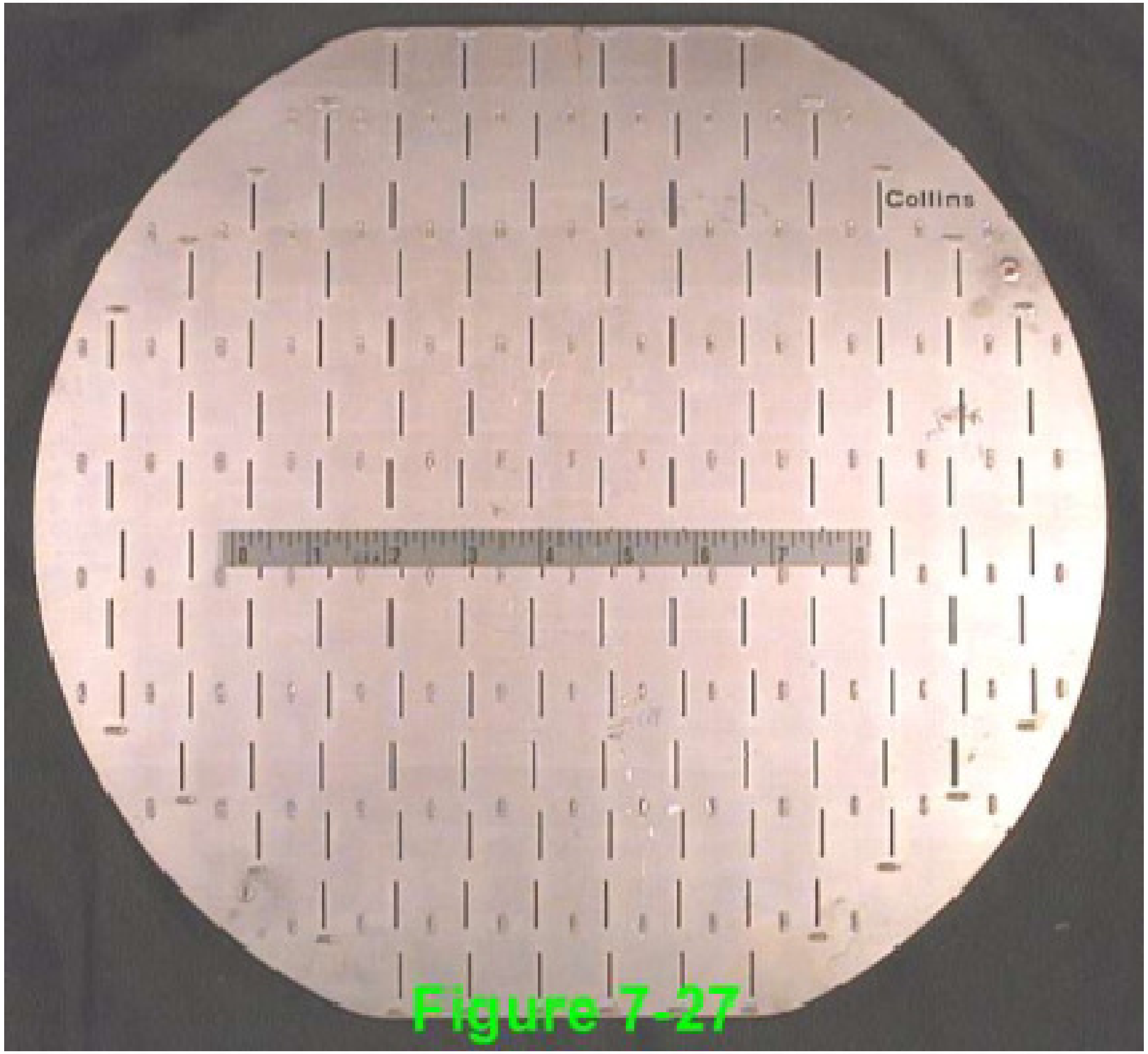
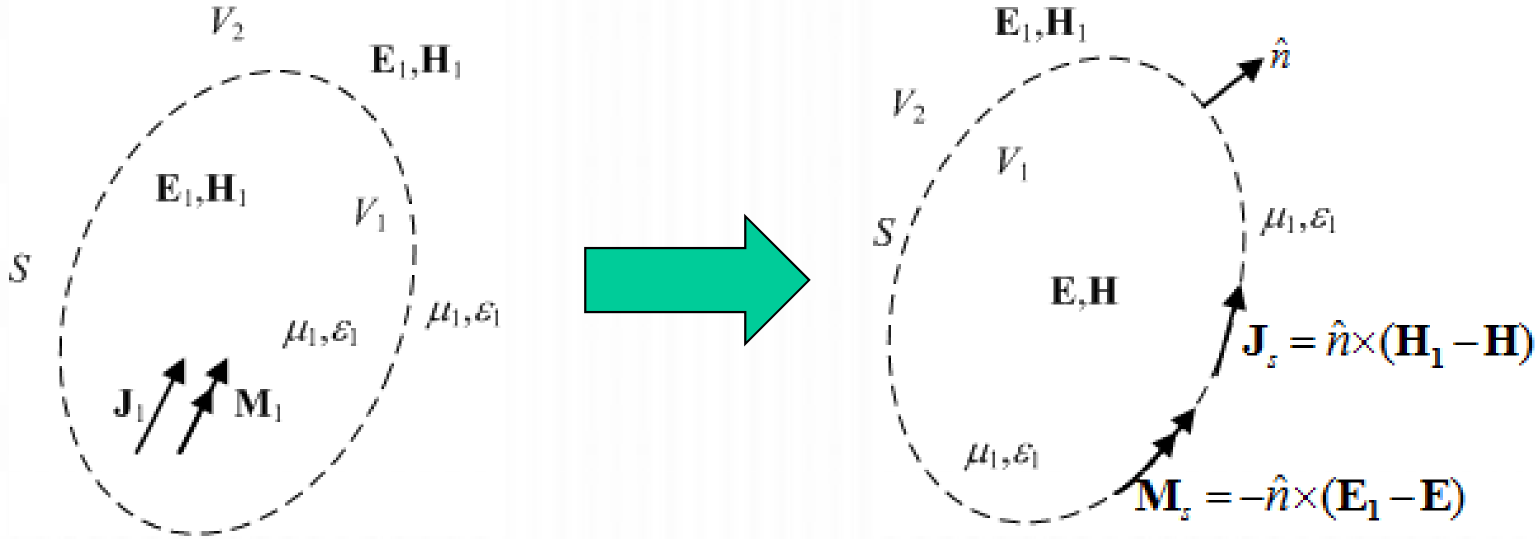
Chapter 12 : Aperture Antenna

- Huygen's Principle
- Application of Huygen's principle
- Rectangular apertures
- Circular apertures

