Chapter 2 :
Fundamental parameters of antennas

- **From Radiation Pattern**
  - Radiation intensity
  - Beamwidth
  - Directivity
  - Antenna efficiency
  - Gain
  - Polarization
- **From Circuit viewpoint**
  - Input Impedance

Chapter 2 : Topics (2)

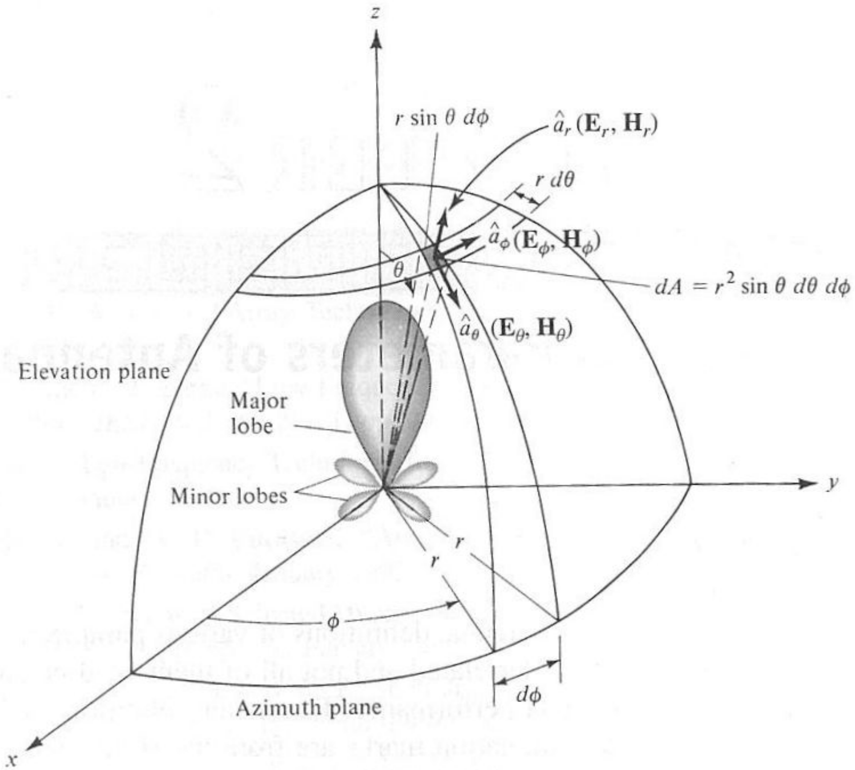
- **Antenna effective length and effective area**
- **Friis transmission equation**
- **Radar range equation**
- **Antenna temperature**

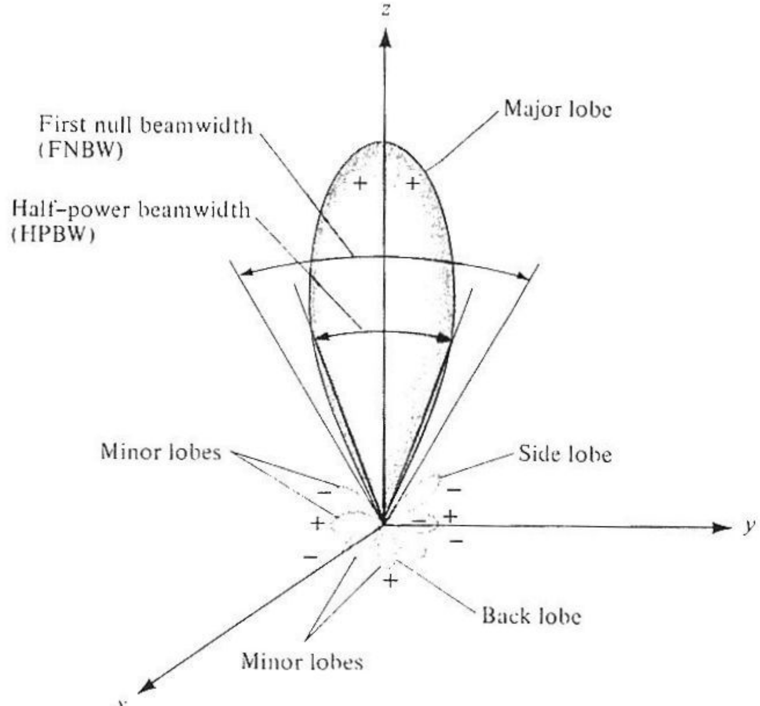
Definition of Radiation Pattern

- **Once the electromagnetic (EM) energy leaves the antenna, the radiation pattern tells us how the energy propagates away from the antenna.**
- **Definition :**

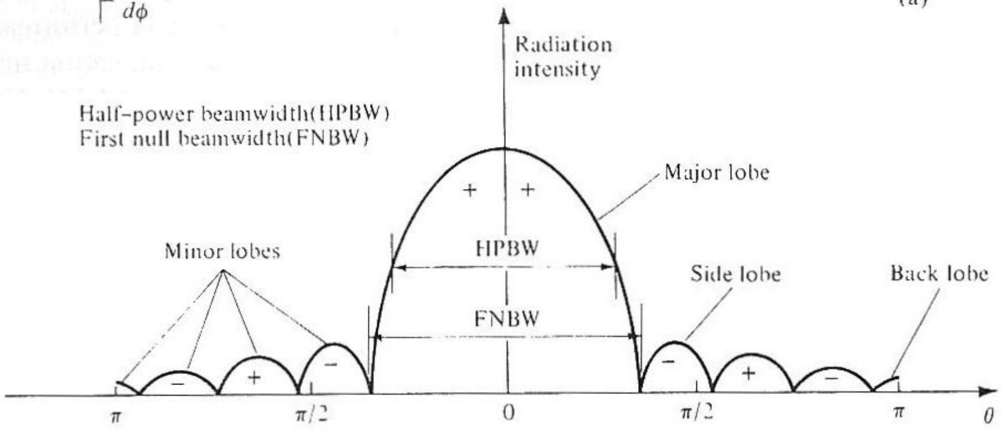
  Mathematical function or a graphical representation of the radiation properties of an antenna as a function of space coordinates

Radiation Pattern Example





(a)



(b)

