37. R. B. Green, "The Effect of Antenna Installations on the Echo Area of an Object," Report No. 1109-3, ElectroScience Laboratory, Ohio State University, Columbus, OH, September 1961.
38. R. B. Green "Scattering from Conjugate-Matched Antennas," *IEEE Trans. Antennas Propagat.*, Vol. AP-14, No. 1, pp. 17–21, January 1966.
39. R. J. Garbacz, "The Determination of Antenna Parameters by Scattering Cross-Section Measurements, III. Antenna Scattering Cross Section," Report No. 1223-10, Antenna Laboratory, Ohio State University, November 1962.
40. R. C. Hansen, "Relationships Between Antennas as Scatterers and as Radiators," *Proc. IEEE*, Vol. 77, No. 5, pp. 659–662, May 1989.
41. S. H. Dike and D. D. King, "Absorption Gain and Backscattering Cross Section of the Cylindrical Antenna," *Proc. IRE*, Vol. 40, 1952.
42. J. Sevick, "Experimental and Theoretical Results on the Backscattering Cross Section of Coupled Antennas," Tech. Report No. 150, Cruft Laboratory, Harvard University, May 1952.
43. D. L. Moffatt, "Determination of Antenna Scattering Properties From Model Measurements," Report No. 1223-12, Antenna Laboratory, Ohio State University, January 1964.
44. J. T. Aberle, Analysis of Probe-Fed Circular Microstrip Antennas, PhD Dissertation, University of Mass., Amherst, MA, 1989.
45. J. T. Aberle, D. M. Pozar, and C. R. Birtcher, "Evaluation of Input Impedance and Radar Cross Section of Probe-Fed Microstrip Patch Elements Using an Accurate Feed Model," *IEEE Trans. Antennas Propagat.*, Vol. 39, No. 12, pp. 1691–1696, December 1991.

PROBLEMS

2.1. An antenna has a beam solid angle that is equivalent to a *trapezoidal* patch (*patch with 4 sides, 2 of which are parallel to each other*) on the surface of a sphere of radius r. The angular space of the patch on the surface of the sphere extends between $\pi/6 \leq \theta \leq \pi/3 (30^\circ \leq \theta \leq 60^\circ)$ in latitude and $\pi/4 \leq \phi \leq \pi/3 (45^\circ \leq \phi \leq 60^\circ)$ in longitude. Find the following:

(a) Equivalent *beam solid angle* [which is equal to number of *square radians/steradians* **or** $(degrees)^2$] of the patch [*in square radians/steradians* ***and*** *in* $(degrees)^2$].

- Exact.
- Approximate using $\Omega_A = \Delta\Theta \cdot \Delta\Phi = (\theta_2 - \theta_1) \cdot (\phi_2 - \phi_1)$. Compare with the exact.

(b) Corresponding antenna *maximum directivities* of part *a* (*dimensionless* ***and*** *in dB*).

2.2. Derive (2-7) given the definitions of (2-5) and (2-6)

2.3. A hypothetical isotropic antenna is radiating in free-space. At a distance of 100 m from the antenna, the total electric field (E_θ) is measured to be 5 V/m. Find the

(a) power density (W_{rad})

(b) power radiated (P_{rad})

2.4. Find the half-power beamwidth (HPBW) and first-null beamwidth (FNBW), *in radians and degrees*, for the following normalized radiation intensities:

$$\left.\begin{array}{ll} \text{(a) } U(\theta) = \cos\theta & \text{(b) } U(\theta) = \cos^2\theta \\ \text{(c) } U(\theta) = \cos(2\theta) & \text{(d) } U(\theta) = \cos^2(2\theta) \\ \text{(e) } U(\theta) = \cos(3\theta) & \text{(f) } U(\theta) = \cos^2(3\theta) \end{array}\right\} (0 \le \theta \le 90^\circ, 0 \le \phi \le 360^\circ)$$

2.5. Find the half-power beamwidth (HPBW) and first-null beamwidth (FNBW), *in radians and degrees*, for the following normalized radiation intensities:

$$\left.\begin{array}{l} \text{(a) } U(\theta) = \cos\theta\cos(2\theta) \\ \text{(b) } U(\theta) = \cos^2\theta\cos^2(2\theta) \\ \text{(c) } U(\theta) = \cos(\theta)\cos(3\theta) \\ \text{(d) } U(\theta) = \cos^2(\theta)\cos^2(3\theta) \\ \text{(e) } U(\theta) = \cos(2\theta)\cos(3\theta) \\ \text{(f) } U(\theta) = \cos^2(2\theta)\cos^2(3\theta) \end{array}\right\} (0 \le \theta \le 90^\circ, 0 \le \phi \le 360^\circ)$$

2.6. The maximum radiation intensity of a 90% efficiency antenna is 200 mW/unit solid angle. Find the directivity and gain (dimensionless and in dB) when the

(a) input power is 125.66 mW

(b) radiated power is 125.66 mW

2.7. The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna are represented by the radiation intensity of

$$\left.\begin{array}{l} \text{(a) } U = B_o\cos^2\theta \\ \text{(b) } U = B_o\cos^3\theta \end{array}\right\} \quad \begin{array}{l} \text{(watts/unit solid angle)} \\ (0 \le \theta \le \pi/2, 0 \le \phi \le 2\pi) \end{array}$$

For each, find the

(a) maximum power density (in watts/square meter) at a distance of 1,000 m (assume far-field distance). Specify the angle where this occurs.

(b) exact and approximate beam solid angle Ω_A.

(c) directivity, exact and approximate, of the antenna (dimensionless and in dB).

(d) gain, exact and approximate, of the antenna (dimensionless and in dB).

