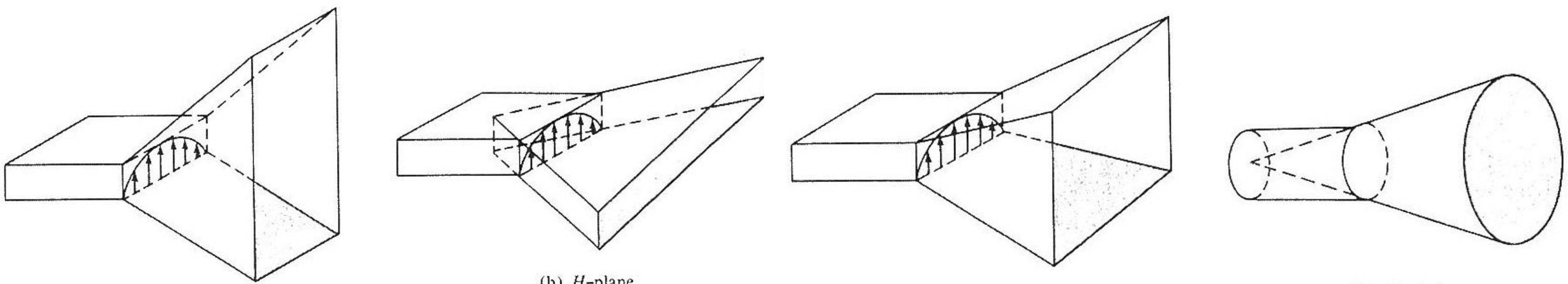
Chapter 13 : Horn Antenna

- E-plane sectoral horn (E-plane horn)
- H-plane sectoral horn (H-plane horn)
- Pyramidal horn
- Corrugated horn


(a) *E*-plane (b) *H*-plane (c) Pyramidal (d) Conical

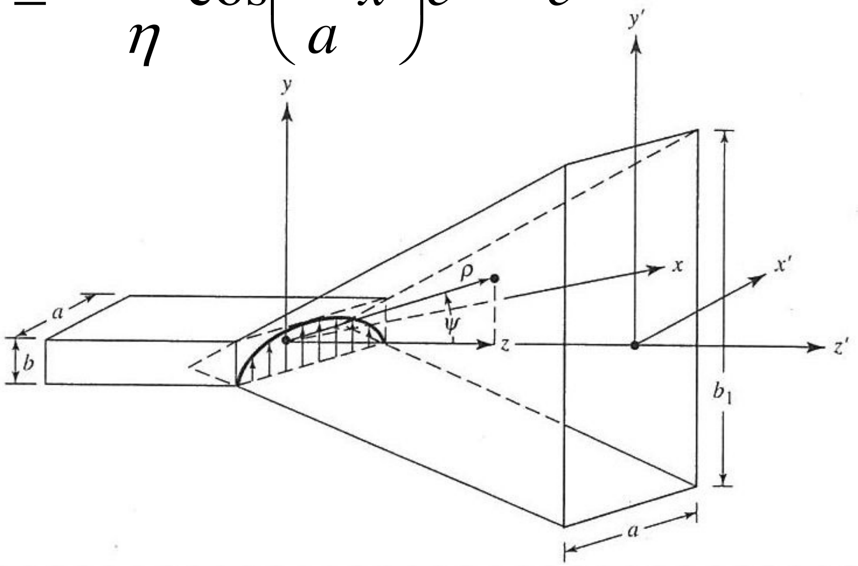
E-plane horn

The aperture fields are given by (for $-a/2 \le x' \le a/2; -b_1/2 \le y' \le b_1/2$)

$$E_y'(x', y') \cong E_1 \cos\left(\frac{\pi}{a} x'\right) e^{-jk\rho_1} e^{-jky'^2/2\rho_1}; E_z' = E_y' = H_y' = 0;$$

$$H_z'(x', y') \cong jE_1 \frac{\pi}{ka\eta} \sin\left(\frac{\pi}{a} x'\right) e^{-jk\rho_1} e^{-jky'^2/2\rho_1};$$

$$H_x'(x', y') \cong -\frac{E_1}{\eta} \cos\left(\frac{\pi}{a} x'\right) e^{-jk\rho_1} e^{-jky'^2/2\rho_1}$$

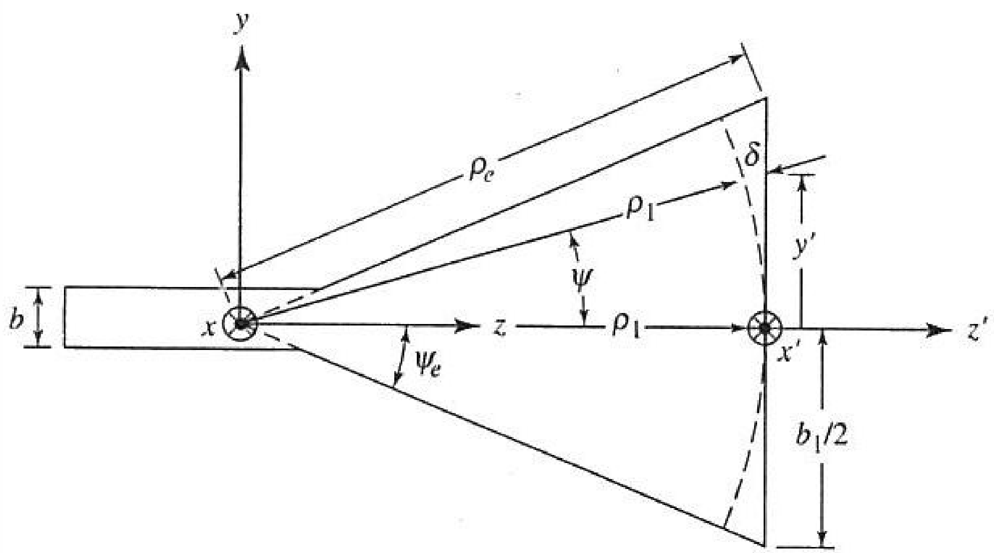


E-plane horn (2)

where $\rho_1 = \rho_e \cos\psi_e ; 2\psi_e$: flare angle

$$[\rho_1 + \delta(y')]^2 = \rho_1^2 + y'^2 \text{ or } \delta(y') = -\rho_1 + [\rho_1^2 + y'^2]^{1/2}$$

Binomial approximation: $\delta(y') = -\rho_1 + \rho_1\left[1 + \frac{1}{2}\left(\frac{y'}{\rho_1}\right)^2\right] = \frac{1}{2}\frac{y'^2}{\rho_1}$



Design example

- **Design an E-plane horn so that the maximum phase deviation at the aperture of the horn is 56.72°. The dimensions of the horn are a=0.5λ, b_1=2.75λ.**

Since
$$\Delta\phi\Big|_{max} = k\delta(y')\Big|_{y'=b_1/2} = \frac{k(b_1/2)^2}{2\rho_1} = 56.72\left(\frac{\pi}{180}\right)$$

or
$$\rho_1 = \left(\frac{2.75}{2}\right)^2 \frac{180}{56.72}\lambda = 6\lambda$$

Thus
$$2\psi_e = 2\tan^{-1}\left(\frac{b_1/2}{\rho_1}\right) = 2\tan^{-1}\left(\frac{2.75/2}{6}\right) = 25.81°$$

Far-zone fields

Assume that closed surface is an infinite plane that passes through the aperture of the horn, i.e., $z'=\rho_1$ plane. On this plane the fields are assumed to be zero everywhere except at the aperture. Thus,

$$\mathbf{J}_s = \hat{n}\times\mathbf{H}_a = \hat{z}\times\mathbf{H}_a ; \mathbf{M}_s = -\hat{z}\times\mathbf{E}_a$$

Hence

$$\left.\begin{aligned} J_y &= -\frac{E_1}{\eta}\cos\left(\frac{\pi}{a}x'\right)e^{-jk\rho_1}e^{-jk\delta(y')} \\ M_x &= E_1\cos\left(\frac{\pi}{a}x'\right)e^{-jk\rho_1}e^{-jk\delta(y')} \end{aligned}\right\} \begin{aligned} -a/2 \le x' \le a/2 \\ -b_1/2 \le y' \le b_1/2 \end{aligned}$$

and

$$\mathbf{J}_s = \mathbf{M}_s = \mathbf{0} \quad \text{elsewhere}$$

Far-zone fields (2)

Recall that the far-zone electric field due to electric and magnetic surface currents are given by:

$$\mathbf{E}(\vec{r}) \approx -\frac{jk}{4\pi}\frac{e^{-jkr}}{r}\iint_S [\mathbf{M}_s(\vec{r}\,')\times\hat{r} - \hat{r}\times\hat{r}\times\eta\mathbf{J}_s(\vec{r}\,')]e^{jk\hat{r}\cdot\vec{r}\,'}ds'$$

Thus

$$\mathbf{E}(\vec{r}) \approx -\frac{jk}{4\pi}\frac{e^{-jkr}}{r}\iint_S [\hat{x}M_x\times\hat{r} - \hat{r}\times\hat{r}\times\hat{y}\eta J_y]e^{jk\hat{r}\cdot\vec{r}\,'}ds'$$

Using

$$\hat{r}\cdot\vec{r}\,' = r'\cos\varsigma = (x'\cos\phi + y'\sin\phi)\sin\theta + \rho_1\cos\theta$$

$$\hat{x}\times\hat{r} = -\hat{\phi}\cos\theta\cos\phi - \hat{\theta}\sin\phi$$

$$\hat{r}\times\hat{r}\times\hat{y} = -\hat{\theta}\cos\theta\sin\phi - \hat{\phi}\cos\phi$$