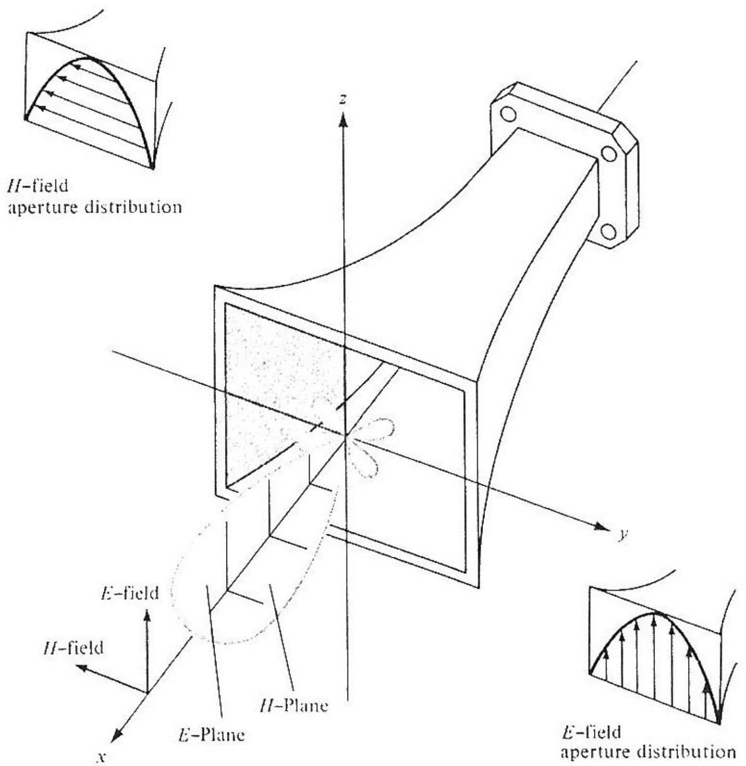
Radiation Pattern (1)

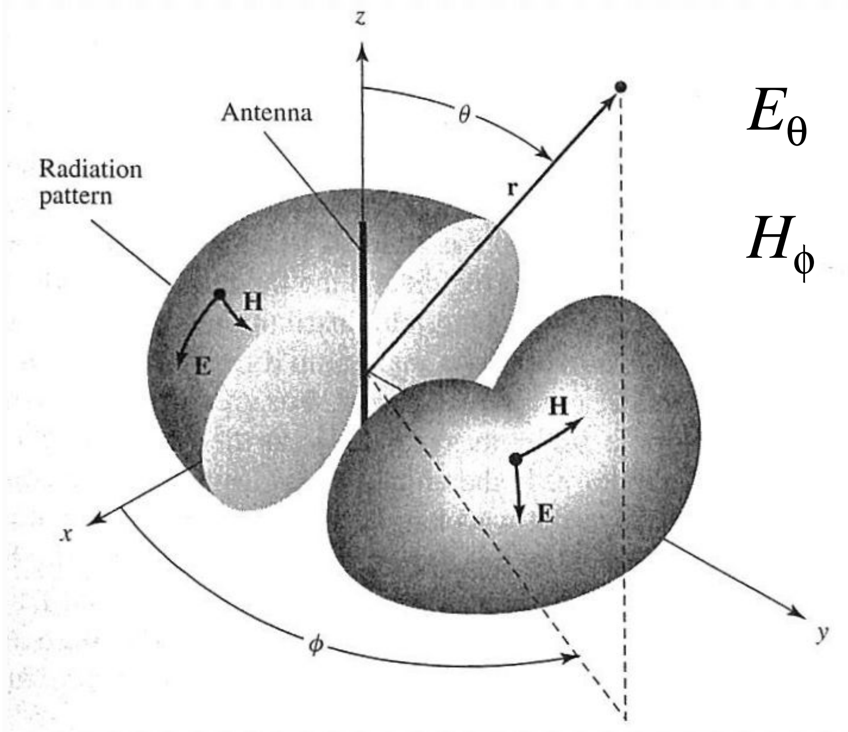
- **Can be classified as:**
  - *Isotropic, directional and omnidirectional*
- **Isotropic: Hypothetical antenna having equal radiation in all directions**
- **Directional: having the property of transmitting or receiving EM energy more effectively in some directions than others**
- **Omnidirectional: having an essentially nondirectional pattern in a given plane and a directional pattern in any orthogonal plane**

Radiation Pattern (2)

- **Principal patterns (or planes):**
  - E-plane : the plane containing the electric field vector and the direction of maximum radiation
  - H-plane : the plane containing the magnetic field vector and the direction of maximum radiation

Radiation Pattern (3)