2.8. You are an antenna engineer and you are asked to design a high directivity/gain antenna for a space-borne communication system operating at 10 GHz. The specifications of the antenna are such that its pattern consists basically of *one major lobe* and, for simplicity, *no minor lobes* (*if there are any minor lobes they are of such very low intensity and you can assume they are negligible/zero*). Also it is desired that the pattern is symmetrical in the azimuthal plane. In order to meet the desired objectives, the main lobe of the pattern should have a *half-power beamwidth* of *10 degrees*. In order to expedite the design, it is assumed that the major lobe of the normalized radiation intensity of the antenna is approximated by

$$U(\theta, \phi) = \cos^n(\theta)$$

and it exists only in the upper hemisphere $(0 \le \theta \le \pi/2, 0 \le \phi \le 2\pi)$. Determine the:

(a) Value of n (*not necessarily an integer*) to meet the specifications of the major lobe. *Keep 5 significant figures in your calculations.*

(b) *Exact* maximum directivity of the antenna (*dimensionless* and *in dB*).

(c) *Approximate* maximum directivity of the antenna *based on Kraus*' formula (*dimensionless* and *in dB*).

(d) *Approximate* maximum directivity of the antenna based on *Tai & Pereira's* formula (*dimensionless* and *in dB*).

2.9. In target-search ground-mapping radars it is desirable to have echo power received from a target, of constant cross section, to be independent of its range. For one such application, the desirable radiation intensity of the antenna is given by

$$U(\theta, \phi) = \left\{ \begin{array}{cc} 1 & 0^\circ \le \theta < 20^\circ \\ 0.342 \csc(\theta) & 20^\circ \le \theta < 60^\circ \\ 0 & 60^\circ \le \theta \le 180^\circ \end{array} \right\} 0^\circ \le \phi \le 360^\circ$$

Find the directivity (in dB) using the exact formula.

2.10. A beam antenna has half-power beamwidths of 30° and 35° in perpendicular planes intersecting at the maximum of the mainbeam. Find its approximate maximum effective aperture (in λ^2) using (a) Kraus' and (b) Tai and Pereira's formulas. The minor lobes are very small and can be neglected.

2.11. The normalized radiation intensity of a given antenna is given by
(a) $U = \sin\theta \sin\phi$ (b) $U = \sin\theta \sin^2\phi$
(c) $U = \sin\theta \sin^3\phi$ (d) $U = \sin^2\theta \sin\phi$
(e) $U = \sin^2\theta \sin^2\phi$ (f) $U = \sin^2\theta \sin^3\phi$
The intensity exists only in the $0 \le \theta \le \pi, 0 \le \phi \le \pi$ region, and it is zero elsewhere. Find the

(a) exact directivity (dimensionless and in dB).

(b) azimuthal and elevation plane half-power beamwidths (in degrees).

2.12. Find the directivity (dimensionless and in dB) for the antenna of Problem 2.11 using

(a) Kraus' approximate formula (2-26)

(b) Tai and Pereira's approximate formula (2-30a)

2.13. For Problem 2.5, determine the approximate directivity (in dB) using

(a) Kraus' formula

(b) Tai and Pereira's formula.

2.14. The normalized radiation intensity of an antenna is rotationally symmetric in ϕ, and it is represented by

$$U = \begin{cases} 1 & 0^\circ \le \theta < 30^\circ \\ 0.5 & 30^\circ \le \theta < 60^\circ \\ 0.1 & 60^\circ \le \theta < 90^\circ \\ 0 & 90^\circ \le \theta \le 180^\circ \end{cases}$$

(a) What is the directivity (above isotropic) of the antenna (in dB)?

(b) What is the directivity (above an infinitesimal dipole) of the antenna (in dB)?

2.15. The radiation intensity of an antenna is given by

$$U(\theta, \phi) = \cos^4 \theta \sin^2 \phi$$

for $0 \le \theta \le \pi/2$ and $0 \le \phi \le 2\pi$ (i.e., in the upper half-space). It is zero in the lower half-space.
Find the

(a) exact directivity (dimensionless and in dB)
(b) elevation plane half-power beamwidth (in degrees)

2.16. The normalized radiation intensity of an antenna is symmetric, and it can be approximated by

$$U(\theta) = \begin{cases} 1 & 0^\circ \le \theta < 30^\circ \\ \dfrac{\cos(\theta)}{0.866} & 30^\circ \le \theta < 90^\circ \\ 0 & 90^\circ \le \theta \le 180^\circ \end{cases}$$

and it is independent of ϕ. Find the

(a) exact directivity by integrating the function
(b) approximate directivity using Kraus' formula

2.17. The maximum gain of a horn antenna is +20 dB, while the gain of its first sidelobe is −15 dB. What is the difference in gain between the maximum and first sidelobe:

(a) in dB
(b) as a ratio of the field intensities.

2.18. The normalized radiation intensity of an antenna is approximated by

$$U = \sin \theta$$

where $0 \le \theta \le \pi$, and $0 \le \phi \le 2\pi$. Determine the directivity using the

(a) exact formula
(b) formulas of (2-33a) by McDonald and (2-33b) by Pozar
(c) computer program ***Directivity*** of this chapter.

2.19. Repeat Problem 2.18 for a $\lambda/2$ dipole whose normalized intensity is approximated by

$$U \simeq \sin^3 \theta$$

Compare the value with that of (4-91) or 1.643 (2.156 dB).

2.20. The radiation intensity of a circular loop of radius a and of constant current is given by

$$U = J_1^2(ka \sin \theta), \quad 0 \le \theta \le \pi \quad \text{and} \quad 0 \le \phi \le 2\pi$$

where $J_1(x)$ is the Bessel function of order 1. For a loop with radii of $a = \lambda/10$ and $\lambda/20$, determine the directivity using the:

(a) formulas (2-33a) by McDonald and (2-33b) by Pozar.

(b) computer program ***Directivity*** of this chapter.

Compare the answers with that of a very small loop represented by 1.5 or 1.76 dB.

2.21. Find the directivity (dimensionless and in dB) for the antenna of Problem 2.11 using numerical techniques with 10° uniform divisions and with the field evaluated at the

(a) midpoint

(b) trailing edge of each division.