Far-zone fields (3)

The far-zone electric field due to the horn antenna can be written as:

$$\mathbf{E}(\vec{r}) \approx -\frac{jk}{4\pi}\frac{e^{-jkr}}{r}\{\mathbf{L}+\mathbf{N}\}$$

where

$$\mathbf{L} = (-\hat{\phi}\cos\theta\cos\phi - \hat{\theta}\sin\phi)e^{jk\rho_1\cos\theta} \times$$

$$\int_{-a/2}^{a/2}\int_{-b_1/2}^{b_1/2} M_x(x',y')e^{jk\sin\theta(x'\cos\phi + y'\sin\phi)}dy'\,dx'$$

and

$$\mathbf{N} = (\hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi)e^{jk\rho_1\cos\theta} \times$$

$$\eta\int_{-a/2}^{a/2}\int_{-b_1/2}^{b_1/2} J_y(x',y')e^{jk\sin\theta(x'\cos\phi + y'\sin\phi)}dy'\,dx'$$

Far-zone fields (4)

Let

$$I_1 = \int_{-a/2}^{a/2} \cos\left(\frac{\pi}{a}x'\right) e^{jkx'\sin\theta\cos\phi} dx'$$

$$I_2 = \int_{-b_1/2}^{b_1/2} e^{-jk\delta(y')} e^{jky'\sin\theta\sin\phi} dy'$$

Then **N** and **L** can be written as:

$$\mathbf{N} = -(\hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi)e^{jk\rho_1(\cos\theta-1)} E_1 I_1 I_2$$

and

$$\mathbf{L} = -(\hat{\phi}\cos\theta\cos\phi + \hat{\theta}\sin\phi)e^{jk\rho_1(\cos\theta-1)} E_1 I_1 I_2$$

Far-zone fields (5)

It can be shown that

$$I_1 = -\frac{\pi a}{2}\left[\frac{\cos\left(\frac{ka}{2}\sin\theta\cos\phi\right)}{\left(\frac{ka}{2}\sin\theta\cos\phi\right)^2-\left(\frac{\pi}{2}\right)^2}\right]$$

$$I_2 = \int_{-b_1/2}^{b_1/2} e^{-jky'^2/2\rho_1} e^{jky'\sin\theta\sin\phi}\,dy'$$

$$\overset{k_y = k\sin\theta\sin\phi}{=} \int_{-b_1/2}^{b_1/2} e^{-j(ky'^2/2\rho_1 - k_y y')}\,dy'$$

$$= e^{jk_y^2\rho_1/2k}\int_{-b_1/2}^{b_1/2} e^{-j(ky'-k_y\rho_1)^2/2k\rho_1}\,dy'$$

Far-zone fields (6)

Using the following change of variable

$$\sqrt{\frac{\pi}{2}}t = \sqrt{\frac{1}{2k\rho_1}}(ky' - k_y\rho_1); dt = \sqrt{\frac{k}{\pi\rho_1}}dy'$$

yields

$$I_2 = \sqrt{\frac{\pi\rho_1}{k}} e^{jk_y^2\rho_1/2k} \int_{t_1}^{t_2} e^{-j(\pi/2)t^2} dt$$

$$= \sqrt{\frac{\pi\rho_1}{k}} e^{jk_y^2\rho_1/2k} \int_{t_1}^{t_2} \left[\cos\left(\frac{\pi}{2}t^2\right) - j\sin\left(\frac{\pi}{2}t^2\right)\right] dt$$

$$= \sqrt{\frac{\pi\rho_1}{k}} e^{jk_y^2\rho_1/2k} \{[C(t_2) - C(t_1)] - j[S(t_2) - S(t_1)]\}$$

Far-zone fields (7)

where

$$t_1 = \sqrt{\frac{1}{\pi k \rho_1}}\left(-\frac{kb_1}{2} - k_y \rho_1\right); t_2 = \sqrt{\frac{1}{\pi k \rho_1}}\left(\frac{kb_1}{2} - k_y \rho_1\right)$$

$$\underbrace{C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt}_{\text{cosine-Fresnel integral}}; \underbrace{S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)dt}_{\text{sine-Fresnel integral}}$$

Thus,

$$\mathbf{N} = (\hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi)e^{jk\rho_1(\cos\theta - 1)}E_1 \times$$

$$\frac{\pi a}{2}\sqrt{\frac{\pi\rho_1}{k}}e^{jk_y^2\rho_1/2k}\left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2 - \left(\frac{\pi}{2}\right)^2}\right]F(t_1, t_2)$$

Far-zone fields (8)

and

$$\mathbf{L} = (\hat{\phi}\cos\theta\cos\phi + \hat{\theta}\sin\phi)e^{jk\rho_1(\cos\theta-1)}E_1 \times$$

$$\frac{\pi a}{2}\sqrt{\frac{\pi\rho_1}{k}}e^{jk_y^2\rho_1/2k}\left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2-\left(\frac{\pi}{2}\right)^2}\right]F(t_1,t_2)$$

where

$$F(t_1,t_2) = [C(t_2)-C(t_1)] - j[S(t_2)-S(t_1)]$$

$$k_x = k\sin\theta\cos\phi;\ k_y = k\sin\theta\sin\phi$$

Far-zone fields (9)

Therefore

$$E_\theta = -j\sqrt{\pi\rho_1 k}\, a \frac{e^{-jkr}}{8r} E_1 e^{jk\rho_1(\cos\theta-1)} \sin\phi(1+\cos\theta)\times$$

$$e^{jk_y^2\rho_1/2k}\left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2-\left(\frac{\pi}{2}\right)^2}\right]F(t_1,t_2)$$

$$E_\phi = -j\sqrt{\pi\rho_1 k}\, a \frac{e^{-jkr}}{8r} E_1 e^{jk\rho_1(\cos\theta-1)} \cos\phi(1+\cos\theta)\times$$

$$e^{jk_y^2\rho_1/2k}\left[\frac{\cos\left(\frac{k_x a}{2}\right)}{\left(\frac{k_x a}{2}\right)^2-\left(\frac{\pi}{2}\right)^2}\right]F(t_1,t_2)$$

E-plane and H-plane

E-Plane
($\phi = \pi/2, 3\pi/2$)
(y-z plane)

$$E_\theta \neq 0, H_\phi = \frac{E_\theta}{\eta};$$

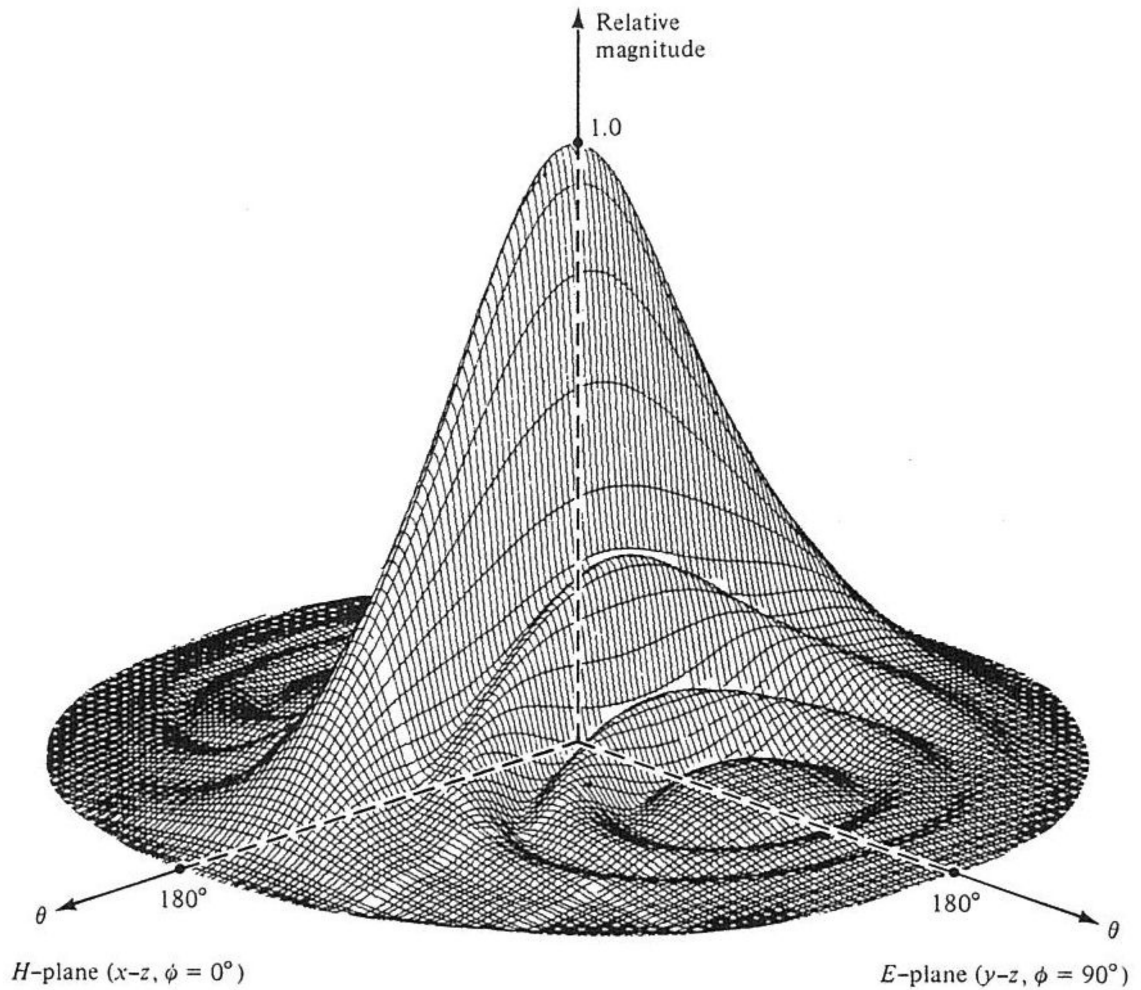
$$E_\phi = H_\theta = 0$$

H-Plane
($\phi = 0, \pi$)
(x-z plane)