Aperture Antenna Example



Huygen's principle



Actual problem | Equivalent problem

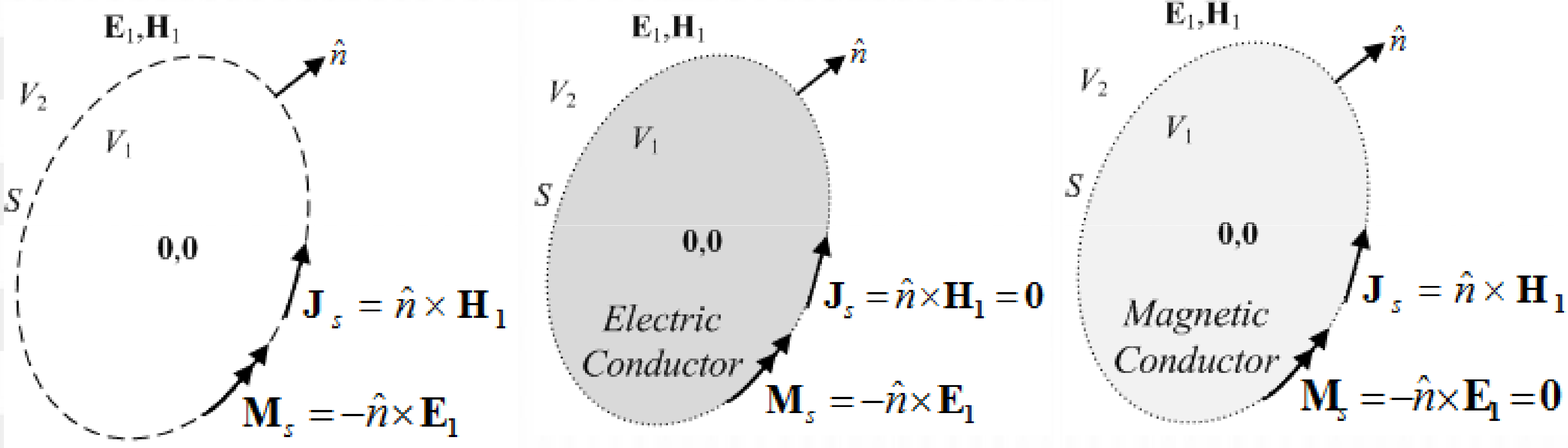
Equivalent surface currents $\mathbf{J}_s$, $\mathbf{M}_s$ radiate fields $\mathbf{E}_1,\mathbf{H}_1$ in V_2 (same as actual problem)

However, $\mathbf{J}_s$, $\mathbf{M}_s$ do NOT radiate fields $\mathbf{E}_1,\mathbf{H}_1$ in V_1.

Thus, actual problem and equivalent problem are equivalent only in V_2 (outside V_1)

Huygen's principle (2)

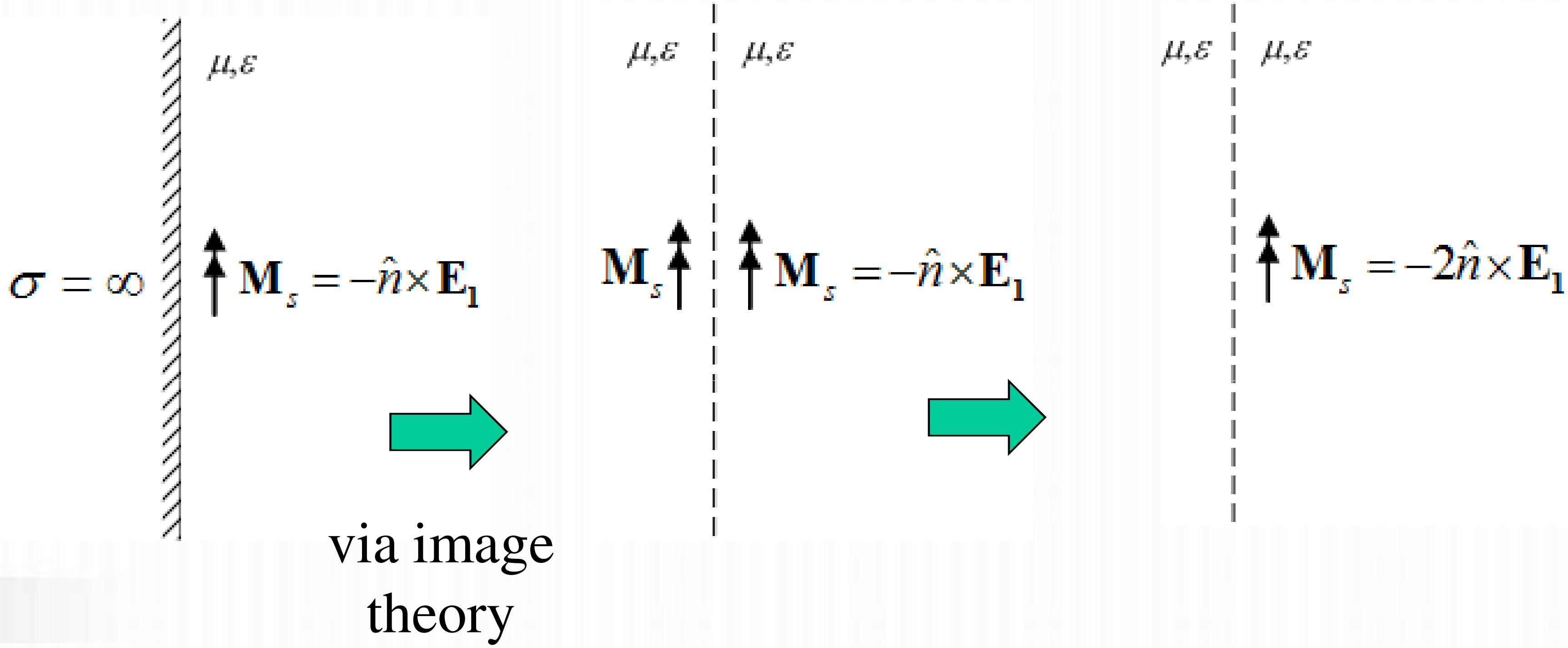
Since the fields **E**,**H** radiated by $\mathbf{J}_s$, $\mathbf{M}_s$ in V_1 are arbitrary, we can assume they are zero.