2.22. Compute the directivity values of Problem 2.11 using the ***Directivity*** computer program of this chapter.

2.23. The far-zone electric-field intensity (array factor) of an end-fire two-element array antenna, placed along the z-axis and radiating into free-space, is given by

$$E = \cos\left[\frac{\pi}{4}(\cos\theta - 1)\right]\frac{e^{-jkr}}{r}, \qquad 0 \le \theta \le \pi$$

Find the directivity using

(a) Kraus' approximate formula

(b) the ***Directivity*** computer program of this chapter.

2.24. Repeat Problem 2.23 when

$$E = \cos\left[\frac{\pi}{4}(\cos\theta + 1)\right]\frac{e^{-jkr}}{r}, \qquad 0 \le \theta \le \pi$$

2.25. The radiation intensity is represented by

$$U = \begin{cases} U_0 \sin(\pi \sin\theta), & 0 \le \theta \le \pi/2 \text{ and } 0 \le \phi \le 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

Find the directivity

(a) exactly

(b) using the computer program ***Directivity*** of this chapter.

2.26. The radiation intensity of an aperture antenna, mounted on an infinite ground plane with z perpendicular to the aperture, is rotationally symmetric (not a function of ϕ), and it is given by

$$U = \left[\frac{\sin(\pi \sin\theta)}{\pi \sin\theta}\right]^2$$

Find the approximate directivity (dimensionless and in dB) using

(a) numerical integration. Use the ***Directivity*** computer program of this chapter.

(b) Kraus' formula

(c) Tai and Pereira's formula.

2.27. The normalized far-zone field pattern of an antenna is given by

$$E = \begin{cases} (\sin\theta \cos^2\phi)^{1/2} & 0 \le \theta \le \pi \text{ and } 0 \le \phi \le \pi/2,\, 3\pi/2 \le \phi \le 2\pi \\ 0 & \text{elsewhere} \end{cases}$$

Find the directivity using
(a) the exact expression
(b) Kraus' approximate formula
(c) Tai and Pereira's approximate formula
(d) the computer program ***Directivity*** of this chapter

2.28. The normalized field pattern of the main beam of a conical horn antenna, mounted on an infinite ground plane with z perpendicular to the aperture, is given by

$$\frac{J_1(ka\sin\theta)}{\sin\theta}$$

where a is its radius at the aperture. Assuming that $a = \lambda$, find the
(a) half-power beamwidth
(b) directivity using Kraus' approximate formula

2.29. A base station cellular communication systems *lossless* antenna has a *maximum gain* of 16 dB (above isotropic) at 1,900 MHz. Assuming the *input power* to the antenna is 8 watts, what is the *maximum* radiated power density (in watts/cm^2) at a distance of 100 meters? This will determine the safe level for human exposure to electromagnetic radiation.

2.30. A uniform plane wave, of a form similar to (2-55), is traveling in the positive z-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), axial ratio (AR), and tilt angle τ (in degrees) when
(a) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = 0$
(b) $E_x \neq E_y$, $\Delta\phi = \phi_y - \phi_x = 0$
(c) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = \pi/2$
(d) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = -\pi/2$
(e) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = \pi/4$
(f) $E_x = E_y$, $\Delta\phi = \phi_y - \phi_x = -\pi/4$
(g) $E_x = 0.5E_y$, $\Delta\phi = \phi_y - \phi_x = \pi/2$
(h) $E_x = 0.5E_y$, $\Delta\phi = \phi_y - \phi_x = -\pi/2$
In all cases, justify the answer.

2.31. Derive (2-66), (2-67), and (2-68).

2.32. Write a general expression for the polarization loss factor (PLF) of two linearly polarized antennas if
(a) both lie in the same plane
(b) both do not lie in the same plane

2.33. A linearly polarized wave traveling in the positive z-direction is incident upon a circularly polarized antenna. Find the polarization loss factor PLF (dimensionless and in dB) when the antenna is (based upon its transmission mode operation)

(a) right-handed (CW)
(b) left-handed (CCW)

2.34. A 300 MHz uniform plane wave, traveling along the *x-axis in the negative x direction*, whose electric field is given by

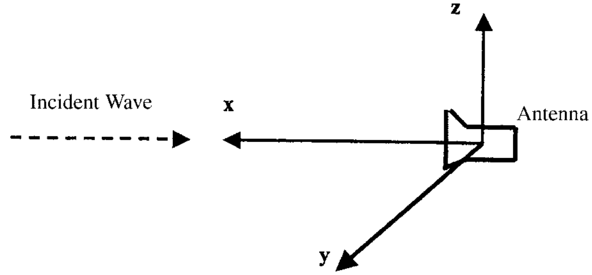
$$\mathbf{E}_w = E_o(j\hat{a}_y + 3\hat{a}_z)e^{+jkx}$$

where E_o is a real constant, impinges upon a dipole antenna that is placed at the origin and whose electric field radiated toward the *x-axis in the positive x direction* is given by

$$\mathbf{E}_a = E_a(\hat{a}_y + 2\hat{a}_z)e^{-jkx}$$

where E_a is a real constant. Determine the following:

(a) Polarization of the incident wave (*including axial ratio and sense of rotation, if any*). You must justify (state why?).
(b) Polarization of the antenna (*including axial ratio and sense of rotation, if any*). You must justify (state why?).
(c) Polarization loss factor (*dimensionless* and *in dB*).



2.35. The electric field of a uniform plane wave traveling along the *negative z* direction is given by

$$\mathbf{E}_w^i = (\hat{a}_x + j\hat{a}_y)E_o e^{+jkz}$$

and is incident upon a receiving antenna placed at the origin and *whose radiated electric field, toward the incident wave*, is given by

$$\mathbf{E}_a = (\hat{a}_x + 2\hat{a}_y)E_1\frac{e^{-jkr}}{r}$$

Determine the following:

(a) Polarization of the *incident wave*, and why?
(b) Sense of rotation of the *incident wave*.
(c) Polarization *of the antenna*, and why?
(d) Sense of rotation of the *antenna polarization*.
(e) Losses (*dimensionless* and *in dB*) due to polarization mismatch between the incident wave and the antenna.