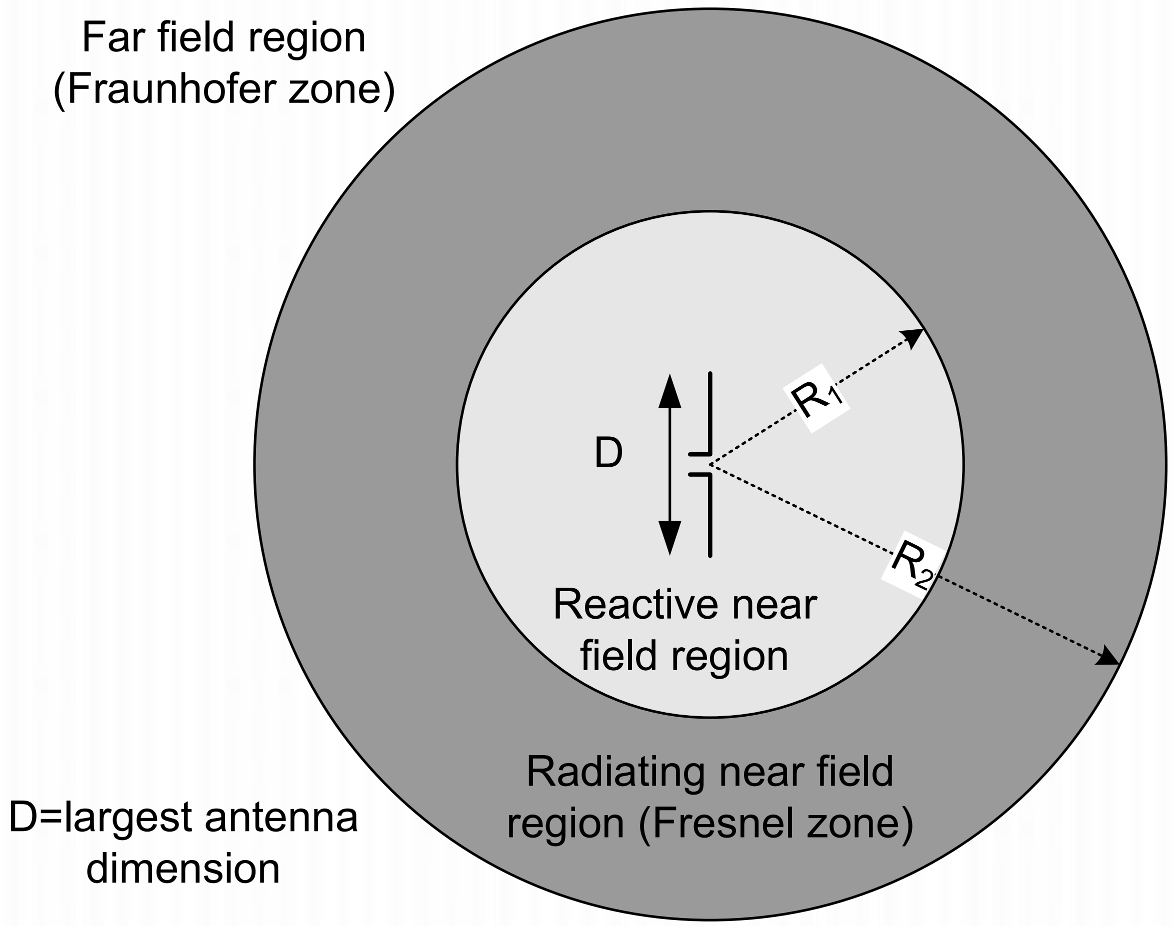


E_θ

H_ϕ

Omnidirectional

Field Regions



Reactive Near Field Region

- **Region surrounding the antenna, wherein the reactive field predominates**

$$\text{For } D > \lambda : R < 0.62\sqrt{\frac{D^3}{\lambda}}$$

$$\text{For } D < \lambda \text{(small antenna)} : R < \frac{\lambda}{2\pi}$$

$$\Rightarrow R < \max[\frac{\lambda}{2\pi}, 0.62\sqrt{\frac{D^3}{\lambda}}]$$

Angular field distribution depends on distance from antenna

Radiating Near Field Region

- **Region between reactive near-field and far-field regions (Fresnel zone)**

$$\text{For } D > \lambda : \frac{2D^2}{\lambda} > R > 0.62\sqrt{\frac{D^3}{\lambda}}$$

$$\text{For } D < \lambda \text{(small antenna)} : 3\lambda > R > \frac{\lambda}{2\pi}$$

$$\Rightarrow \max[3\lambda, \frac{2D^2}{\lambda}] > R > \max[\frac{\lambda}{2\pi}, 0.62\sqrt{\frac{D^3}{\lambda}}]$$

Radiation fields predominate but angular field distribution still depends on distance from antenna

Far Field Region

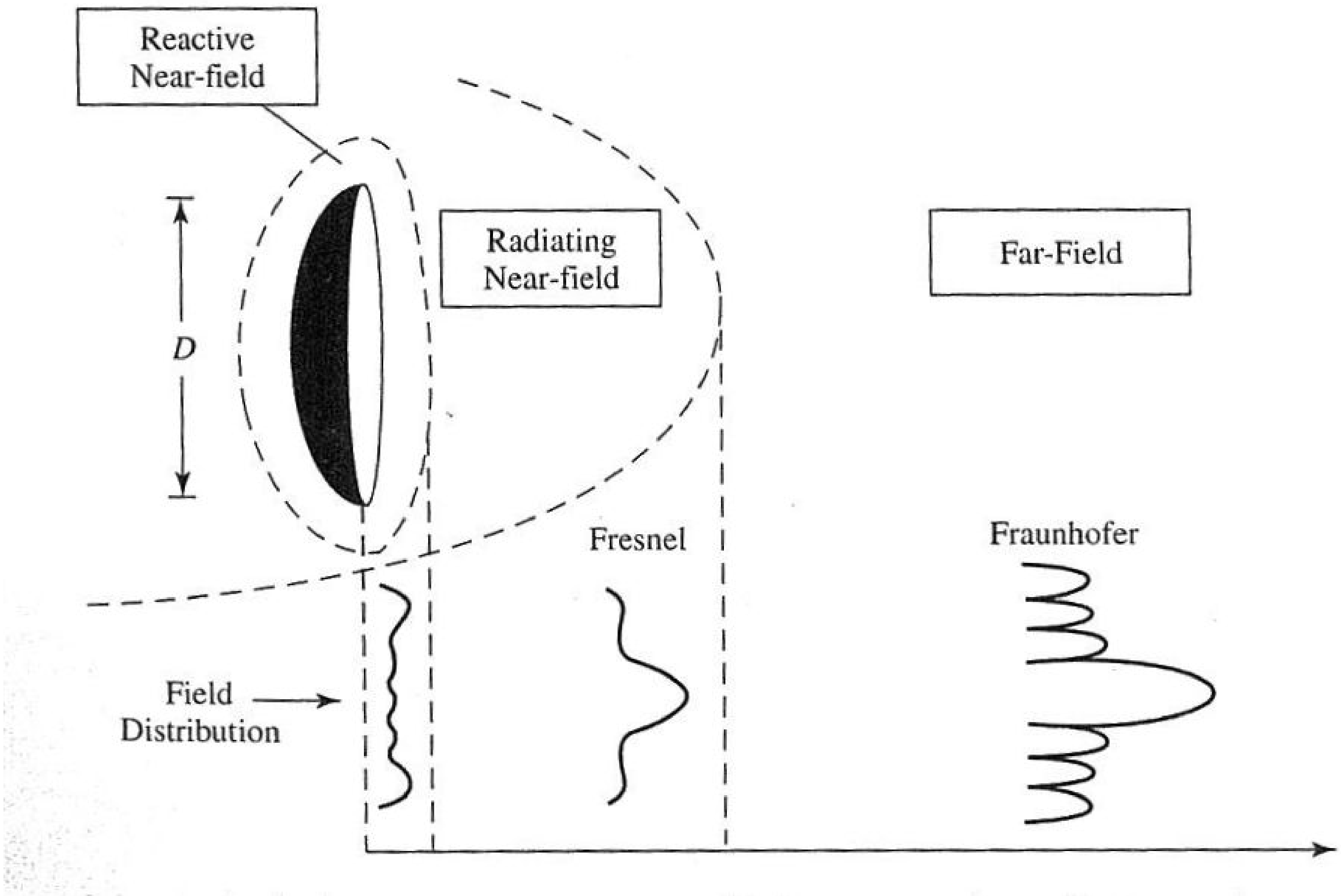
- **Region where angular field distribution is essentially independent of the distance from antenna (Fraunhofer zone)**