Love's equivalent	Electric Conductor equivalent	Magnetic Conductor equivalent

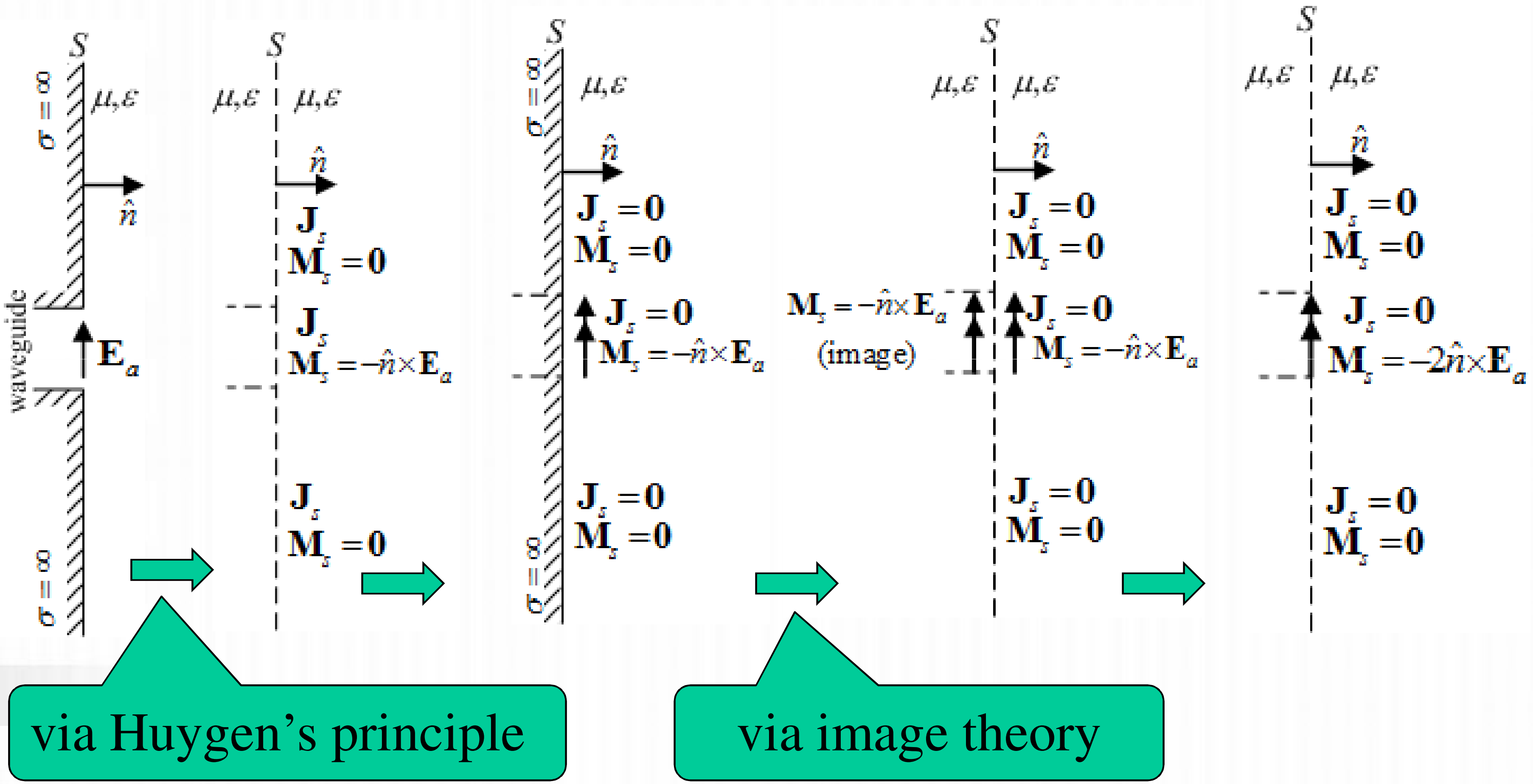
Equivalence principle models

Equivalent model example



Equivalent models for magnetic source radiation near a perfect electric conductor

Waveguide aperture example



Aperture fields $(\mathbf{E}_a, \mathbf{H}_a)$ are known.

Far-zone fields

Far-zone fields due to electric and magnetic surface currents are given by:

$$\mathbf{E}(\vec{r}) \approx -\frac{jk}{4\pi}\frac{e^{-jkr}}{r}\iint_S [\mathbf{M}_s(\vec{r}\,')\times\hat{r} - \hat{r}\times\hat{r}\times\eta\mathbf{J}_s(\vec{r}\,')]e^{jk\hat{r}\cdot\vec{r}\,'}ds'$$

$$\mathbf{H}(\vec{r}) \approx \frac{jk}{4\pi}\frac{e^{-jkr}}{r}\iint_S [\mathbf{J}_s(\vec{r}\,')\times\hat{r} + \hat{r}\times\hat{r}\times\frac{1}{\eta}\mathbf{M}_s(\vec{r}\,')]e^{jk\hat{r}\cdot\vec{r}\,'}ds'$$

Recall that: $\mathbf{H} = \dfrac{\hat{r}\times\mathbf{E}}{\eta};\mathbf{E} = \eta\mathbf{H}\times\hat{r}$

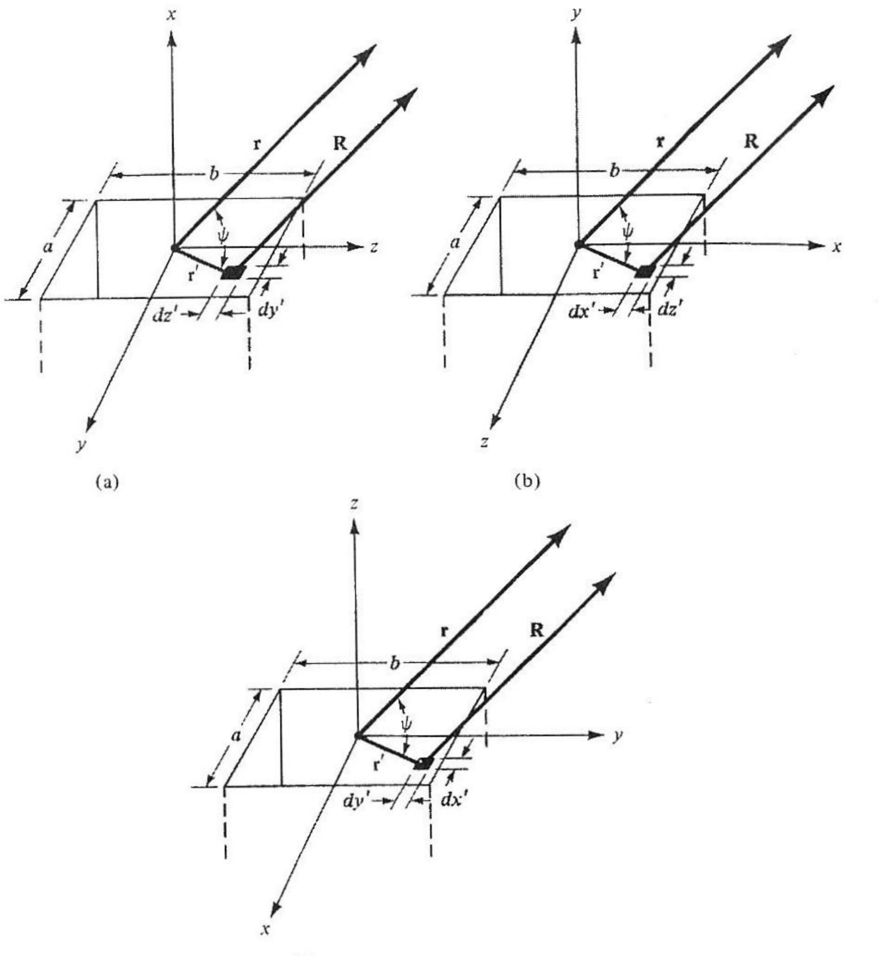
and

$$\hat{r}\cdot\vec{r}\,' = r'\cos\varsigma = (x'\cos\phi + y'\sin\phi)\sin\theta + z'\cos\theta$$

$$= \rho'\cos(\phi-\phi')\sin\theta + z'\cos\theta$$

$$= r'[\cos(\phi-\phi')\sin\theta\sin\theta' + \cos\theta\cos\theta']$$

Rectangular Apertures



Rectangular aperture positions
for
antenna system analysis

In general, the non-zero components of $\mathbf{J}_s$ and $\mathbf{M}_s$ are:

J_y, J_z, M_y, M_z Figure (a)

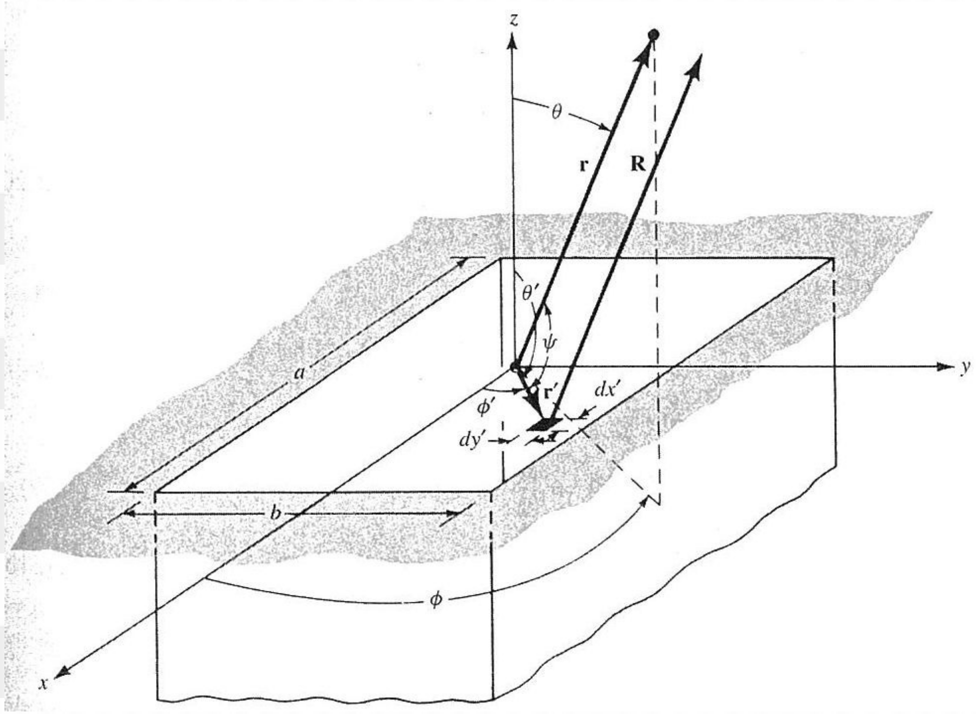
J_x, J_z, M_x, M_z Figure (b)

J_x, J_y, M_x, M_y Figure (c)

Rectangular Apertures (2)

Rectangular aperture with uniform distribution on an infinite ground plane: find far-zone fields for $z > 0$.