2.36. A ground-based helical antenna is placed at the origin of a coordinate system and it is used as a receiving antenna. The normalized far-zone electric-field pattern of the helical antenna in the transmitting mode is represented in the direction θ_o, ϕ_o by

$$\mathbf{E}_a = E_o(j\hat{a}_\theta + 2\hat{a}_\phi) f_o(\theta_o, \phi_o)\frac{e^{-jkr}}{r}$$

The far-zone electric field transmitted by an antenna on a flying aircraft towards θ_o, ϕ_o, which is received by the ground-based helical antenna, is represented by

$$\mathbf{E}_w = E_1(2\hat{a}_\theta + j\hat{a}_\phi) f_1(\theta_o, \phi_o)\frac{e^{+jkr}}{r}$$

Determine the following:

(a) Polarization (*linear, circular, or elliptical*) of the helical antenna in the transmitting mode. *State also the sense of rotation, if any.*

(b) Polarization (*linear, circular, or elliptical*) of the incoming wave that impinges upon the helical antenna. *State also the sense of rotation, if any.*

(c) Polarization loss (*dimensionless and in dB*) due to match/mismatch of the polarizations of the antenna and incoming wave.

2.37. A circularly polarized wave, traveling in the positive z-direction, is incident upon a circularly polarized antenna. Find the polarization loss factor PLF (dimensionless and in dB) for right-hand (CW) and left-hand (CCW) wave and antenna.

2.38. The electric field radiated by a rectangular aperture, mounted on an infinite ground plane with z perpendicular to the aperture, is given by

$$\mathbf{E} = [\hat{\mathbf{a}}_\theta \cos\phi - \hat{\mathbf{a}}_\phi \sin\phi\cos\theta] f(r, \theta, \phi)$$

where $f(r, \theta, \phi)$ is a scalar function which describes the field variation of the antenna. Assuming that the receiving antenna is linearly polarized along the x-axis, find the polarization loss factor (PLF).

2.39. A circularly polarized wave, traveling in the $+z$-direction, is received by an elliptically polarized antenna whose reception characteristics near the main lobe are given approximately by

$$\mathbf{E}_a \simeq [2\hat{\mathbf{a}}_x + j\hat{\mathbf{a}}_y] f(r, \theta, \phi)$$

Find the polarization loss factor PLF (dimensionless and in dB) when the incident wave is

(a) right-hand (CW)

(b) left-hand (CCW)

circularly polarized. Repeat the problem when

$$\mathbf{E}_a \simeq [2\hat{\mathbf{a}}_x - j\hat{\mathbf{a}}_y] f(r, \theta, \phi)$$

In each case, what is the polarization of the antenna? How does it match with that of the wave?

2.40. A linearly polarized wave traveling in the negative z-direction has a tilt angle (τ) of 45°. It is incident upon an antenna whose polarization characteristics are given by

$$\hat{\boldsymbol{\rho}}_a = \frac{4\hat{\mathbf{a}}_x + j\hat{\mathbf{a}}_y}{\sqrt{17}}$$

Find the polarization loss factor PLF (dimensionless and db).

2.41. An elliptically polarized wave traveling in the negative z-direction is received by a circularly polarized antenna whose main lobe is along the $\theta = 0$ direction. The unit vector describing the polarization of the incident wave is given by

$$\hat{\boldsymbol{\rho}}_w = \frac{2\hat{\mathbf{a}}_x + j\hat{\mathbf{a}}_y}{\sqrt{5}}$$

Find the polarization loss factor PLF (dimensionless and in dB) when the wave that would be transmitted by the antenna is

(a) right-hand CP
(b) left-hand CP

2.42. A CW circularly polarized uniform plane wave is traveling in the $+z$ direction. Find the polarization loss factor PLF (dimensionless and in dB) assuming the receiving antenna (in its transmitting mode) is

(a) CW circularly polarized
(b) CCW circularly polarized

2.43. A linearly polarized uniform plane wave traveling in the $+z$ direction, with a power density of 10 milliwatts per square meter, is incident upon a CW circularly polarized antenna whose gain is 10 dB at 10 GHz. Find the

(a) maximum effective area of the antenna (in square meters)
(b) power (in watts) that will be delivered to a load attached directly to the terminals of the antenna.

2.44. A linearly polarized plane wave traveling along the negative z-axis is incident upon an elliptically polarized antenna (either CW or CCW). The axial ratio of the antenna polarization ellipse is 2:1 and its major axis coincides with the principal x-axis. Find the polarization loss factor (PLF) assuming the incident wave is linearly polarized in the

(a) x-direction
(b) y-direction

2.45. A wave traveling normally outward from the page (toward the reader) is the resultant of two elliptically polarized waves, one with components of **E** given by:

$$\mathscr{E}'_y = 3\cos\omega t$$

$$\mathscr{E}'_x = 7\cos\left(\omega t + \frac{\pi}{2}\right)$$

and the other with components given by:

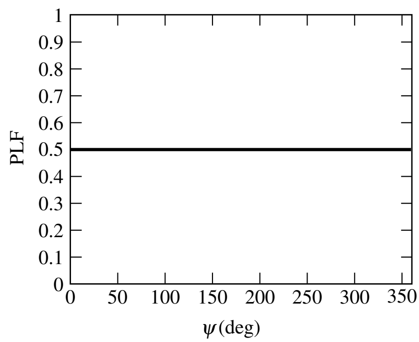
$$\mathscr{E}''_y = 2\cos\omega t$$
$$\mathscr{E}''_x = 3\cos\left(\omega t - \frac{\pi}{2}\right)$$

(a) What is the axial ratio of the resultant wave?

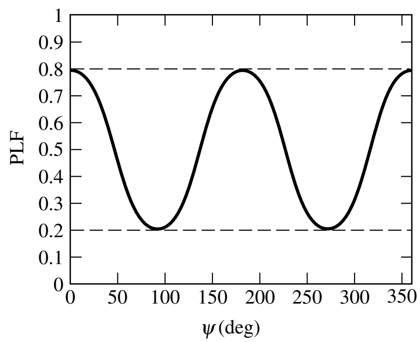
(b) Does the resultant vector **E** rotate clockwise or counterclockwise?