$$\text{For } D > \lambda : R > \frac{2D^2}{\lambda}$$

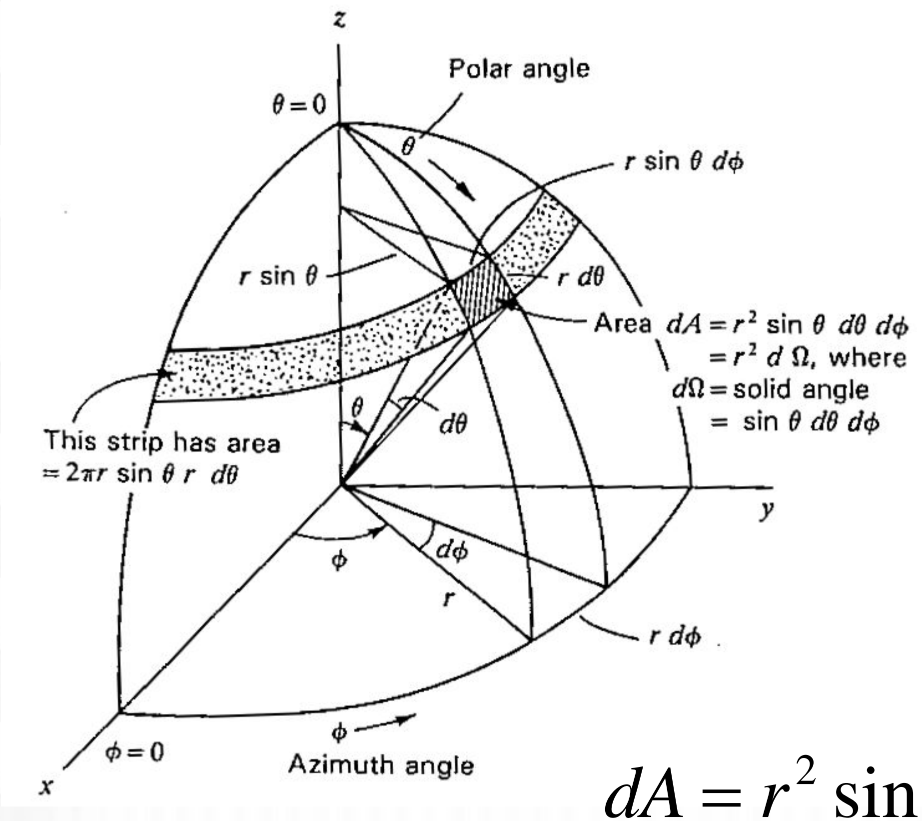
$$\text{For } D < \lambda \,(\text{small antenna}) : R > 3\lambda$$

$$\Rightarrow R > \max[3\lambda, \frac{2D^2}{\lambda}]$$

Change of antenna amplitude pattern shape



Spherical coordinate and Solid Angle : Steradian



Measure of solid angle: 1 steradian = solid angle with its vertex at the center of a sphere of radius *r* that is subtended by a surface of area r^2

$$dA = r^2 \sin\theta d\theta d\phi = r^2 d\Omega;$$

$$d\Omega = \sin\theta d\theta d\phi = \text{solid angle}$$

Quiz: What’s the solid angle subtended by a sphere?

Radiation Power Density

- **Poynting vector = Power density**

$$\vec{\mathscr{W}} = \vec{\mathscr{E}} \times \vec{\mathscr{H}}$$

$\vec{\mathscr{W}}$: instantaneous Poynting vector [W/m^2]

$\vec{\mathscr{E}}$: instantaneous electric field Intensity [V/m]

$\vec{\mathscr{H}}$: instantaneous magnetic field Intensity [A/m]

- **Total power:** $\mathscr{P} = \oiint_S \vec{\mathscr{W}} \cdot ds = \oiint_S \vec{\mathscr{W}} \cdot \hat{n} da$

$\mathscr{P}$: instantaneous total power [W]

$\hat{n}$: unit vector normal to the surface

da : infinitesimal area of the closed surface [m^2]

Radiation Power Density (2)

- **For time-harmonic EM fields**

$$\vec{\mathscr{E}}(x,y,z;t)=\mathrm{Re}[\vec{\mathbf{E}}(x,y,z)e^{j\omega t}]$$

$$\vec{\mathscr{H}}(x,y,z;t)=\mathrm{Re}[\vec{\mathbf{H}}(x,y,z)e^{j\omega t}]$$

- **Poynting vector**

$$\vec{\mathscr{W}}=\vec{\mathscr{E}}\times\vec{\mathscr{H}}=\frac{1}{2}\mathrm{Re}[\vec{\mathbf{E}}\times\vec{\mathbf{H}}^{*}]+\frac{1}{2}\mathrm{Re}[\vec{\mathbf{E}}\times\vec{\mathbf{H}}e^{j2\omega t}]$$

- **Time average Poynting vector (average power density or radiation density)**

$$\vec{\mathbf{W}}_{av}(x,y,z)=[\vec{\mathscr{W}}(x,y,z;t)]_{av}=\frac{1}{2}\mathrm{Re}[\vec{\mathbf{E}}\times\vec{\mathbf{H}}^{*}]$$