Aperture field: $\mathbf{E}_a = \hat{y}E_0 \quad -a/2 \le x' \le a/2; -b/2 \le y' \le b/2$



Equivalent problem for $z > 0$

$$\mathbf{M}_s = -2\hat{n} \times \mathbf{E}_a$$
$$= -2\hat{z} \times \hat{y}E_0$$
$$= \hat{x}2E_0 \text{ on aperture}$$

$\mathbf{M}_s = \mathbf{J}_s = \mathbf{0}$ outside aperture

Rectangular Apertures (3)

The far-zone electric field can then be given by

$$\mathbf{E}(\vec{r}) \approx -\frac{jk}{4\pi}\frac{e^{-jkr}}{r}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\mathbf{M_s}(\vec{r}\,')\times\hat{r}e^{jk\hat{r}\cdot\vec{r}\,'}dy'\,dx'$$

Since $\quad \mathbf{M}_s \times \hat{r} = \hat{x}2E_0 \times \hat{r}$

$$= 2E_0(-\hat{\phi}\cos\theta\cos\phi - \hat{\theta}\sin\phi)$$

and $\quad \hat{r}\cdot\vec{r}\,' = (x'\cos\phi + y'\sin\phi)\sin\theta\,(\text{for } z'=0)$

$$\mathbf{E}(\vec{r}) \approx \frac{jk}{4\pi}\frac{e^{-jkr}}{r}2E_0(\hat{\phi}\cos\theta\cos\phi + \hat{\theta}\sin\phi)\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}e^{jk\hat{r}\cdot\vec{r}\,'}dy'\,dx'$$

Rectangular Apertures (4)

Since

$$\int_{-a/2}^{a/2}\int_{-b/2}^{b/2} e^{jk\hat{r}\cdot\vec{r}\,'}dy'\,dx' = \int_{-a/2}^{a/2} e^{jkx'\sin\theta\cos\phi}dx'\int_{-b/2}^{b/2} e^{jky'\sin\theta\sin\phi}dy'$$

$$= ab\,\mathrm{sinc}\left(\frac{ka}{2}\sin\theta\cos\phi\right)\mathrm{sinc}\left(\frac{kb}{2}\sin\theta\sin\phi\right)$$

The far-zone fields are given by

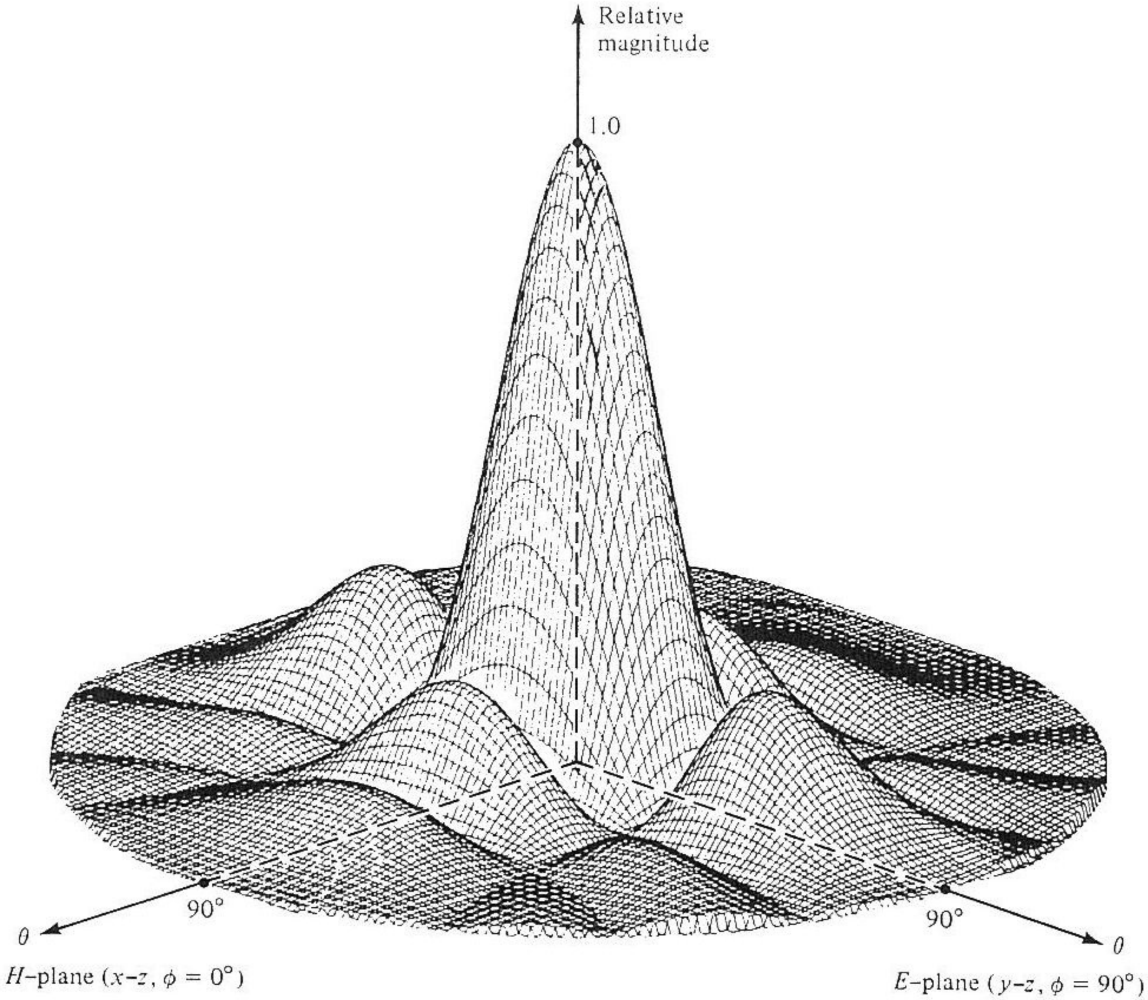
$$\mathbf{E}(\vec{r}) = \frac{jk}{4\pi}\frac{e^{-jkr}}{r}2E_0 ab\,\mathrm{sinc}\left(\frac{ka}{2}\sin\theta\cos\phi\right)\mathrm{sinc}\left(\frac{kb}{2}\sin\theta\sin\phi\right)$$

$$\times(\hat{\phi}\cos\theta\cos\phi + \hat{\theta}\sin\phi)$$

$$\mathbf{H}(\vec{r}) = \frac{jk}{4\pi}\frac{e^{-jkr}}{\eta r}2E_0 ab\,\mathrm{sinc}\left(\frac{ka}{2}\sin\theta\cos\phi\right)\mathrm{sinc}\left(\frac{kb}{2}\sin\theta\sin\phi\right)$$