2.46. A linearly polarized antenna lying in the x-y plane is used to determine the polarization axial ratio of incoming plane waves traveling in the negative z-direction. The polarization of the antenna is described by the unit vector

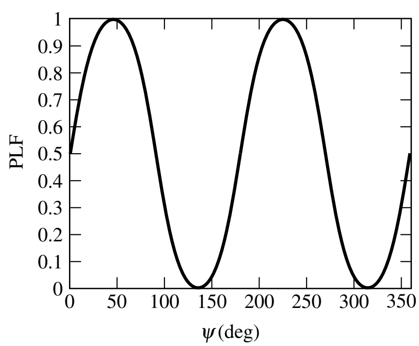
$$\hat{\boldsymbol{\rho}}_a = \hat{\mathbf{a}}_x \cos\psi + \hat{\mathbf{a}}_y \sin\psi$$



(a) PLF versus ψ



(b) PLF versus ψ



(c) PLF versus ψ

where ψ is an angle describing the orientation in the x-y plane of the receiving antenna. Above are the polarization loss factor (PLF) versus receiving antenna orientation curves obtained for three different incident plane waves. For each curve determine the axial ratio of the incident plane wave.

2.47. A $\lambda/2$ dipole, with a total loss resistance of 1 ohm, is connected to a generator whose internal impedance is $50 + j25$ ohms. Assuming that the peak voltage of the generator is 2 V and the impedance of the dipole, excluding the loss resistance, is $73 + j42.5$ ohms, find the power

(a) supplied by the source (real)
(b) radiated by the antenna
(c) dissipated by the antenna

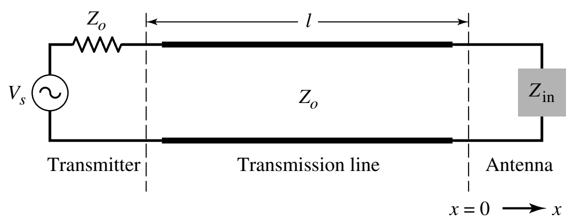
2.48. The antenna and generator of Problem 2.47 are connected via a 50-ohm $\lambda/2$-long lossless transmission line. Find the power

(a) supplied by the source (real)
(b) radiated by the antenna
(c) dissipated by the antenna

2.49. An antenna with a radiation resistance of 48 ohms, a loss resistance of 2 ohms, and a reactance of 50 ohms is connected to a generator with open-circuit voltage of 10 V and internal impedance of 50 ohms via a $\lambda/4$-long transmission line with characteristic impedance of 100 ohms.

(a) Draw the equivalent circuit
(b) Determine the power supplied by the generator
(c) Determine the power radiated by the antenna

2.50. A transmitter, with an internal impedance Z_0 (real), is connected to an antenna through a lossless transmission line of length l and characteristic impedance Z_0. Find a *simple* expression for the ratio between the antenna gain and its realized gain.



$$V(x) = A\,[e^{-jkx} + \Gamma(0)e^{+jkx}]$$
$$I(x) = \frac{A}{Z_0}\,[e^{-jkx} - \Gamma(0)e^{+jkx}]$$

V_s = strength of voltage source
$Z_{in} = R_{in} + jX_{in}$ = input impedance of the antenna
$Z_0 = R_0$ = characteristic impedance of the line
P_{accepted} = power accepted by the antenna $\{P_{\text{accepted}} = \text{Re}[V(0)I^*(0)]\}$
$P_{\text{available}}$ = power delivered to a matched load [i.e., $Z_{in} = Z_0^* = Z_0$]

2.51. The input reactance of an infinitesimal linear dipole of length $\lambda/60$ and radius $a = \lambda/200$ is given by

$$X_{in} \simeq -120\frac{[\ln(\ell/2a) - 1]}{\tan(k\ell/2)}$$

Assuming the wire of the dipole is copper with a conductivity of 5.7×10^7 S/m, determine at $f = 1$ GHz the

(a) loss resistance
(b) radiation resistance

(c) radiation efficiency
(d) VSWR when the antenna is connected to a 50-ohm line

2.52. A dipole antenna consists of a circular wire of length l. Assuming the current distribution on the wire is cosinusoidal, i.e.,

$$I_z(z) = I_0 \cos\left(\frac{\pi}{l} z'\right) \quad -l/2 \leq z' \leq l/2$$

where I_0 is a constant, derive an expression for the loss resistance R_L, which is one-half of (2-90b).

2.53. The E-field pattern of an antenna, independent of ϕ, varies as follows:

$$E = \begin{cases} 1 & 0^\circ \leq \theta \leq 45^\circ \\ 0 & 45^\circ < \theta \leq 90^\circ \\ \frac{1}{2} & 90^\circ < \theta \leq 180^\circ \end{cases}$$

(a) What is the directivity of this antenna?
(b) What is the radiation resistance of the antenna at 200 m from it if the field is equal to 10 V/m (rms) for $\theta = 0^\circ$ at that distance and the terminal current is 5 A (rms)?

2.54. The far-zone field radiated by a rectangular aperture mounted on a ground plane, with dimensions a and b and uniform aperture distribution, is given by (see Table 12.1)

$$E \approx \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

$$\left.\begin{aligned} E_\theta &= C \sin\phi \frac{\sin X}{X} \frac{\sin Y}{Y} \\ E_\phi &= C \cos\theta \cos\phi \frac{\sin X}{X} \frac{\sin Y}{Y} \end{aligned}\right\} \begin{aligned} X &= \frac{ka}{2} \sin\theta\cos\phi; \quad 0 \leq \theta \leq 90^\circ \\ Y &= \frac{kb}{2} \sin\theta\sin\phi; \quad 0 \leq \phi \leq 180^\circ \end{aligned}$$

where C is a constant and $0 \leq \theta \leq 90^\circ$ and $0 \leq \phi \leq 180^\circ$. For an aperture with $a = 3\lambda, b = 2\lambda$, determine the

(a) maximum partial directivities D_θ, D_ϕ (*dimensionless* and *in dB*) and
(b) total maximum directivity D_o (*dimensionless* and *in dB*). Compare with that computed using the equation in Table 12.1.