$$E_\phi \neq 0; H_\theta = -\frac{E_\phi}{\eta};$$

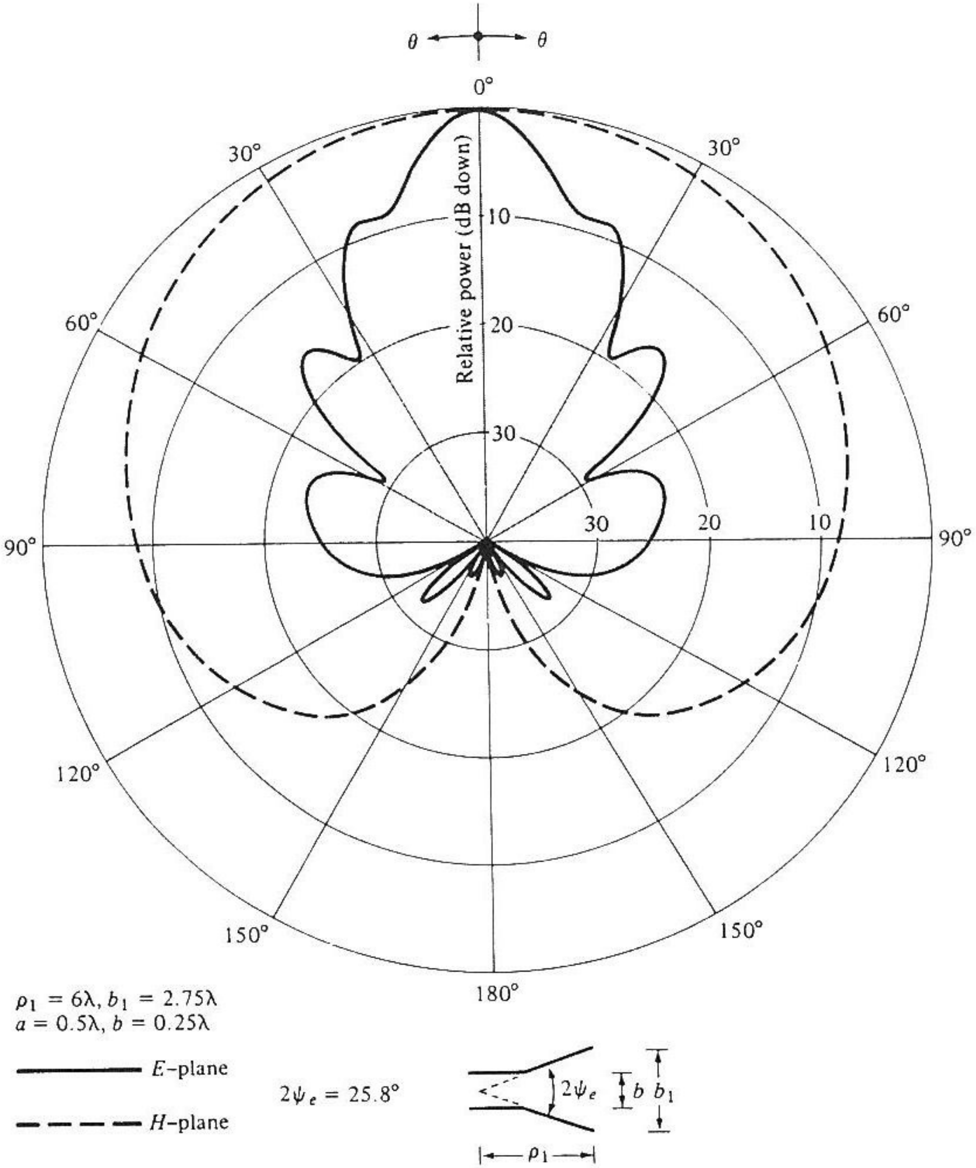
$$E_\theta = H_\phi = 0$$

Radiation Pattern

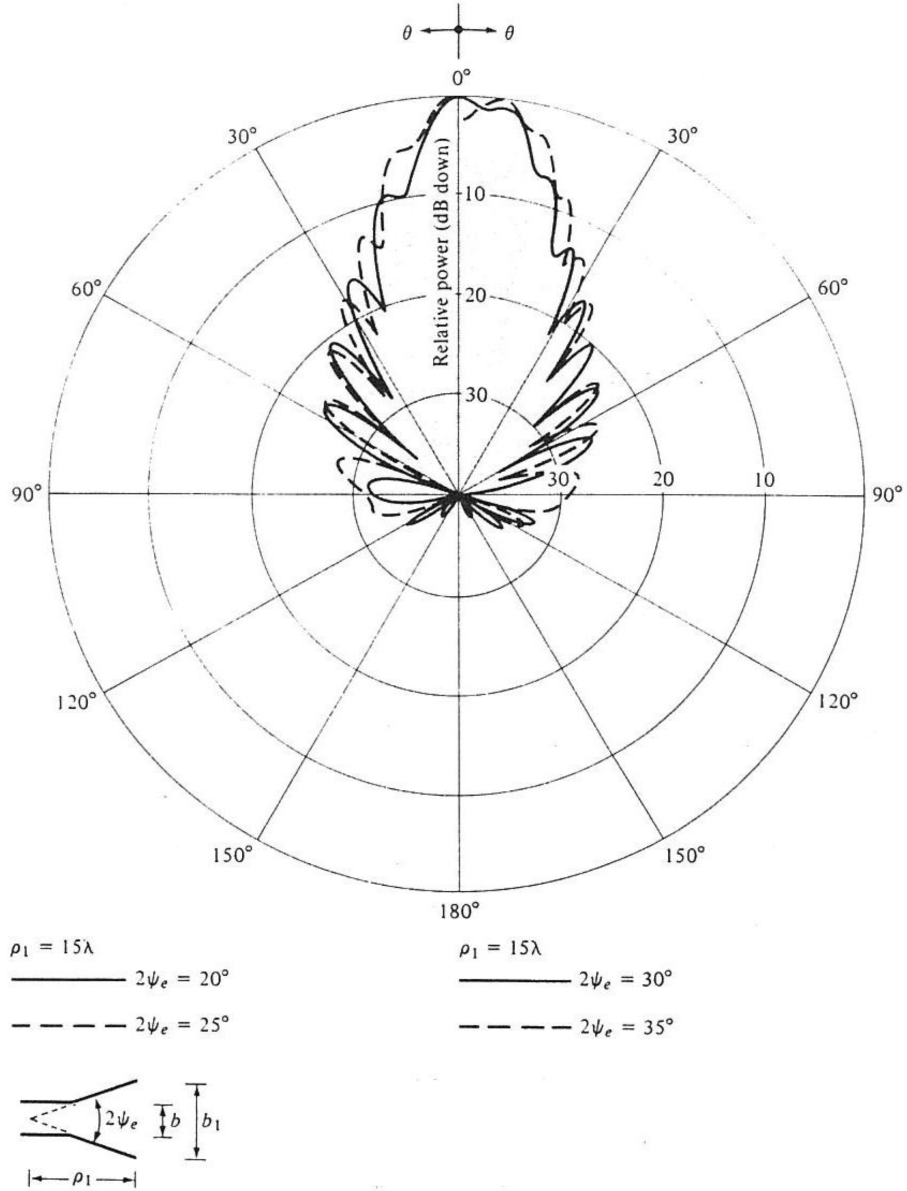


Normalized field pattern: $\rho_1 = 6\lambda$, $a = 0.5\lambda$, $b_1 = 2.75\lambda$

Radiation Pattern (2)



Radiation Pattern (3)



Directivity

The radiated power can be obtained from the power into the aperture. For uniform distribution, the Poynting vector is given by

$$\mathbf{W}_{av} = \frac{1}{2}\mathrm{Re}(\mathbf{E}\times\mathbf{H}^*) = \hat{z}\frac{|E_1|^2}{2\eta}\cos^2\left(\frac{\pi}{2}x'\right)$$

$$\text{for } -a/2 \le x' \le a/2; -b_1/2 \le y' \le b_1/2$$

thus

$$P_{rad} = \int_{-a/2}^{a/2}\int_{-b_1/2}^{b_1/2}\frac{|E_1|^2}{2\eta}\cos^2\left(\frac{\pi}{2}x'\right)dy'\,dx' = \frac{|E_1|^2}{4\eta}ab_1$$

Recall that $$D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_{rad}} \text{ and}$$

$$U(\theta,\phi) = \frac{r^2}{2\eta}\left(\left|E_\theta\right|^2 + \left|E_\phi\right|^2\right)$$

Directivity (2)

The maximum radiation intensity is given by

$$U_{\max} = U(\theta,\phi)\Big|_{\theta=0} = \frac{r^2}{2\eta}\left|\mathbf{E}\right|^2_{\max}$$