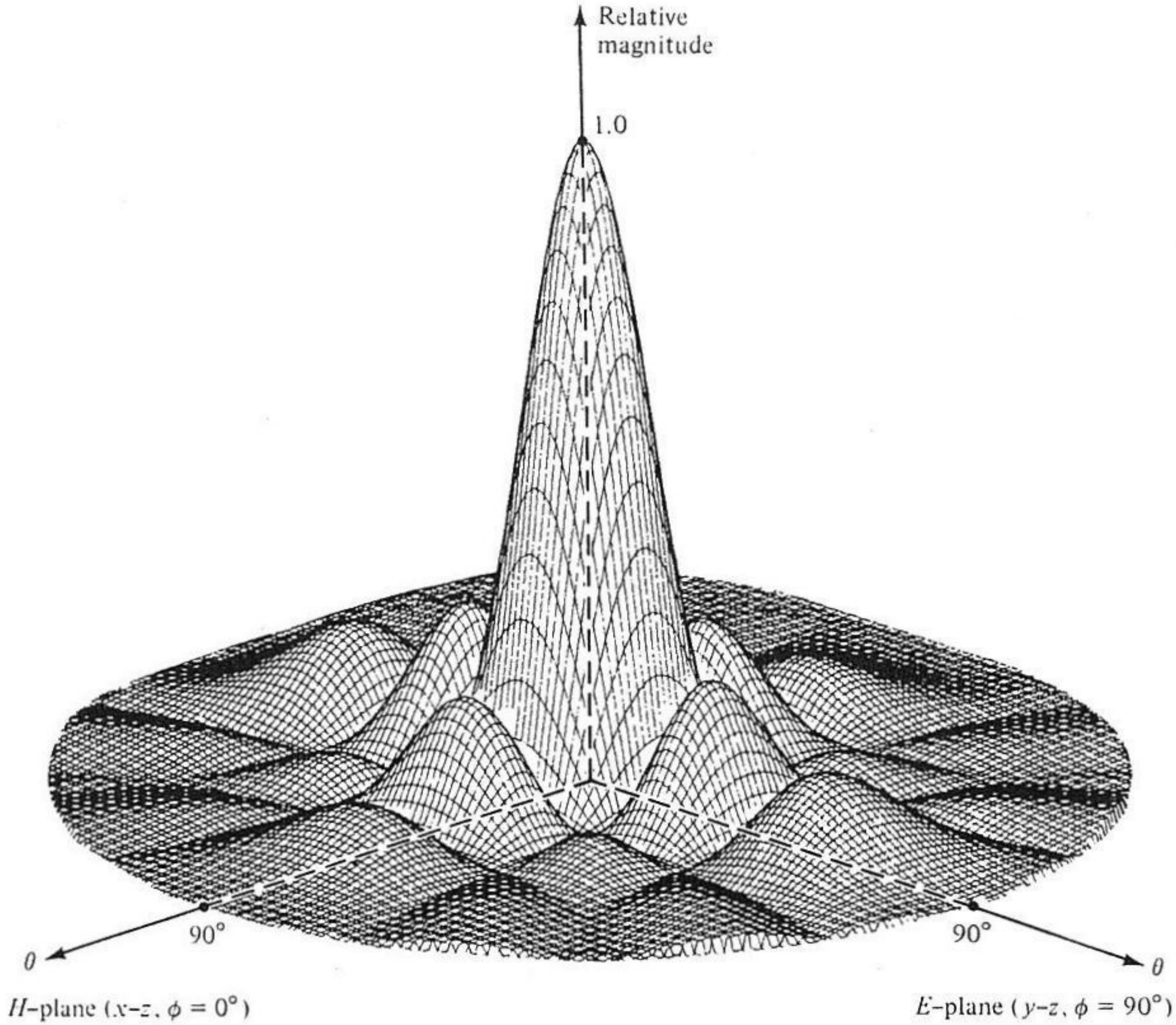
$$\times(-\hat{\theta}\cos\theta\cos\phi + \hat{\phi}\sin\phi)$$

Radiation Pattern



Normalized field pattern: $a = 3\lambda$, $b = 2\lambda$

Radiation Pattern



Normalized field pattern: $a = 3\lambda$, $b = 3\lambda$

Radiation Pattern (3)

In many applications, only principle plane patterns are usually sufficient.

E-Plane
($\phi=\pi/2, 3\pi/2$)
(y-z plane)

$$E_\theta = \frac{jk}{4\pi}\frac{e^{-jkr}}{r}2E_0 ab \operatorname{sinc}\left(\frac{kb}{2}\sin\theta\right)\sin\phi$$

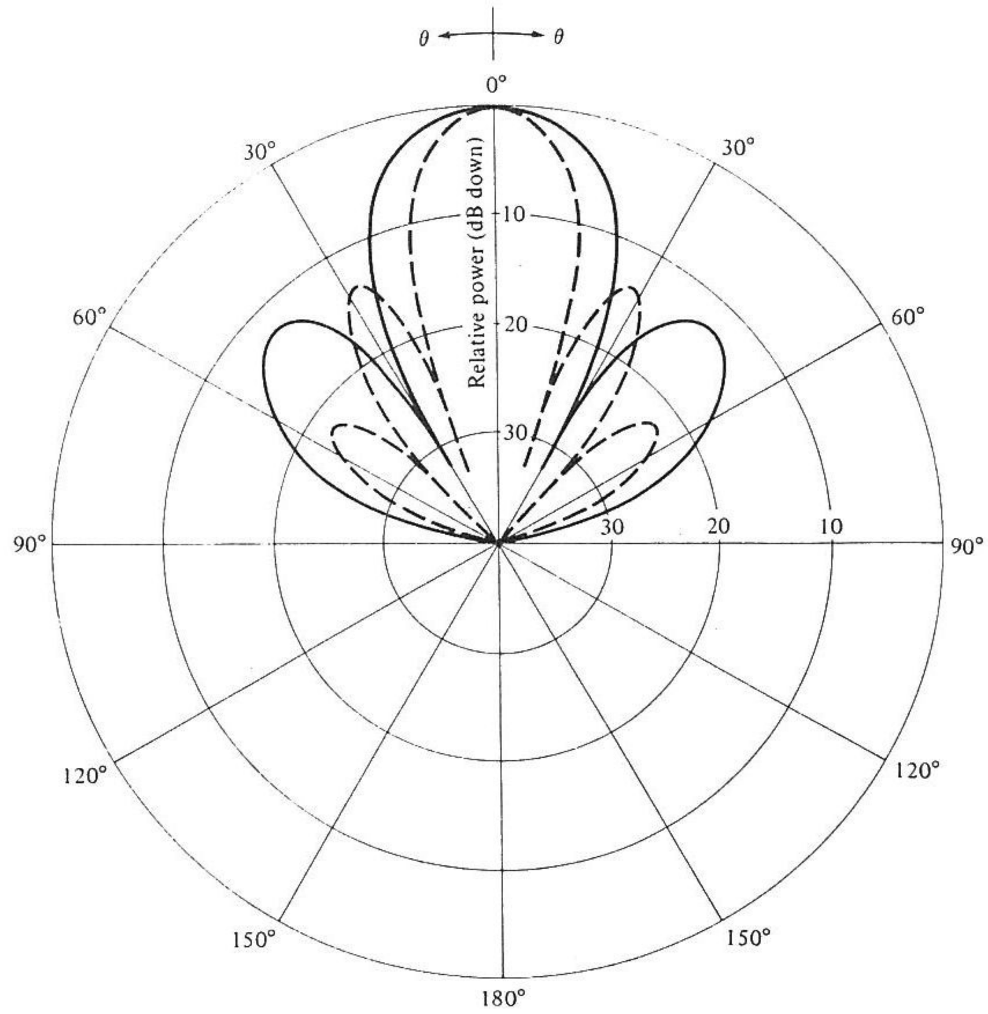
$$H_\phi = \frac{E_\theta}{\eta}; E_\phi = H_\theta = 0$$

H-Plane
($\phi=0, \pi$)
(x-z plane)

$$E_\phi = \frac{jk}{4\pi}\frac{e^{-jkr}}{r}2E_0 ab \operatorname{sinc}\left(\frac{ka}{2}\sin\theta\right)\cos\theta\cos\phi$$

$$H_\theta = -\frac{E_\phi}{\eta}; E_\theta = H_\phi = 0$$

Radiation Pattern (4)



E-plane and H-plane amplitude pattern for uniform distribution aperture mounted on an infinite ground plane ($a=3\lambda$, $b=2\lambda$)

Beamwidth

For the E-plane, maximum at $\theta=0$.