Use the computer program ***Directivity*** of this chapter.

2.55. Repeat Problem 2.54 when the aperture distribution is that of the dominant TE_{10} mode of a rectangular waveguide, or from Table 12.1

$$E \approx \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

$$\left.\begin{aligned} E_\theta &= -\frac{\pi}{2} C \sin\phi \frac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin Y}{Y} \\ E_\phi &= -\frac{\pi}{2} C \cos\theta\cos\phi \frac{\cos X}{(X)^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin Y}{Y} \end{aligned}\right\} \begin{aligned} X &= \frac{ka}{2} \sin\theta\cos\phi \\ Y &= \frac{kb}{2} \sin\theta\sin\phi \end{aligned}$$

2.56. Repeat Problem 2.55 when the aperture dimensions are those of an X-band rectangular waveguide with $a = 2.286$ cm (0.9 in.), $b = 1.016$ cm (0.4 in.) and frequency of operation is 10 GHz.

2.57. Repeat Problem 2.54 for a circular aperture with a uniform distribution and whose far-zone fields are, from Table 12.2

$$E \approx \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

$$\left.\begin{aligned} E_\theta &= jC_1 \sin\phi \frac{J_1(Z)}{Z} \\ E_\phi &= jC_1 \cos\theta \cos\phi \frac{J_1(Z)}{Z} \end{aligned}\right\} \begin{aligned} Z = ka\sin\theta; \quad & 0 \le \theta \le 90^\circ \\ & 0 \le \phi \le 180^\circ \end{aligned}$$

where C_1 is a constant and $J_1(Z)$ is the Bessel function of the first kind. Assume $a = 1.5\lambda$.

2.58. Repeat Problem 2.57 when the aperture distribution is that of the dominant TE_{11} mode of a circular waveguide, or from Table 12.2

$$E \approx \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi$$

$$\left.\begin{aligned} E_\theta &= C_2 \sin\phi \frac{J_1(Z)}{Z} \\ E_\phi &= C_2 \cos\theta \cos\phi \frac{J_z'(Z)}{(1) - \left(\dfrac{Z}{\chi_{11}'}\right)^2} \end{aligned}\right\} \begin{aligned} Z &= ka\sin\theta; & 0 \le \theta \le 90^\circ \\ J_z'(Z) &= J_o(Z) & 0 \le \phi \le 180^\circ \\ &\quad - J_1(Z)/Z; & \end{aligned}$$

where C_2 is a constant, $J_1'(Z)$ is the derivative of $J_1(Z)$, $\chi_{11}' = 1.841$ is the first zero of $J_1'(Z)$, and $J_o(Z)$ is the Bessel function of the first kind of order zero.

2.59. Repeat 2.58 when the radius of the aperture is $a = 1.143$ cm (0.45 in.) and the frequency of operation is 10 GHz.

2.60. A 1-m long dipole antenna is driven by a 150 MHz source having a source resistance of 50 ohms and a voltage of 100 V. If the ohmic resistance of the antennas is given by $R_L = 0.625$ ohms, find the:

(a) Current going into the antenna (I_{ant})
(b) Power dissipated by the antenna
(c) Power radiated by the antenna
(d) Radiation efficiency of the antenna

2.61. The field radiated by an infinitesimal dipole of very small length ($\ell \le \lambda/50$), and of uniform current distribution I_o, is given by (4-26a) or

$$\mathbf{E} = \hat{a}_\theta E_\theta \approx \hat{a}_\theta j\eta \frac{kI_o\ell}{4\pi r} e^{-jkr} \sin\theta$$

Determine the

(a) vector effective length
(b) maximum value of the vector effective length. Specify the angle.

(c) ratio of the maximum effective length to the physical length ℓ.

2.62. The field radiated by a half-wavelength dipole ($\ell = \lambda/2$), with a sinusoidal current distribution, is given by (4-84) or

$$\mathbf{E} = \hat{a}_\theta E_\theta \approx \hat{a}_\theta j\eta \frac{I_o}{2\pi r} e^{-jkr} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]$$

where I_o is the maximum current. Determine the

(a) vector effective length

(b) maximum value of the vector effective length. Specify the angle.

(c) ratio of the maximum effective length to the physical length ℓ.

2.63. A uniform plane wave, of 10^{-3}watts/cm^2 power density, is incident upon an infinitesimal dipole of length $\ell = \lambda/50$ and uniform current distribution, as shown in Figure 2.29(a). For a frequency of 10 GHz, determine the maximum open-circuited voltage at the terminals of the antenna. See Problem 2.61.

2.64. Repeat Problem 2.63 for a small dipole with triangular current distribution and length $\ell = \lambda/10$. See Example 2.14.

2.65. Repeat Problem 2.63 for a half-wavelength dipole ($\ell = \lambda/2$) with sinusoidal current distribution. See Problem 2.62.

2.66. Show that the effective length of a linear antenna can be written as

$$l_e = \sqrt{\frac{A_e |Z_t|^2}{\eta R_T}}$$

which for a lossless antenna and maximum power transfer reduces to

$$l_e = 2\sqrt{\frac{A_{em} R_r}{\eta}}$$

A_e and A_{em} represent, respectively, the effective and maximum effective apertures of the antenna while η is the intrinsic impedance of the medium.

2.67. An antenna has a maximum effective aperture of 2.147 m^2 at its operating frequency of 100 MHz. It has no conduction or dielectric losses. The input impedance of the antenna itself is 75 ohms, and it is connected to a 50-ohm transmission line. Find the directivity of the antenna system ("system" meaning includes any effects of connection to the transmission line). Assume no polarization losses.