Since $$\left|\mathbf{E}\right|^2_{\max} = \left|E_\theta\right|^2_{\max} + \left|E_\phi\right|^2_{\max} = \frac{4a^2 k\rho_1}{\pi^3 r^2}\left|E_1\right|^2\left|F(t)\right|^2$$

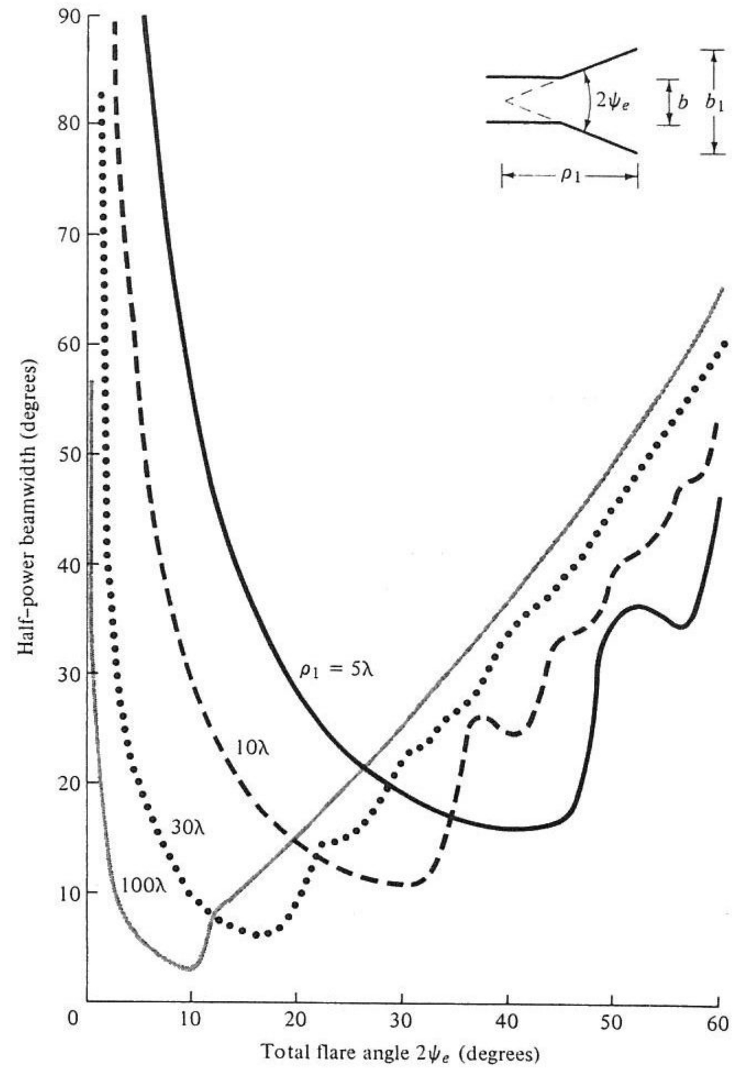
where $$F(t) = C(t) - jS(t); t = \frac{b_1}{\sqrt{2\lambda\rho_1}}$$

Therefore, the maximum directivity is given by

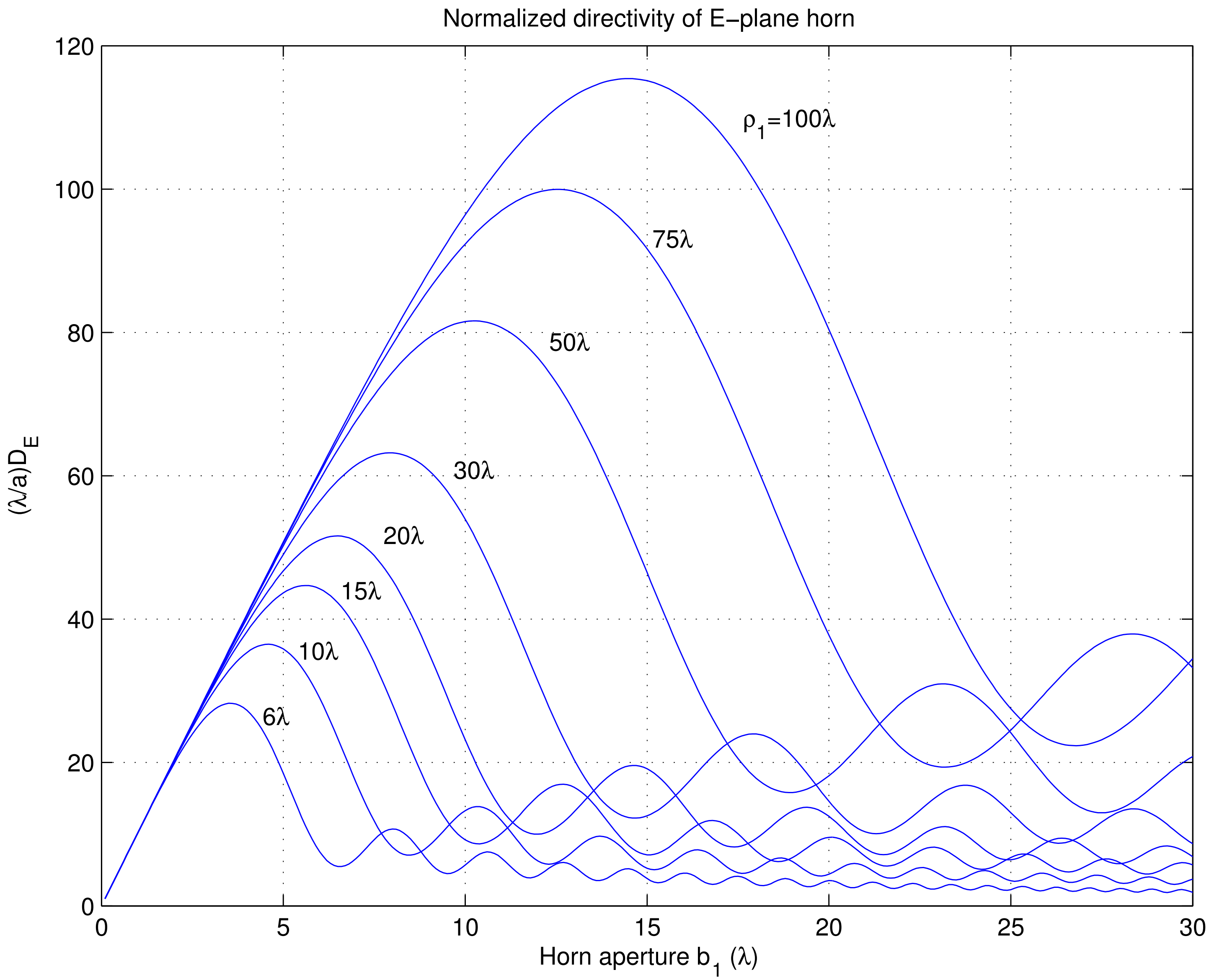
$$D_{\max} = \frac{4\pi U_{\max}}{P_{rad}} = \frac{64a\rho_1}{\pi\lambda b_1}\left|F(t)\right|^2$$