$\frac{1}{2}$ appears because $\vec{\mathbf{E}}, \vec{\mathbf{H}}$ fields represent peak values

Radiation Power Density (3)

- **Average power radiated power**

$$P_{rad} = P_{av} = \oiint_S \vec{\mathbf{W}}_{av} \cdot d\mathbf{s} = \oint_S \vec{\mathbf{W}}_{av} \cdot \hat{n} da = \frac{1}{2} \oiint_S \mathrm{Re}[\vec{\mathbf{E}} \times \vec{\mathbf{H}}^*] \cdot d\mathbf{s}$$

Example 2.1: The average power density is given by

$$\vec{\mathbf{W}}_{av} = \hat{r} W_r = \hat{r} A_0 \frac{\sin\theta}{r^2} [\mathrm{W/m^2}]$$

The total radiated power becomes

$$P_{rad} = \oiint_S \vec{\mathbf{W}}_{av} \cdot \hat{n} da$$

$$= \int_0^{2\pi} \int_0^{\pi} (\hat{r} A_0 \frac{\sin\theta}{r^2}) \cdot \hat{r} r^2 \sin\theta d\theta d\phi = \pi^2 A_0 [\mathrm{W}]$$

Radiation Power Density (4)

- **For an isotropic antenna**

$$P_{rad} = \oiint_S \vec{\mathbf{W}}_0 \cdot d\vec{\mathbf{s}}$$

$$= \int_0^{2\pi}\int_0^{\pi} [\hat{r}W_0(r)] \cdot \hat{r}r^2 \sin\theta d\theta d\phi = 4\pi r^2 W_0 \ [\mathrm{W}]$$

The power density is then given by

$$\vec{\mathbf{W}}_0 = \hat{r}W_0 = \hat{r}\frac{P_{rad}}{4\pi r^2} [\mathrm{W/m^2}]$$

Radiation Intensity

- **Definition : The power radiated from an antenna per unit solid angle**

$$U = r^2 W_{rad}$$

U : radiation intensity [W/unit solid angle]

W_{rad} : radiation density [$\mathrm{W/m^2}$]

Total power can be given by

$$P_{rad} = \oiint_{\Omega} U d\Omega = \int_0^{2\pi} \int_0^{\pi} U \sin\theta d\theta d\phi$$

Radiation Intensity (2)

- **Radiation intensity is related to the far-zone electric field of antenna**

$$U(\theta,\phi) = \frac{r^2}{2\eta}|\vec{\mathbf{E}}(r,\theta,\phi)|^2 \cong \frac{r^2}{2\eta}[|E_\theta(r,\theta,\phi)|^2 + |E_\phi(r,\theta,\phi)|^2]$$

$$\cong \frac{1}{2\eta}[|E_\theta^o(\theta,\phi)|^2 + |E_\phi^o(\theta,\phi)|^2]$$

$\vec{\mathbf{E}}(r,\theta,\phi)$: far-zone electric-field intensity of the antenna $= \vec{\mathbf{E}}^o(\theta,\phi)\frac{e^{-jkr}}{r}$

E_θ, E_ϕ : far-zone electric-field components of the antenna

η : intrinsic impedance of the medium ($\approx 377\,\Omega$ in free space)

Radiation Intensity (3)

Example 2.2: The radiation intensity is given by

$$U = r^2 W_{rad} = A_0 \sin\theta$$

The total radiated power becomes

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta d\theta d\phi$$

$$= A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta d\theta d\phi = \pi^2 A_0 [\mathrm{W}]$$

For an isotropic antenna

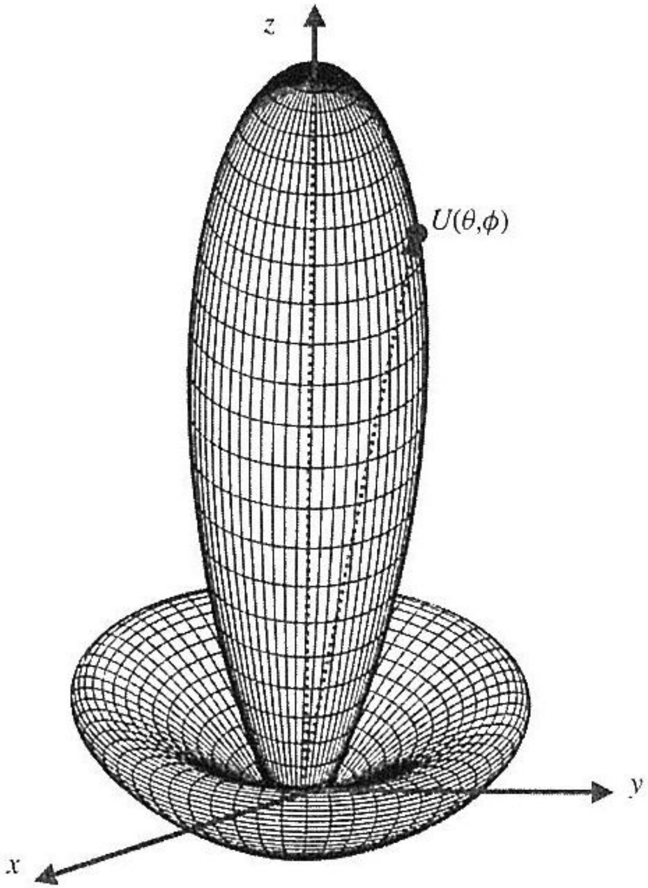
$$P_{rad} = \oiint_{\Omega} U_0 d\Omega = U_0 \oiint_{\Omega} d\Omega = 4\pi U_0$$