2.68. A small circular parabolic reflector, often referred to as dish, is now being advertised as a TV antenna for direct broadcast. Assuming the diameter of the antenna is 1 meter, the frequency of operation is 3 GHz, and its aperture efficiency is 68%, determine the following:

(a) Physical area of the reflector (*in m*^{2}).

(b) Maximum effective area of the antenna (*in* m^2).

(c) Maximum directivity (*dimensionless* and *in dB*).

(d) Maximum power (*in watts*) that can be delivered to the TV if the power density of the wave incident upon the antenna is *10* μ*watts*/m^2. Assume *no losses* between the incident wave and the receiver (TV).

2.69. An incoming wave, with a uniform power density equal to 10^{-3} W/m^2 is incident normally upon a lossless horn antenna whose directivity is 20 dB. At a frequency of 10 GHz, determine the very maximum possible power that can be expected to be delivered to a receiver or a load connected to the horn antenna. There are no losses between the antenna and the receiver or load.

2.70. A linearly polarized aperture antenna, with a uniform field distribution over its area, is used as a receiving antenna. The antenna physical area over its aperture is 10 cm^2, and it is operating at 10 GHz. The antenna is illuminated with a circularly polarized plane wave whose incident power density is 10 mwatts/cm^2. Assuming the antenna element itself is lossless, determine its

(a) gain (*dimensionless* and *in dB*).

(b) maximum power (*in watts*) that can be delivered to a load connected to the antenna. Assume no other losses between the antenna and the load.

2.71. The *far-zone power density* radiated by a helical antenna can be approximated by

$$\mathbf{W}_{rad} = \mathbf{W}_{ave} \approx \hat{a}_r C_o \frac{1}{r^2} \cos^4 \theta$$

The radiated power density is symmetrical with respect to ϕ, and *it exists only in the upper hemisphere* $(0 \leq \theta \leq \pi/2, 0 \leq \phi \leq 2\pi)$; C_o is a constant. Determine the following:

(a) Power radiated by the antenna (*in watts*).

(b) Maximum directivity of the antenna (*dimensionless* and *in dB*)

(c) Direction (*in degrees*) along which the maximum directivity occurs.

(d) Maximum effective area (*in* m^2) at 1 GHz.

(e) Maximum power (*in watts*) received by the antenna, assuming no losses, at 1 GHz when the antenna is used as a receiver and the incident power density is *10 mwatts/m*^{2}.

2.72. For an X-band (8.2–12.4 GHz) rectangular horn, with aperture dimensions of 5.5 cm and 7.4 cm, find its maximum effective aperture (*in* cm^2) when its gain (over isotropic) is

(a) 14.8 dB at 8.2 GHz

(b) 16.5 dB at 10.3 GHz

(c) 18.0 dB at 12.4 GHz

2.73. For Problem 2.54 compute the

(a) maximum effective area (in λ^2) using the computer program ***Directivity*** of this chapter. Compare with that computed using the equation in Table 12.1.

(b) aperture efficiencies of part (a). Are they smaller or larger than unity and why?

2.74. Repeat Problem 2.73 for Problem 2.55.

2.75. Repeat Problem 2.73 for Problem 2.56.

2.76. Repeat Problem 2.73 for Problem 2.57. Compare with those in Table 12.2.

2.77. Repeat Problem 2.73 for Problem 2.58. Compare with those in Table 12.2.

2.78. Repeat Problem 2.73 for Problem 2.59. Compare with those in Table 12.2.

2.79. A 30-dB, right-circularly polarized antenna in a radio link radiates 5 W of power at 2 GHz. The receiving antenna has an impedance mismatch at its terminals, which leads to a VSWR of 2. The receiving antenna is about 95% efficient and has a field pattern near the beam maximum given by $\mathbf{E}_r = (2\hat{\mathbf{a}}_x + j\hat{\mathbf{a}}_y)F_r(\theta, \phi)$. The distance between the two antennas is 4,000 km, and the receiving antenna is required to deliver 10^{-14} W to the receiver. Determine the maximum effective aperture of the receiving antenna.

2.80. The radiation intensity of an antenna can be approximated by

$$U(\theta, \phi) = \begin{cases} \cos^4(\theta) & 0^\circ \le \theta < 90^\circ \\ 0 & 90^\circ \le \theta \le 180^\circ \end{cases} \qquad \text{with } 0^\circ \le \phi \le 360^\circ$$

Determine the maximum effective aperture (*in* m^2) of the antenna if its frequency of operation is $f = 10$ GHz.

2.81. A communication satellite is in stationary (synchronous) orbit about the earth (assume altitude of 22,300 statute miles). Its transmitter generates 8.0 W. Assume the transmitting antenna is isotropic. Its signal is received by the 210-ft diameter tracking paraboloidal antenna on the earth at the NASA tracking station at Goldstone, California. Also assume no resistive losses in either antenna, perfect polarization match, and perfect impedance match at both antennas. At a frequency of 2 GHz, determine the:

(a) power density (*in watts/m*^{2}) incident on the receiving antenna.

(b) power received by the ground-based antenna whose gain is 60 dB.

2.82. A lossless ($e_{cd} = 1$) antenna is operating at 100 MHz and its maximum effective aperture is 0.7162 m^2 at this frequency. The input impedance of this antenna is 75 ohms, and it is attached to a 50-ohm transmission line. Find the directivity (dimensionless) of this antenna if it is polarization-matched.

2.83. A resonant, lossless ($e_{cd} = 1.0$) half-wavelength dipole antenna, having a directivity of 2.156 dB, has an input impedance of 73 ohms and is connected to a lossless, 50 ohms transmission line. A wave, having the same polarization as the antenna, is incident upon the antenna with a power density of 5 W/m^2 at a frequency of 10 MHz. Find the received power available at the end of the transmission line.