$$= \frac{64a\rho_1}{\pi\lambda b_1}\left[C^2(t) + S^2(t)\right]$$

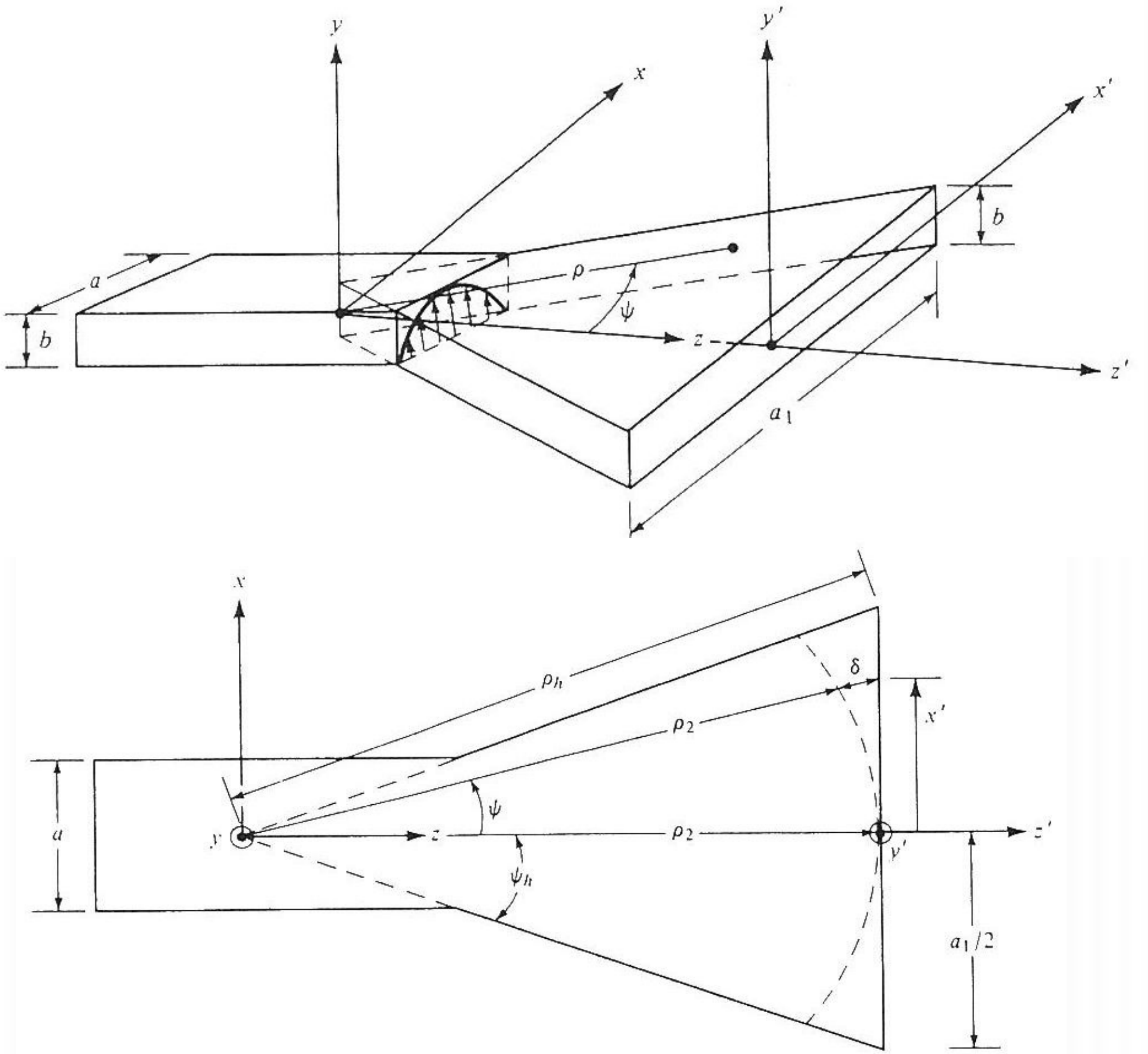
Half-power beamwidth



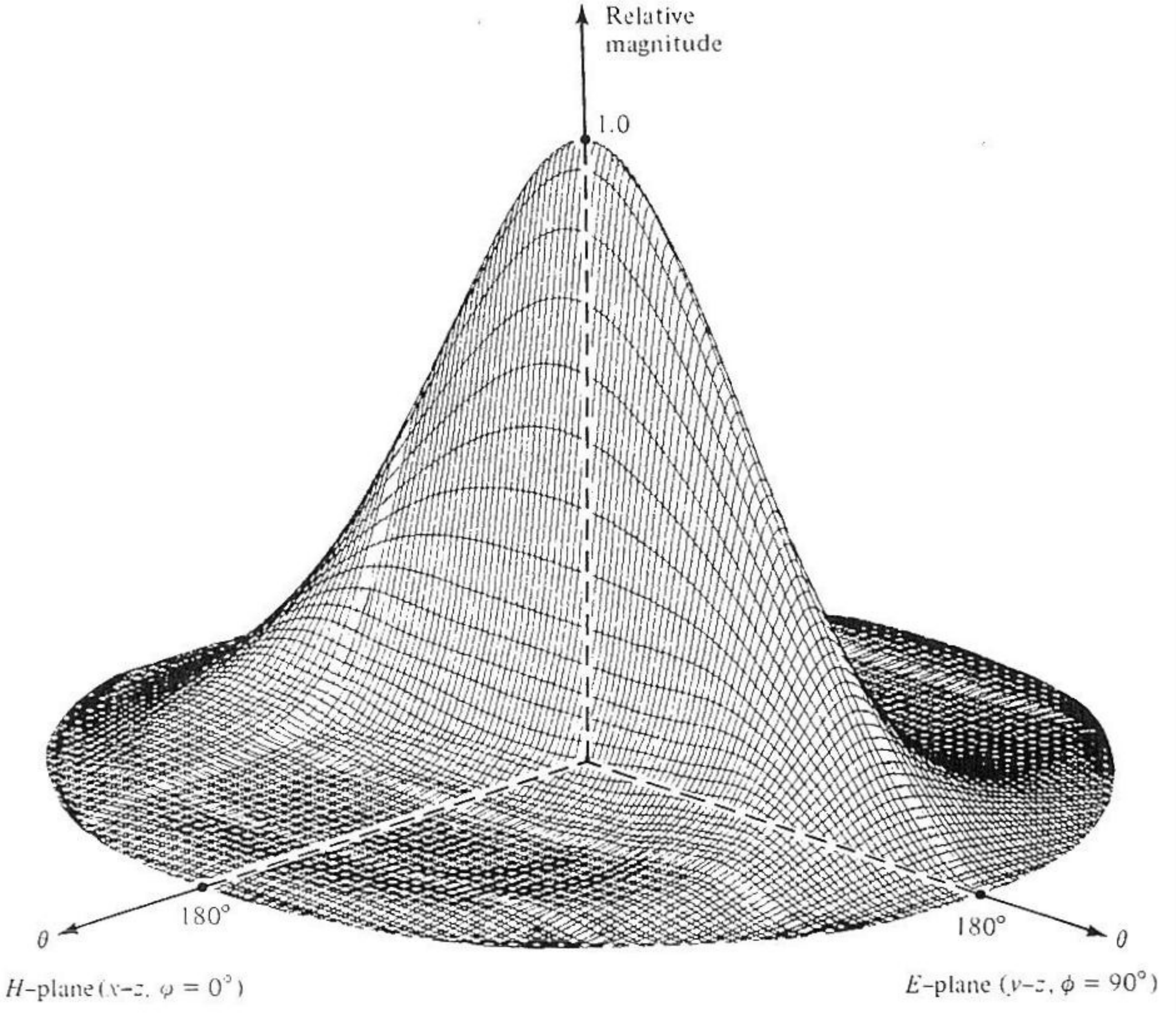
Normalized Directivity



H-plane sectoral horn

