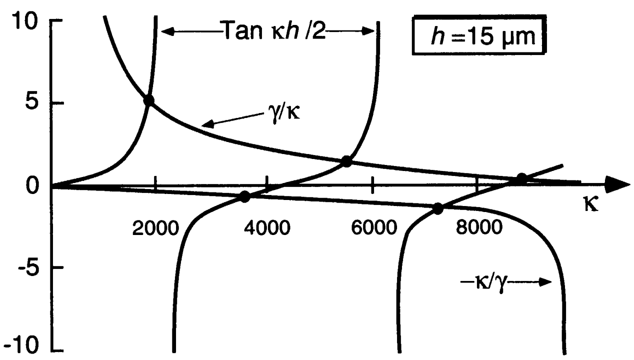

Figure 2.11 The thick waveguide supports both even and odd modes.