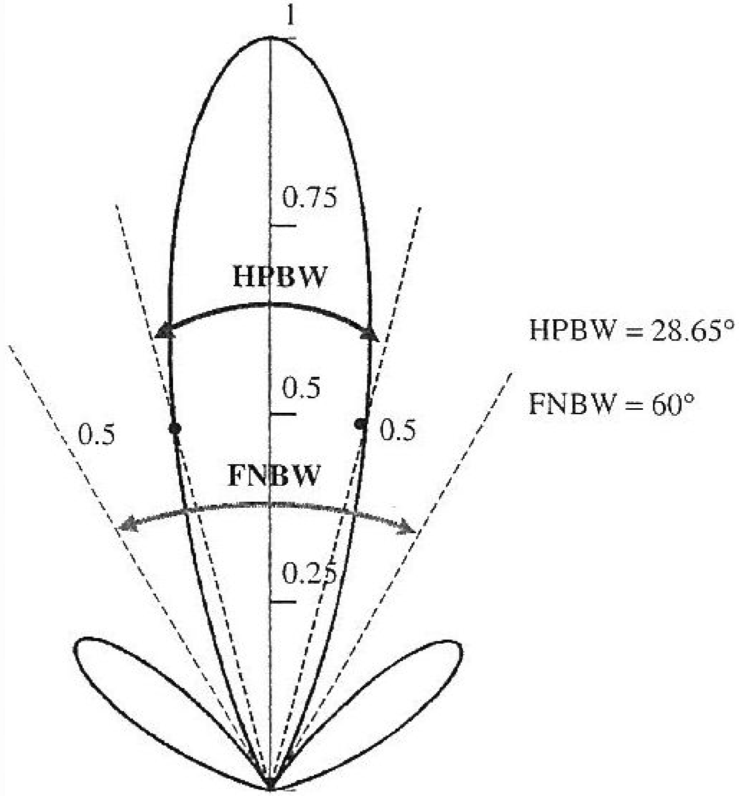
$$\Rightarrow U_0 = \frac{P_{rad}}{4\pi}$$

Beamwidth

- **Beamwidth is the angular separation between two identical points on opposite site of the pattern maximum**
- **Half-power beamwidth (HPBW): in a plane containing the direction of the maximum of a beam, the angle between the two directions in which the radiation intensity is one-half value of the beam**
- **First-Null beamwidth (FNBW): angular separation between the first nulls of the pattern**

Beamwidth (2)





Beamwidth (3)

Example 2.3: The normalized radiation intensity of an antenna is represented by

$$U(\theta) = \cos^2\theta \ (0 \le \theta \le \pi, 0 \le \phi \le 2\pi)$$

The angle θ_h at which the function equal to half of its maximum can be found by

$$U(\theta)|_{\theta=\theta_h} = \cos^2\theta = 0.5 \Rightarrow \cos\theta_h = 0.707$$

$$\theta_h = \cos^{-1}(0.707) = \frac{\pi}{4}$$

Since the pattern is symmetric with respect to the maximum, HPBW = 2 θ_h = $\pi/2$

Likewise, FNBW = $2\theta_n = \pi$ since $U(\theta)|_{\theta=\theta_n} = 0 \Rightarrow \theta_n = \cos^{-1}(0) = \frac{\pi}{2}$

Directivity

- **Ratio of radiation intensity in a given direction from the antenna to the average radiation intensity**

$$D(\theta,\phi) = \frac{U(\theta,\phi)}{U_0} = \frac{4\pi U}{P_{rad}} \text{(dimension - less)}$$

Note that the average radiation intensity equals to the radiation intensity of an isotropic source.

Directivity (2)

Since $P_{rad} = \oiint_{\Omega'} U(\theta', \phi') d\Omega'$

$$D(\theta,\phi) = 4\pi \frac{U(\theta,\phi)}{\oiint_{\Omega'} U(\theta', \phi') d\Omega'}$$

$$D_{\max} = 4\pi \frac{U_{\max}}{\oiint_{\Omega'} U(\theta', \phi') d\Omega'} = \frac{4\pi}{\Omega_A}$$

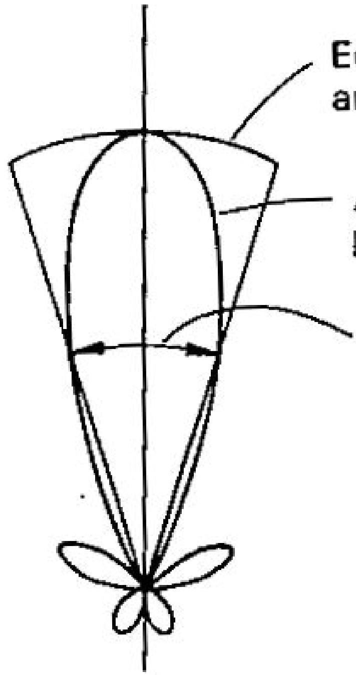
$D_{\max}$: maximum directivity

Ω_A is called beam solid angle, and is defined as "solid angle through which all the power of the antenna would flow if its radiation intensity were constant and equal to $U_{\max}$ for all angles within Ω_A

$$\Omega_A = \oiint_{\Omega'} \frac{U(\theta', \phi')}{U_{\max}} d\Omega' \Rightarrow P_{rad} = \Omega_A U_{\max}$$

Directivity (3)

- **If the direction is not specified, it implies the directivity of maximum radiation intensity (maximum directivity) expressed as**



$$D_{\max} = D_0 = \frac{U_{\max}}{U_0} = \frac{4\pi U_{\max}}{P_{rad}} \text{(dimension - less)}$$

Directivity (4)

Example 2.4: The radial component of the radiated power density of an infinitesimal linear dipole is given by

$$W_{av} = \hat{r}W_r = \hat{r}A_0 \frac{\sin^2\theta}{r^2} [\text{W/m}^2]$$

where A_0 is the peak value of the power density. The radiation intensity is given by

$$U = r^2 W_r = A_0 \sin^2\theta$$

The maximum radiation is directed along $\theta = \pi/2$ and $U_{\max} = A_0$.