Nulls occur when $$\frac{kb}{2}\sin\theta_n = n\pi \quad n=1,2,\cdots$$

thus $$\theta_n = \sin^{-1}\frac{n\lambda}{b}$$

Beamwidth between nulls $$\Theta_N = 2\sin^{-1}\frac{n\lambda}{b},\ n=1,2,\ldots$$

FNBW: $$\Theta_{FNBW} = 2\sin^{-1}\frac{\lambda}{b}$$

Half-power angles:

$$\mathrm{sinc}^2(x) = 1/2 \Rightarrow x = 1.391 \quad \Longrightarrow \quad \theta_h = \sin^{-1}\frac{0.443\lambda}{b}$$

HPBW: $$\Theta_{HPBW} = 2\sin^{-1}\frac{0.443\lambda}{b}$$

Directivity

The radiated power can be obtained from the power into the aperture. For uniform distribution, the Poynting vector is given by

$$\mathbf{W}_{av} = \frac{1}{2}\mathrm{Re}(\mathbf{E}\times\mathbf{H}^*) = \hat{z}\frac{|E_0|^2}{2\eta} \quad -a/2 \le x' \le a/2; -b/2 \le y' \le b/2$$

thus $$P_{rad} = \int_S \mathbf{W}_{av}\cdot d\mathbf{s} = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\frac{|E_0|^2}{2\eta}dy'\,dx' = \frac{|E_0|^2}{2\eta}ab$$

Recall that $$D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{rad}}$$ and

$$U(\theta,\phi) = \frac{r^2}{2\eta}\left(|E_\theta|^2 + |E_\phi|^2\right)$$

Directivity (2)

The maximum radiation intensity is given by

$$U_{\max} = U(\theta,\phi)\big|_{\theta=0} = \frac{(ab)^2}{2\eta}\frac{k^2|E_0|^2}{4\pi^2} = \left(\frac{ab}{\lambda}\right)^2 \frac{|E_0|^2}{2\eta}$$

thus

$$D_{\max} = \frac{4\pi U_{\max}}{P_{rad}} = 4\pi\left(\frac{ab}{\lambda}\right)^2 \frac{1}{ab} = \frac{4\pi}{\lambda^2}ab = \frac{4\pi}{\lambda^2}A_p$$

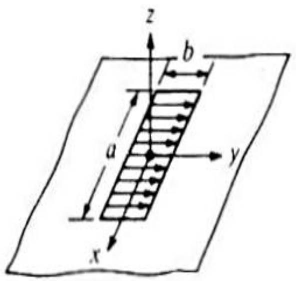
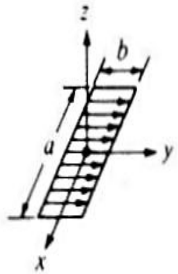
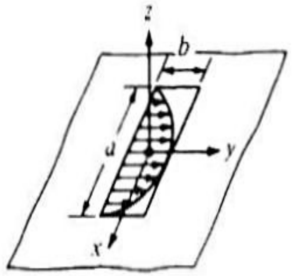
where A_p is the physical aperture (=*ab*).

Recall that the maximum effective aperture is given by

$$A_{em} = \frac{4\pi}{\lambda^2}D_{\max} = A_p$$

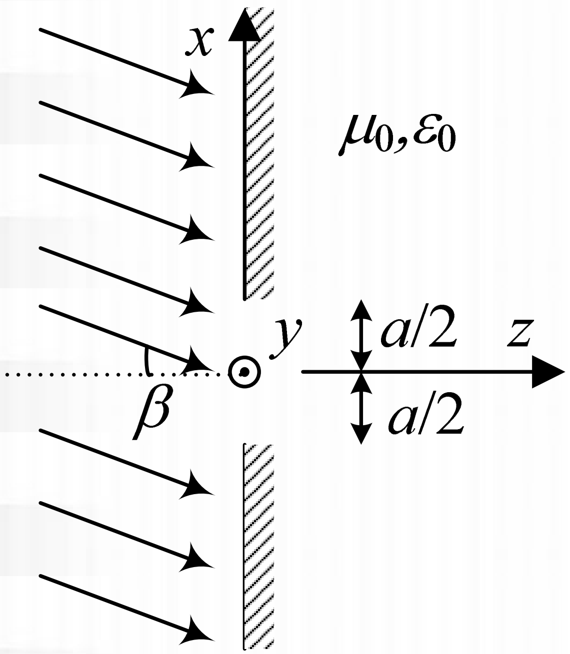
The aperture efficiency $\varepsilon_{ap} = \dfrac{A_{em}}{A_p} = 1$

TABLE 12.1 Equivalents, Fields, Beamwidths, Side Lobe Levels, and Directivities of Rectangular Apertures

	Uniform Distribution Aperture on Ground Plane	**Uniform Distribution Aperture in Free-Space**	**TE_{10}-Mode Distribution Aperture on Ground Plane**
Aperture distribution of tangential components (analytical)	$\mathbf{E}_a = \hat{\mathbf{a}}_y E_0 \left\} \begin{matrix} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{matrix} \right.$	$\left.\begin{matrix} \mathbf{E}_a = \hat{\mathbf{a}}_y E_0 \\ \mathbf{H}_a = -\hat{\mathbf{a}}_x \dfrac{E_0}{\eta} \end{matrix} \right\} \begin{matrix} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{matrix}$	$\mathbf{E}_a = \hat{\mathbf{a}}_y E_0 \cos\left(\dfrac{\pi}{a} x'\right) \left\} \begin{matrix} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{matrix} \right.$
Aperture distribution of tangential components (graphical)			
Equivalent	$\mathbf{M}_s = \begin{cases} -2\hat{\mathbf{n}} \times \mathbf{E}_a & \begin{matrix} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{matrix} \\ 0 & \text{elsewhere} \end{cases}$ $\mathbf{J}_s = 0$ everywhere	$\left.\begin{matrix} \mathbf{M}_s = -\hat{\mathbf{n}} \times \mathbf{E}_a \\ \mathbf{J}_s = \hat{\mathbf{n}} \times \mathbf{H}_a \end{matrix} \right\} \begin{matrix} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{matrix}$ $\mathbf{M}_s \simeq \mathbf{J}_s \simeq 0$ elsewhere	$\mathbf{M}_s = \begin{cases} -2\hat{\mathbf{n}} \times \mathbf{E}_a & \begin{matrix} -a/2 \le x' \le a/2 \\ -b/2 \le y' \le b/2 \end{matrix} \\ 0 & \text{elsewhere} \end{cases}$ $\mathbf{J}_s = 0$ everywhere
Far-zone fields $X = \dfrac{ka}{2} \sin\theta \cos\phi$ $Y = \dfrac{kb}{2} \sin\theta \sin\phi$ $C = j \dfrac{abkE_0 e^{-jkr}}{2\pi r}$	$E_r = H_r = 0$ $E_\theta = C \sin\phi \dfrac{\sin X}{X} \dfrac{\sin Y}{Y}$ $E_\phi = C \cos\theta \cos\phi \dfrac{\sin X}{X} \dfrac{\sin Y}{Y}$ $H_\theta = -E_\phi/\eta$ $H_\phi = E_\theta/\eta$	$E_r = H_r = 0$ $E_\theta = \dfrac{C}{2} \sin\phi(1+\cos\theta) \dfrac{\sin X}{X} \dfrac{\sin Y}{Y}$ $E_\phi = \dfrac{C}{2} \cos\phi(1+\cos\theta) \dfrac{\sin X}{X} \dfrac{\sin Y}{Y}$ $H_\theta = -E_\phi/\eta$ $H_\phi = E_\theta/\eta$	$E_r = H_r = 0$ $E_\theta = -\dfrac{\pi}{2} C \sin\phi \dfrac{\cos X}{(X)^2 - \left(\dfrac{\pi}{2}\right)^2} \dfrac{\sin Y}{Y}$ $E_\phi = -\dfrac{\pi}{2} C \cos\theta \cos\phi \dfrac{\cos X}{(X)^2 - \left(\dfrac{\pi}{2}\right)^2} \dfrac{\sin Y}{Y}$ $H_\theta = -E_\phi/\eta$ $H_\phi = E_\theta/\eta$