2.84. Two X-band (8.2–12.4 GHz) rectangular horns, with aperture dimensions of 5.5 cm and 7.4 cm and each with a gain of 16.3 dB (over isotropic) at 10 GHz,

are used as transmitting and receiving antennas. Assuming that the input power is 200 mW, the VSWR of each is 1.1, the conduction-dielectric efficiency is 100%, and the antennas are polarization-matched, find the maximum received power when the horns are separated in air by
(a) 5 m (b) 50 m (c) 500 m

2.85. Transmitting and receiving antennas operating at 1 GHz with gains (over isotropic) of 20 and 15 dB, respectively, are separated by a distance of 1 km. Find the maximum power delivered to the load when the input power is 150 W. Assume that the

(a) antennas are polarization-matched

(b) transmitting antenna is circularly polarized (either right- or left-hand) and the receiving antenna is linearly polarized.

2.86. Two lossless, polarization-matched antennas are aligned for maximum radiation between them, and are separated by a distance of 50λ. The antennas are matched to their transmission lines and have directivities of 20 dB. Assuming that the power at the input terminals of the transmitting antenna is 10 W, find the power at the terminals of the receiving antenna.

2.87. Repeat Problem 2.86 for two antennas with 30 dB directivities and separated by 100λ. The power at the input terminals is 20 W.

2.88. Transmitting and receiving antennas operating at 1 GHz with gains of 20 and 15 dB, respectively, are separated by a distance of 1 km. Find the power delivered to the load when the input power is 150 W. Assume the PLF = 1.

2.89. A series of microwave repeater links operating at 10 GHz are used to relay television signals into a valley that is surrounded by steep mountain ranges. Each repeater consists of a receiver, transmitter, antennas, and associated equipment. The transmitting and receiving antennas are identical horns, each having gain over isotropic of 15 dB. The repeaters are separated in distance by 10 km. For acceptable signal-to-noise ratio, the power received at each repeater must be greater than 10 nW. Loss due to polarization mismatch is not expected to exceed 3 dB. Assume matched loads and free-space propagation conditions. Determine the minimum transmitter power that should be used.

2.90. A one-way communication system, operating at 100 MHz, uses two identical $\lambda/2$ vertical, resonant, and lossless dipole antennas as transmitting and receiving elements separated by 10 km. In order for the signal to be detected by the receiver, the power level at the receiver terminals must be at least 1 μW. Each antenna is connected to the transmitter and receiver by a lossless 50-Ω transmission line. Assuming the antennas are polarization-matched and are aligned so that the maximum intensity of one is directed toward the maximum radiation intensity of the other, determine the minimum power that must be generated by the transmitter so that the signal will be detected by the receiver. Account for the proper losses from the transmitter to the receiver.

2.91. In a long-range microwave communication system operating at 9 GHz, the transmitting and receiving antennas are identical, and they are separated by

10,000 m. To meet the signal-to-noise ratio of the receiver, the received power must be at least 10 μW. Assuming the two antennas are aligned for maximum reception to each other, including being polarization-matched, what should the gains (in dB) of the transmitting and receiving antennas be when the input power to the transmitting antenna is 10 W?

2.92. A mobile wireless communication system operating at 2 GHz utilizes two antennas, one at the base station and the other at the mobile unit, which are separated by *16 kilometers*. The transmitting antenna, at the base station, is circularly-polarized while the receiving antenna, at the mobile station, is linearly polarized. The *maximum gain of the transmitting antenna is 20 dB* while the gain of the receiving antennas is unknown. The input power to the transmitting antenna is *100 watts* and the power received at the receiver, which is connected to the receiving antenna, is *5 nanowatts*. Assuming that the two antennas are aligned so that the maximum of one is directed toward the maximum of the other, *and also assuming no reflection/mismatch losses at the transmitter or the receiver*, what is the maximum gain of the receiving antenna (*dimensions* and *in dB*)?

2.93. A rectangular X-band horn, with aperture dimensions of 5.5 cm and 7.4 cm and a gain of 16.3 dB (over isotropic) at 10 GHz, is used to transmit and receive energy scattered from a perfectly conducting sphere of radius $a = 5\lambda$. Find the maximum scattered power delivered to the load when the distance between the horn and the sphere is
(a) 200λ (b) 500λ
Assume that the input power is 200 mW, and the radar cross section is equal to the geometrical cross section.

2.94. A radar antenna, used for both transmitting and receiving, has a gain of 150 (dimensionless) at its operating frequency of 5 GHz. It transmits 100 kW, and is aligned for maximum directional radiation and reception to a target 1 km away having a radar cross section of 3 m^2. The received signal matches the polarization of the transmitted signal. Find the received power.

2.95. In an experiment to determine the radar cross section of a Tomahawk cruise missile, a 1,000 W, 300 MHz signal was transmitted toward the target, and the received power was measured to be 0.1425 mW. The same antenna, whose gain was 75 (*dimensionless*), was used for both transmitting and receiving. The polarizations of both signals were identical (PLF $= 1$), and the distance between the antenna and missile was 500 m. What is the radar cross section of the cruise missile?

2.96. Repeat Problem 2.95 for a radar system with 1,000 W, 100 MHz transmitted signal, 0.01 W received signal, an antenna with a gain of 75 (*dimensionless*), and separation between the antenna and target of 700 m.

2.97. The maximum radar cross section of a resonant linear $\lambda/2$ dipole is approximately $0.86\lambda^2$. For a monostatic system (i.e., transmitter and receiver at the same location), find the received power (in W) if the transmitted power is 100 W, the distance of the dipole from the transmitting and receiving antennas is 100 m, the gain of the transmitting and receiving antennas is 15 dB each, and the frequency of operation is 3 GHz. Assume a polarization loss factor of -1 dB.

2.98. The effective antenna temperature of an antenna looking toward zenith is approximately 5 K. Assuming that the temperature of the transmission line (waveguide) is 72 °F, find the effective temperature at the receiver terminals when the attenuation of the transmission line is 4 dB/100 ft and its length is
(a) 2 ft (b) 100 ft
Compare it to a receiver noise temperature of about 54 K.

2.99. Derive (2-146). Begin with an expression that assumes that the physical temperature and the attenuation of the transmission line are not constant.