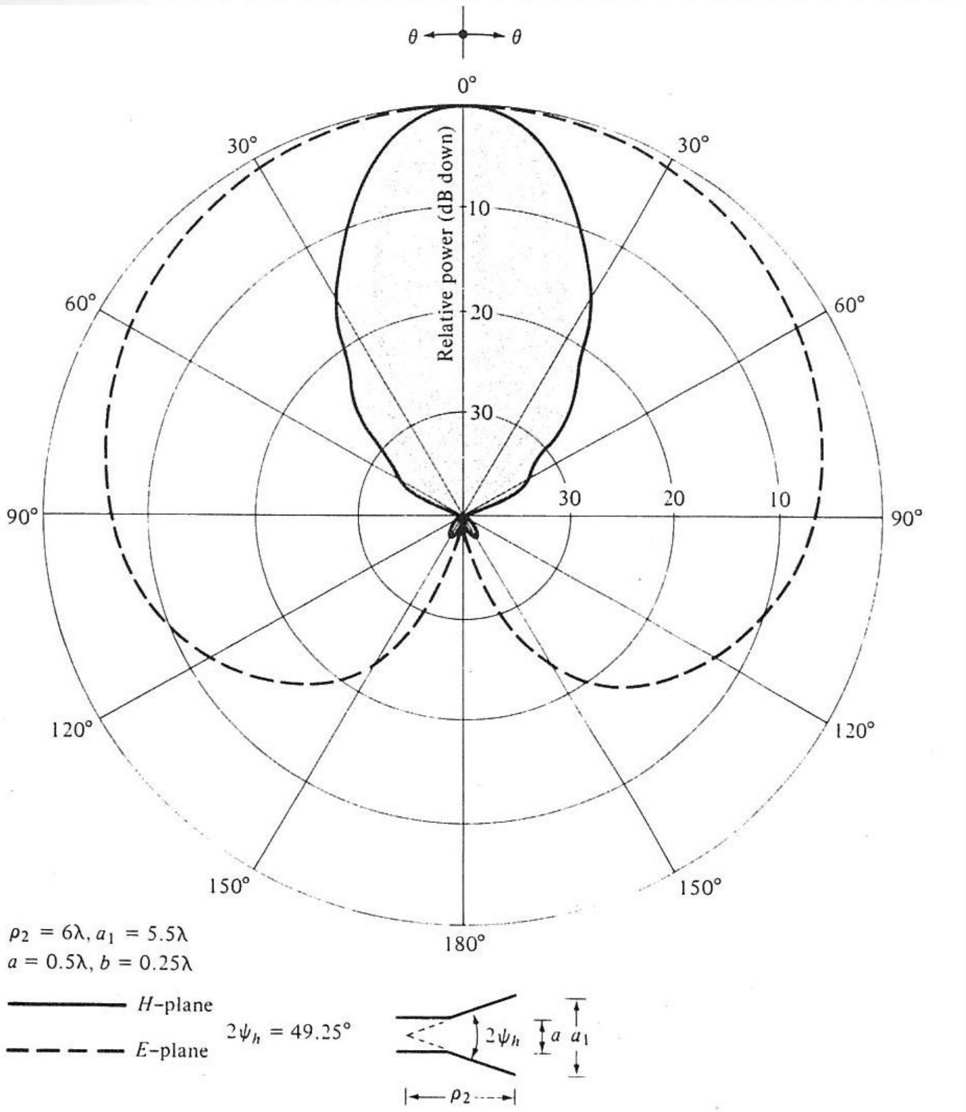


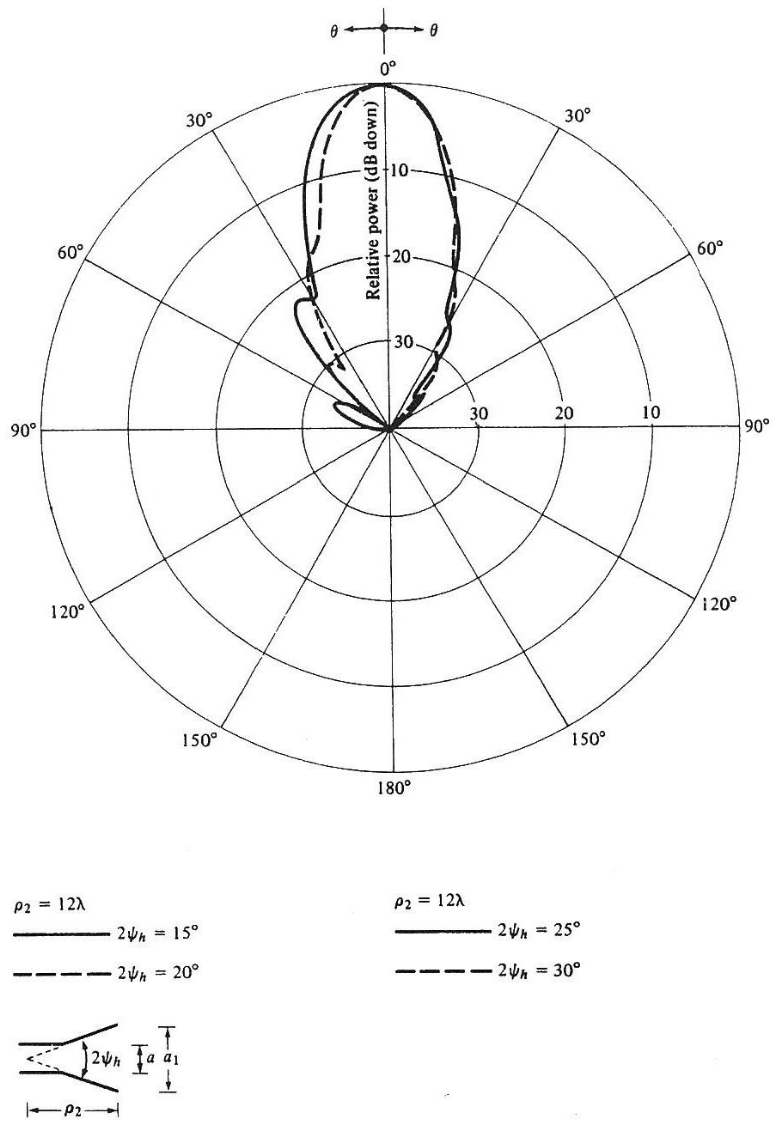
Radiation Pattern



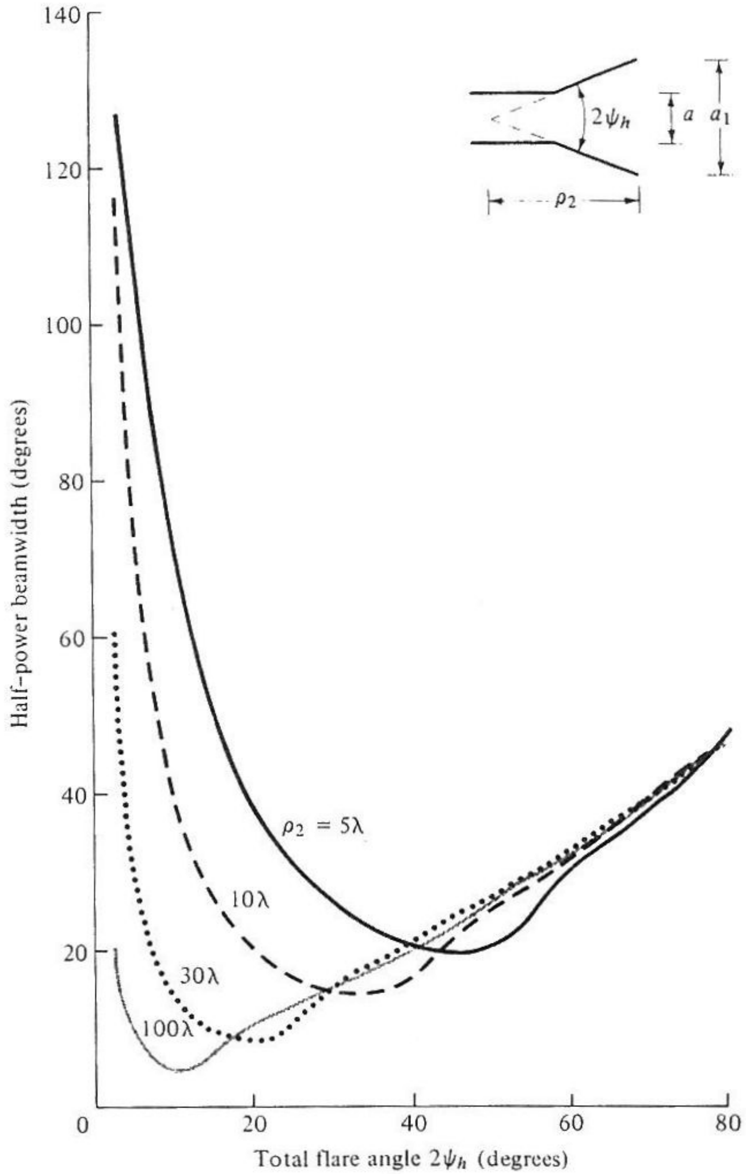
Normalized field pattern: $\rho_2 = 6\lambda$, $a = 5.5\lambda$, $b_1 = 0.25\lambda$

Radiation Pattern (2)