Half-power beamwidth (degrees)	E-plane $b \gg \lambda$	$\frac{50.6}{b/\lambda}$	$\frac{50.6}{b/\lambda}$	$\frac{50.6}{b/\lambda}$
	H-plane $a \gg \lambda$	$\frac{50.6}{a/\lambda}$	$\frac{50.6}{a/\lambda}$	$\frac{68.8}{a/\lambda}$
First null beamwidth (degrees)	E-plane $b \gg \lambda$	$\frac{114.6}{b/\lambda}$	$\frac{114.6}{b/\lambda}$	$\frac{114.6}{b/\lambda}$
	H-plane $a \gg \lambda$	$\frac{114.6}{a/\lambda}$	$\frac{114.6}{a/\lambda}$	$\frac{171.9}{a/\lambda}$
First side lobe max. (to main max.) (dB)	E-plane	-13.26	-13.26	-13.26
	H-plane	-13.26 $a \gg \lambda$	-13.26 $a \gg \lambda$	-23 $a \gg \lambda$
Directivity D_0 (dimensionless)		$\frac{4\pi}{\lambda^2}(\text{area}) = 4\pi\left(\frac{ab}{\lambda^2}\right)$	$\frac{4\pi}{\lambda^2}(\text{area}) = 4\pi\left(\frac{ab}{\lambda^2}\right)$	$\frac{8}{\pi^2}\left[4\pi\left(\frac{ab}{\lambda^2}\right)\right] = 0.81\left[4\pi\left(\frac{ab}{\lambda^2}\right)\right]$

Quiz



A uniform plane wave traveling in the direction as shown. The field is given by

$$\text{TE case}: \mathbf{E}^i = \hat{y} E_0 e^{-j\bar{k}\cdot\bar{r}} \quad z \le 0$$

$$\text{TM case}: \mathbf{H}^i = \hat{y} H_0 e^{-j\bar{k}\cdot\bar{r}} \quad z \le 0$$

(a) Find the propagation direction.

(b) Find the aperture field.

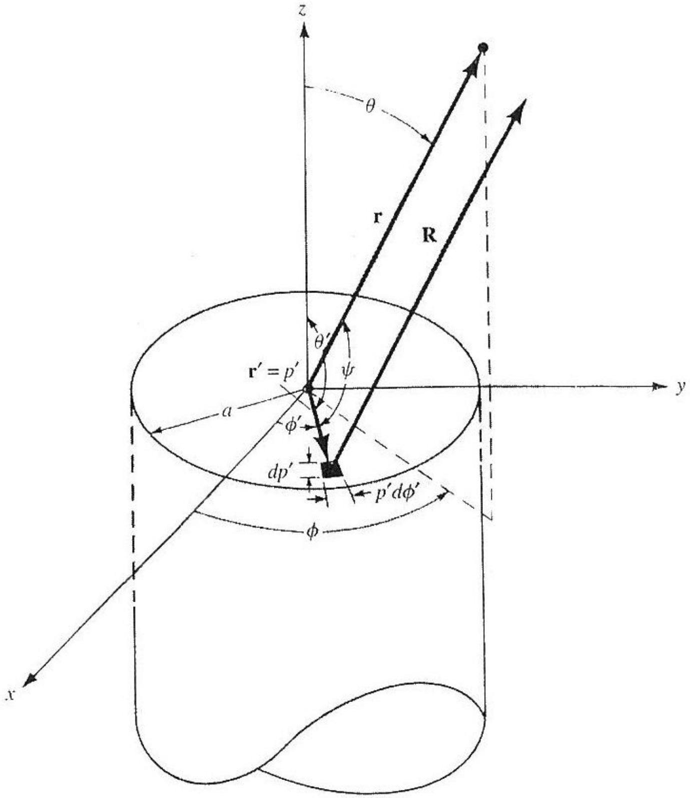
(c) Find the equivalent current.

Assume that the aperture dimension in the *y* direction is *b*.

Circular Apertures

Circular aperture with uniform distribution on an infinite ground plane: find far-zone fields for $z > 0$.

Aperture field: $\mathbf{E}_a = \hat{y}E_0 \quad \rho' \leq a$



Equivalent problem for $z > 0$

$$\mathbf{M}_s = -2\hat{n} \times \mathbf{E}_a$$
$$= -2\hat{z} \times \hat{y}E_0$$
$$= \hat{x}2E_0 \quad \rho' \leq a$$

$\mathbf{M}_s = \mathbf{J}_s = \mathbf{0}$ outside aperture

Circular Apertures (2)

The far-zone electric field can then be given by

$$\mathbf{E}(\vec{r}) \approx -\frac{jk}{4\pi}\frac{e^{-jkr}}{r}\int_0^a\int_0^{2\pi}\mathbf{M}_s(\vec{r}\,')\times\hat{r}e^{jk\hat{r}\cdot\vec{r}\,'}\rho' d\phi' d\rho'$$

Since $\quad \mathbf{M}_s \times \hat{r} = \hat{x}2E_0 \times \hat{r}$

$$= 2E_0(-\hat{\phi}\cos\theta\cos\phi - \hat{\theta}\sin\phi)$$

and $\quad \hat{r}\cdot\vec{r}\,' = (x'\cos\phi + y'\sin\phi)\sin\theta \,(z'=0)$

$$= \rho'\sin\theta\cos(\phi-\phi')$$

$$\mathbf{E}(\vec{r}) \approx \frac{jk}{4\pi}\frac{e^{-jkr}}{r}2E_0(\hat{\phi}\cos\theta\cos\phi + \hat{\theta}\sin\phi)\int_0^a\int_0^{2\pi}e^{jk\hat{r}\cdot\vec{r}\,'}\rho' d\phi' d\rho'$$

Circular Apertures (3)