The total radiated power is given by

$$P_{rad} = \oiint_{\Omega} U d\Omega = A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \sin\theta d\theta d\phi = A_0 \frac{8\pi}{3}$$

Thus,

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \frac{3}{2} \text{ and } D = D_0 \sin^2\theta = 1.5\sin^2\theta$$

Antenna Efficiency

- **The overall antenna efficiency take into the following losses:**
  - Reflections because of the mismatch between the transmission line and the antenna
  - Conduction and dielectric losses

$$e_0 = e_{cd} e_r = e_{cd} (1 - |\Gamma|^2)$$

e_0 : total efficiency

e_r : reflection (mismatch) efficiency

e_{cd} : antenna radiation efficiency $= e_c e_d$

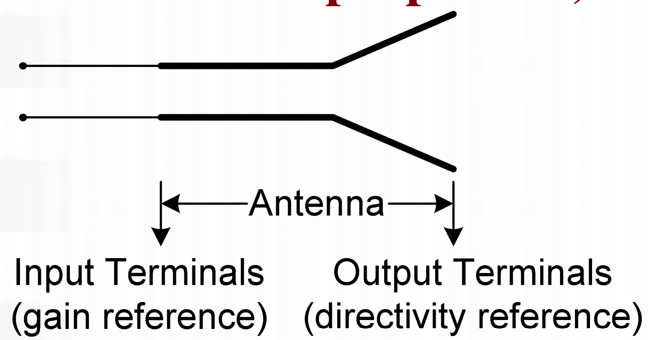
e_c, e_d : conduction, dielectric efficiencies

Γ : voltage reflection coefficient at the input terminal

Gain

- **It takes into account the efficiency of the antenna as well as its directional properties. (Directivity only measures directional properties.)**

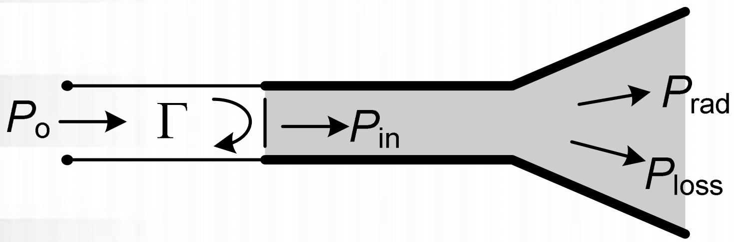
$$G(\theta,\phi) = 4\pi \frac{U(\theta,\phi)}{P_{in}}$$



P_{in} : Power input to antenna;

$$P_{in} = (1 - |\Gamma|^2) P_o = P_{rad} + P_{loss}$$

(P_{loss} : Ohmic and dielectric power loss)



Gain : ratio of radiation intensity in a given direction to the average radiation intensity that would be obtained if all the power input to the antenna were radiated isotropically

Gain (2)

- **Using e_{cd}, $P_{rad} = e_{cd} P_{in}$ and**

$$G(\theta,\phi) = 4\pi \frac{U(\theta,\phi)}{P_{rad}} e_{cd} = e_{cd} D(\theta,\phi)$$

Relative gain: ratio of power gain in a given direction to the power gain of a reference antenna in the same direction. The power input must be the same for both antennas. If the reference antenna is a lossless isotropic source, then

$$G(\theta,\phi) = 4\pi \frac{U(\theta,\phi)}{P_{in}(\text{lossless isotropic source})}$$

Gain (3)

- **Absolute gain takes into account impedance mismatch losses at the input terminals in addition to losses within antenna**

$$G_{abs}(\theta,\phi) = 4\pi \frac{U(\theta,\phi)}{P_o} = 4\pi \frac{U(\theta,\phi)}{P_{in}}(1-|\Gamma|^2)$$

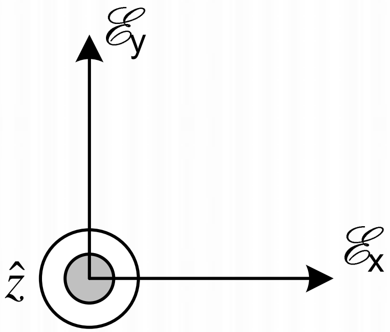
$$= 4\pi \frac{U(\theta,\phi)}{P_{rad}}(1-|\Gamma|^2)e_{cd} = e_0 D(\theta,\phi)$$

Polarization

- **Property of an EM wave describing the time varying direction and relative magnitude of the electric field. The figure traced as a function of time by the tip of the electric field and the sense in which is traced, as observed along direction of propagation.**

Wave propagating in –z direction

$$\vec{\mathscr{E}}(z;t) = \hat{x}\,\mathscr{E}_x(z;t) + \hat{y}\,\mathscr{E}_y(z;t)$$



$$\text{Let } E_x(z) = E_{xo} e^{j\phi_x} e^{jkz},$$

$$E_y(z) = E_{yo} e^{j\phi_y} e^{jkz} \text{ where } E_{xo}, E_{yo} \geq 0$$

$$\mathscr{E}_x(z;t) = E_{xo}\cos(\omega t + kz + \phi_x)$$

$e^{j\omega t}$ time dependence

$$\mathscr{E}_y(z;t) = E_{yo}\cos(\omega t + kz + \phi_y)$$

Polarization (2)

A. Linear Polarization

$$\text{(i)}E_{xo} = 0 \text{ or } E_{yo} = 0$$

or

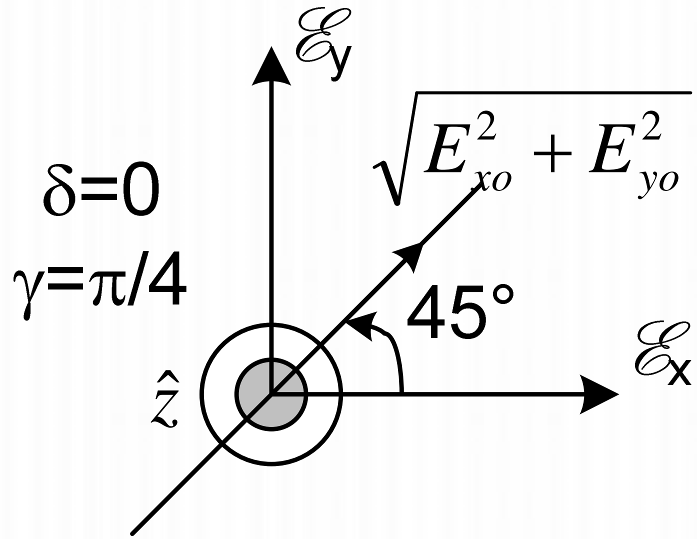
$$\text{(ii)}\delta = \phi_y - \phi_x = n\pi$$

where $n = 0, \pm1, \pm2, \ldots$

$$\gamma = \tan^{-1}\left(\frac{E_{yo}}{E_{xo}}\right), 0 \le \gamma \le \frac{\pi}{2}$$

δ, γ determine polarization state

Example



Polarization (3)