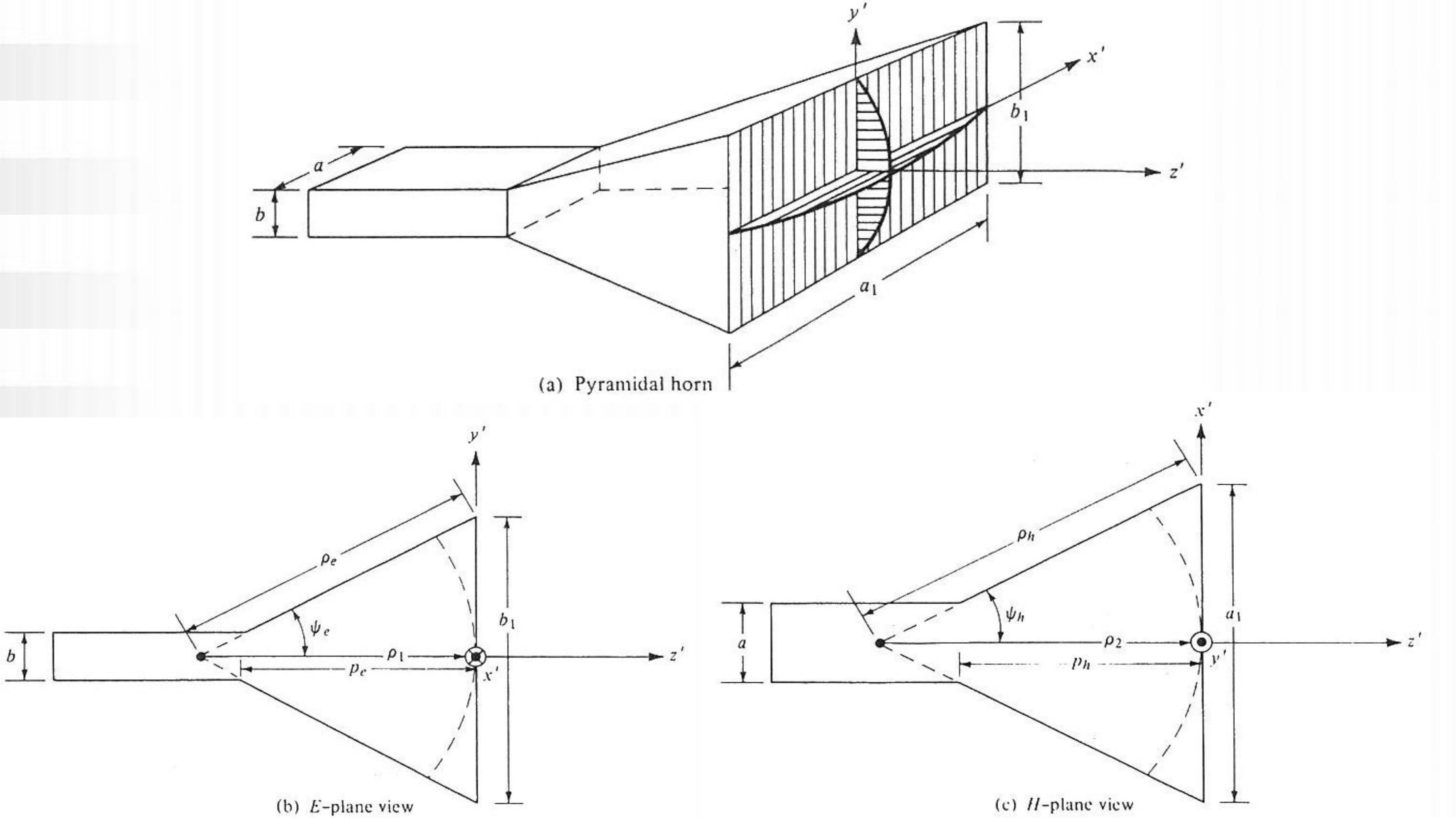




Half-power beamwidth



Pyramidal Horn

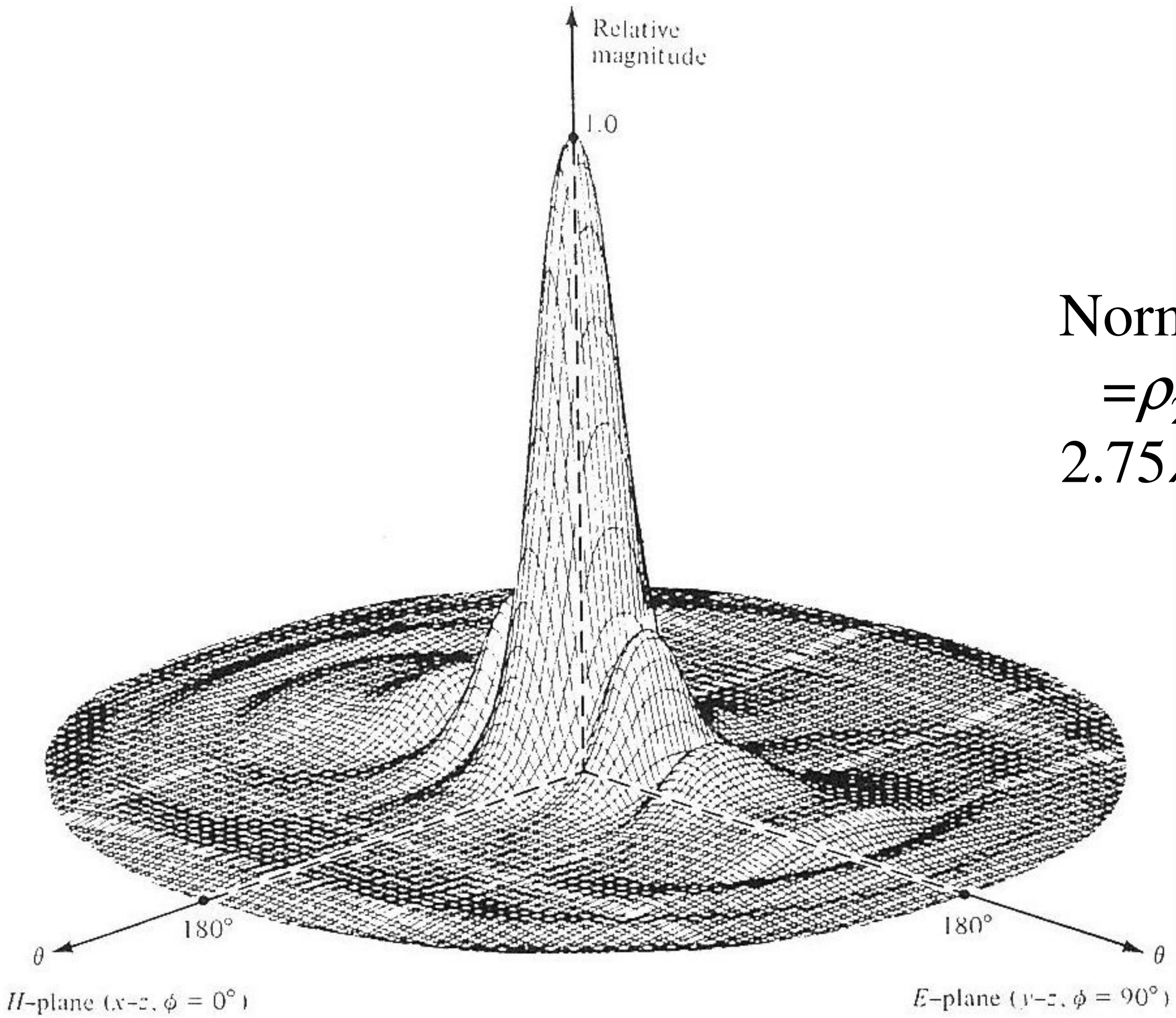


(a) Pyramidal horn

(b) *E*-plane view

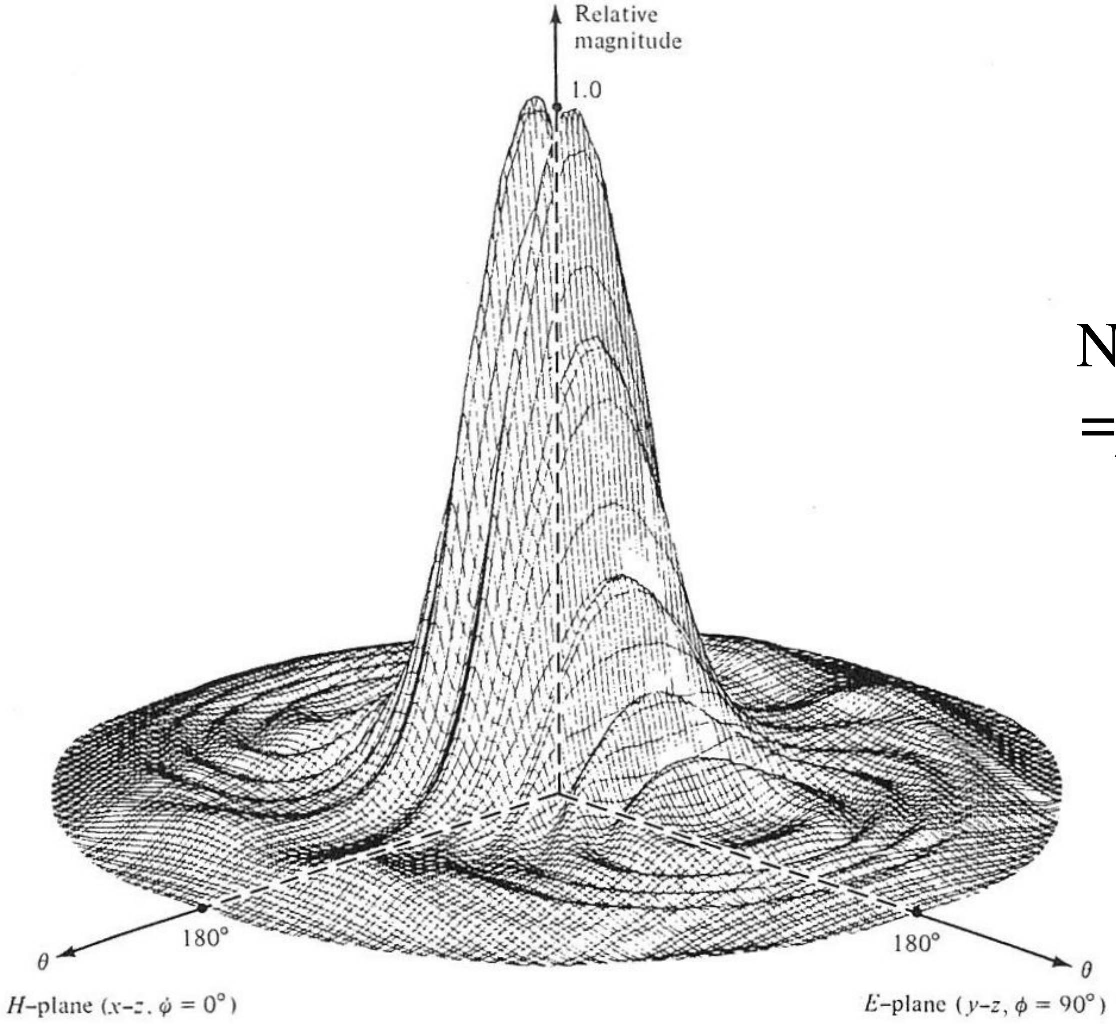
(c) *H*-plane view

Radiation Pattern



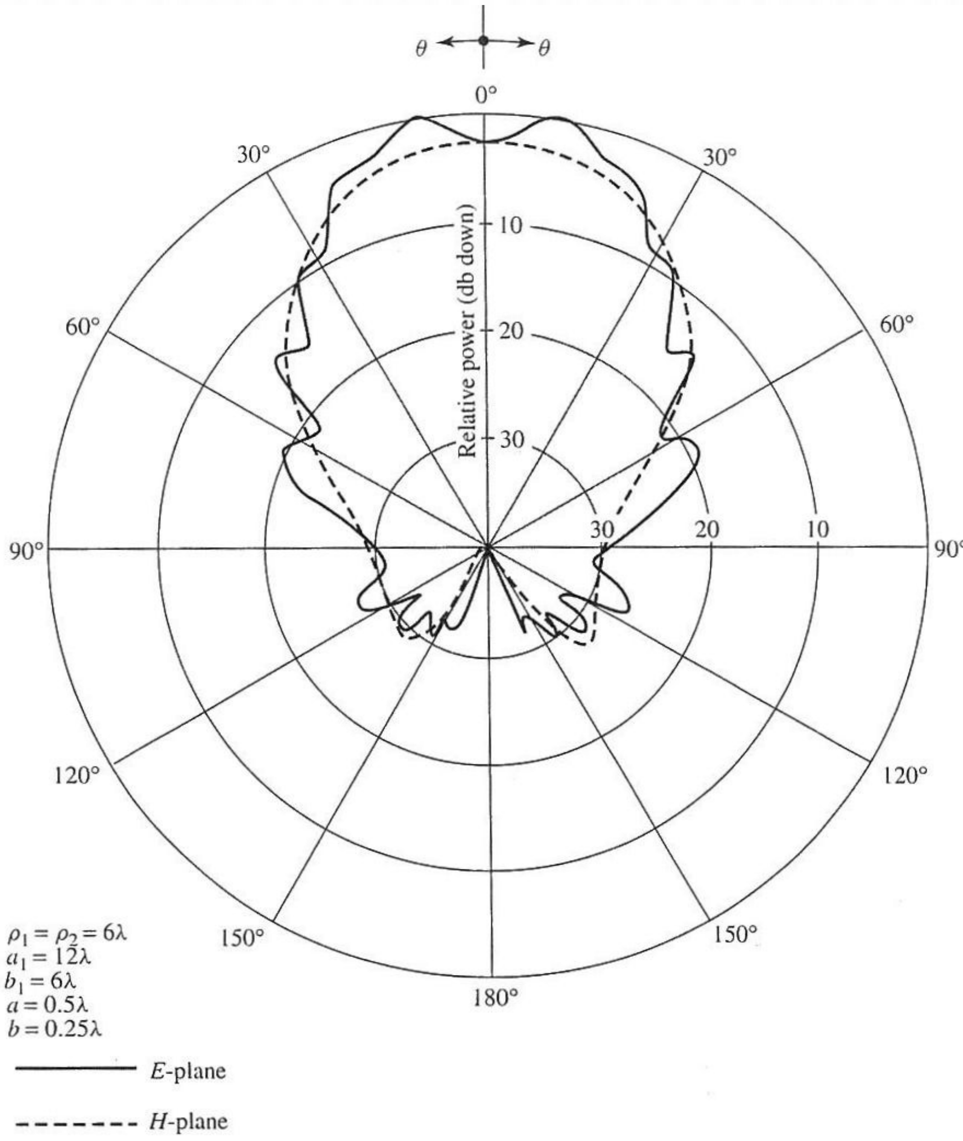
Normalized field pattern: $\rho_1 = \rho_2 = 6\lambda$, $a_1 = 5.5\lambda$, $b_1 = 2.75\lambda$, $a_1 = 0.5\lambda$, $b_1 = 0.25\lambda$