Using

$$\int_0^{2\pi} e^{jk\rho'\sin\theta\cos(\phi-\phi')}d\phi' = 2\pi J_0(k\rho'\sin\theta)$$

and

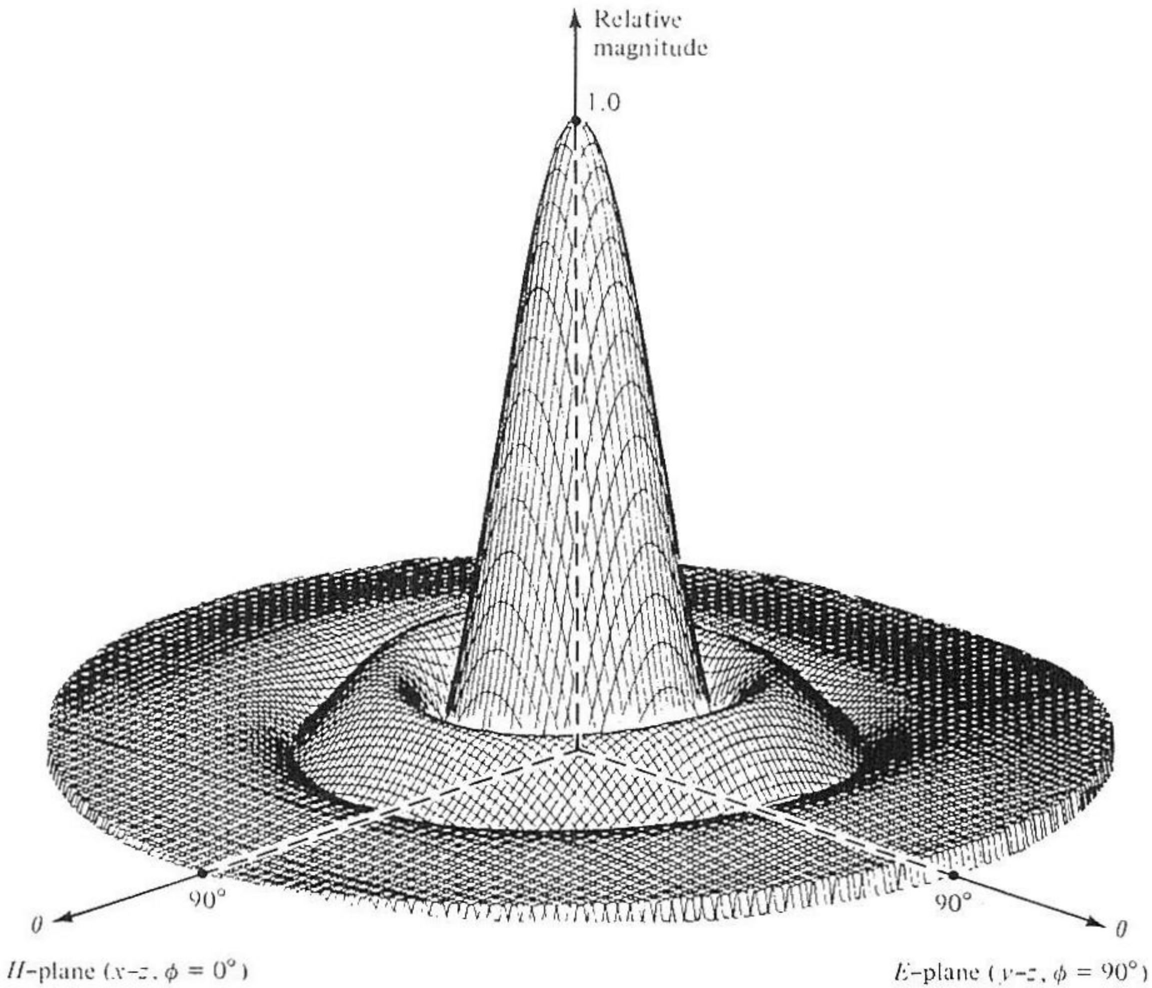
$$\int_0^{\beta} zJ_0(z)dz = zJ_1(z)\Big|_0^{\beta} = \beta J_1(\beta)$$

the far-zone fields can be found to be

$$\mathbf{E}(\vec{r}) = \frac{jka^2E_0e^{-jkr}}{r}\frac{J_1(ka\sin\theta)}{ka\sin\theta}(\hat{\phi}\cos\theta\cos\phi + \hat{\theta}\sin\phi)$$

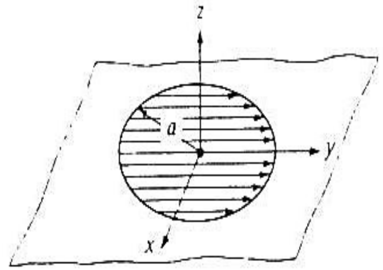
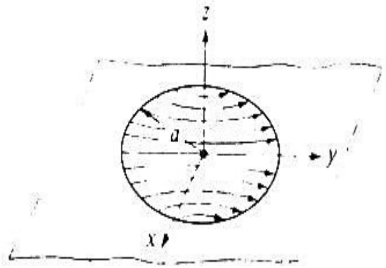
$$\mathbf{H}(\vec{r}) = \frac{jka^2E_0e^{-jkr}}{\eta r}\frac{J_1(ka\sin\theta)}{ka\sin\theta}(-\hat{\theta}\cos\theta\cos\phi + \hat{\phi}\sin\phi)$$

Radiation Pattern



Normalized field pattern: $a = 1.5\lambda$

TABLE 12.2 Equivalents, Fields, Beamwidths, Side Lobe Levels, and Directivities of Circular Apertures

	Uniform Distribution Aperture on Ground Plane	TE_{11}-Mode Distribution Aperture on Ground Plane
Aperture distribution of tangential components (analytical)	$\mathbf{E}_a = \hat{\mathbf{a}}_y E_0 \quad \rho' \le a$	$\left.\begin{aligned} \mathbf{E}_a &= \hat{\mathbf{a}}_\rho E_\rho + \hat{\mathbf{a}}_\phi E_\phi \\ E_\rho &= E_0 J_1(\chi'_{11}\rho'/a)\sin\phi'/\rho' \\ E_\phi &= E_0 J_1{}'(\chi'_{11}\rho'/a)\cos\phi' \end{aligned}\right\}$ $\rho' \le a$, $\chi'_{11} = 1.841$, $' = \dfrac{\partial}{\partial \rho'}$
Aperture distribution of tangential components (graphical)		
Equivalent	$\mathbf{M}_s = \begin{cases} -2\hat{\mathbf{n}} \times \mathbf{E}_a & \rho' \le a \\ 0 & \text{elsewhere} \end{cases}$ $\mathbf{J}_s = 0 \quad \text{everywhere}$	$\mathbf{M}_s = \begin{cases} -2\hat{\mathbf{n}} \times \mathbf{E}_a & \rho' \le a \\ 0 & \text{elsewhere} \end{cases}$ $\mathbf{J}_s = 0 \quad \text{everywhere}$
Far-zone fields $Z = ka\sin\theta$ $C_1 = j\dfrac{ka^2 E_0 e^{-jkr}}{r}$ $C_2 = j\dfrac{kaE_0 J_1(\chi'_{11})e^{-jkr}}{r}$ $\chi'_{11} = 1.841$	$E_r = H_r = 0$ $E_\theta = jC_1 \sin\phi \dfrac{J_1(Z)}{Z}$ $E_\phi = jC_1\cos\theta\cos\phi\dfrac{J_1(Z)}{Z}$ $H_\theta = -E_\phi/\eta$ $H_\phi = E_\theta/\eta$	$E_r = H_r = 0$ $E_\theta = C_2\sin\phi\dfrac{J_1(Z)}{Z}$ $E_\phi = C_2\cos\theta\cos\phi\dfrac{J_1{}'(Z)}{1-(Z/\chi'_{11})^2}$ $H_\theta = -E_\phi/\eta$ $H_\phi = E_\theta/\eta$ $J_1{}'(Z) = J_0(Z) - J_1(Z)/Z$