Radiation Pattern (2)

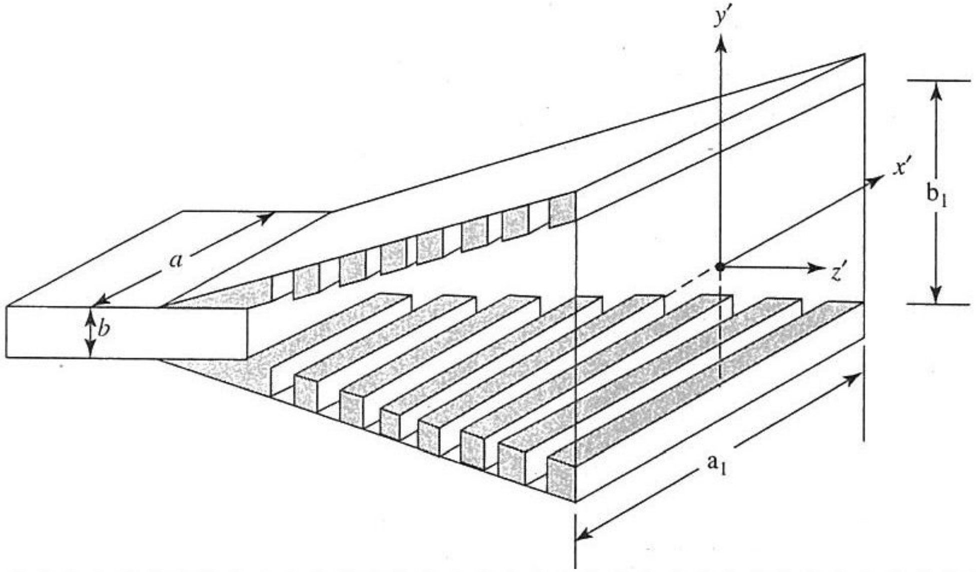


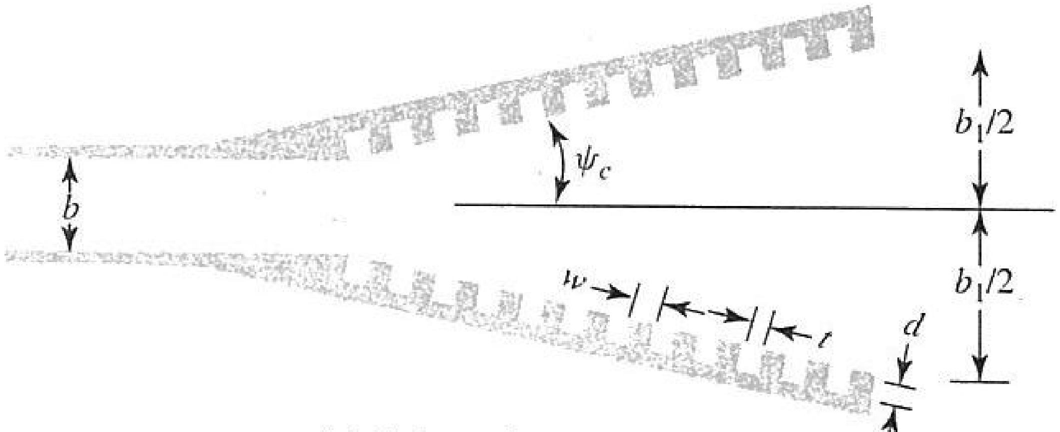
Normalized field pattern: $\rho_1 = \rho_2 = 6\lambda$, $a_1 = 12\lambda$, $b_1 = 6\lambda$, $a_1 = 0.5\lambda$, $b_1 = 0.25\lambda$

Radiation Pattern (3)



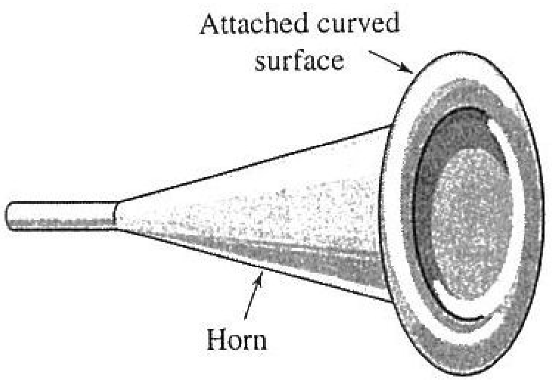
Corrugated Horns

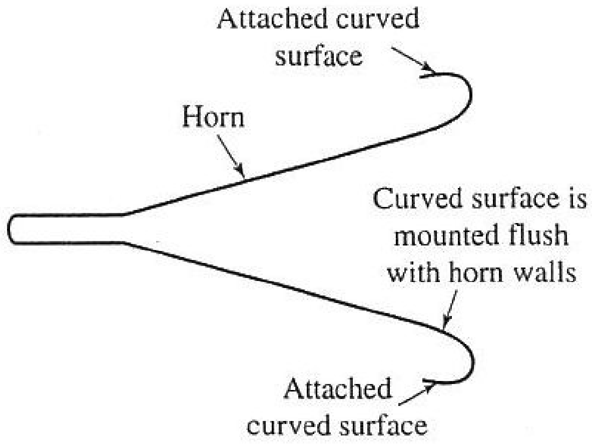


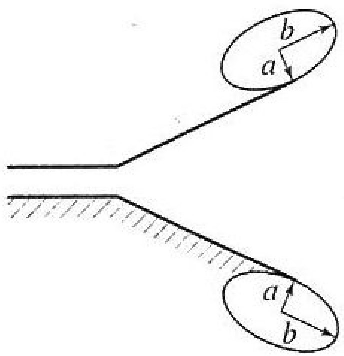


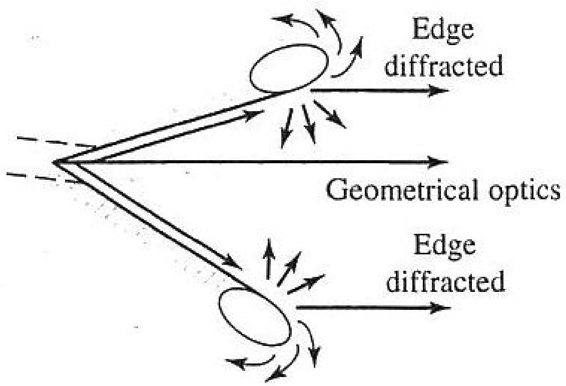
(b) *E*-plane view

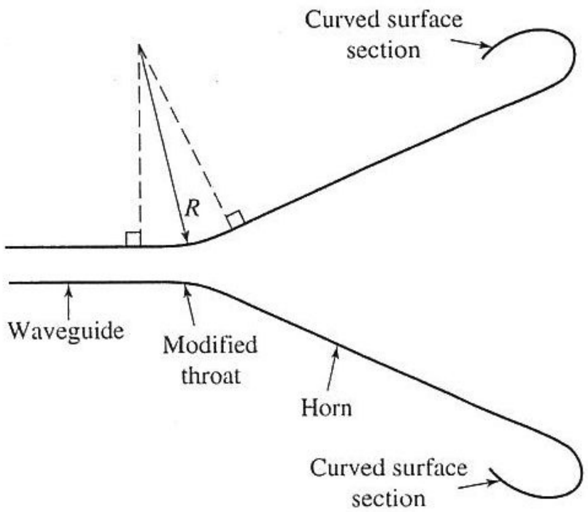
Aperture-matched horns











Quiz

- **Derive the formula for the maximum directivity of E-plane horn (in slide 19).**

HINT: $C(-x) = -C(x)$, $S(-x) = -S(x)$.