B. Circular Polarization

$$\text{(i)}\, E_{xo} = E_{yo} = E_o \Rightarrow \gamma = \tan^{-1}(1) = \pi/4$$

and

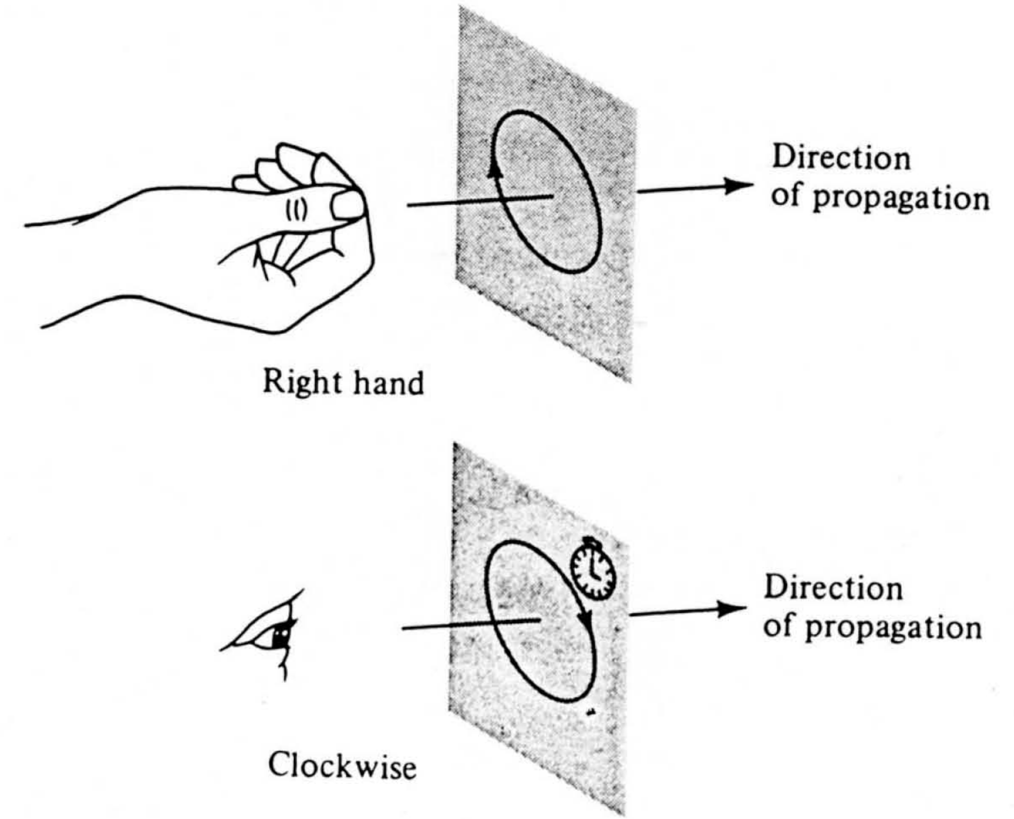
$$\text{(ii)}\, \delta = \phi_y - \phi_x = \begin{cases} \left(2\text{n} + \frac{1}{2}\right)\pi;\ \text{CW/RCP} \\ -\left(2\text{n} + \frac{1}{2}\right)\pi;\ \text{CCW/LCP} \end{cases}$$

where $n = 0,1,2,\ldots$

Note that the sense of rotation is observed *along the direction of propagation.*

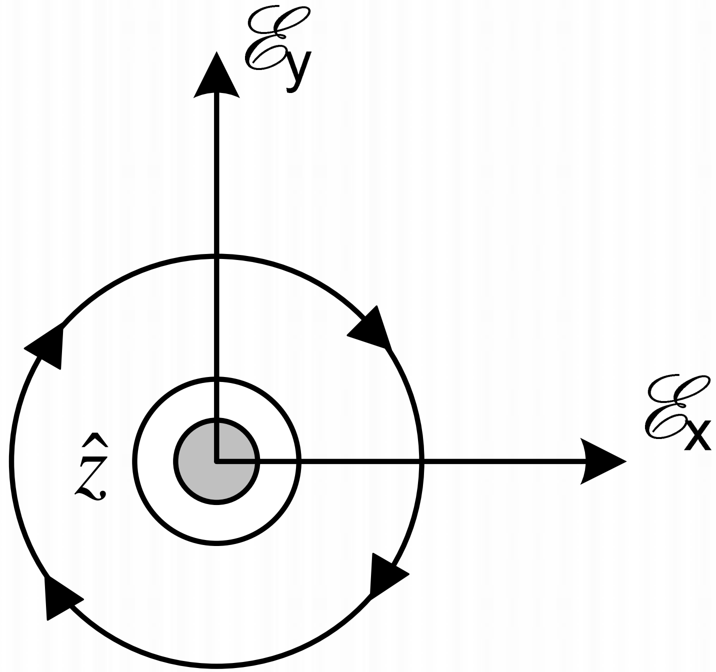
Polarization Ellipse & Sense of Rotation for Antenna Coordinate System

Sense Of Rotation



Polarization (4)

Example: RCP



$$\gamma = \pi/4, \delta = \pi/2$$

$$\mathscr{E}_x(z;t) = E_o \cos(\omega t + kz + \phi_x)$$

$$\mathscr{E}_y(z;t) = E_o \cos(\omega t + kz + \phi_x + \pi/2)$$

$$= -E_o \sin(\omega t + kz + \phi_x)$$

Polarization (5)

C. Elliptic Polarization

- A wave is elliptically polarized if it is not linearly or circularly polarized.
- Linear and circular polarization are special cases of elliptic polarization.

To have elliptic polarization:

1. Field must have two orthogonal linear components.
2. The two components can be of the same or different magnitude.

Polarization (6)

C. Elliptic Polarization

(i)if $\delta = \phi_y - \phi_x = \begin{cases} \left(2\text{n} + \dfrac{1}{2}\right)\pi; \text{ CW/REP} \\ -\left(2\text{n} + \dfrac{1}{2}\right)\pi; \text{ CCW/LEP} \end{cases}$

where $n = 0,1,2,\ldots$ AND $E_{xo} \neq E_{yo}$

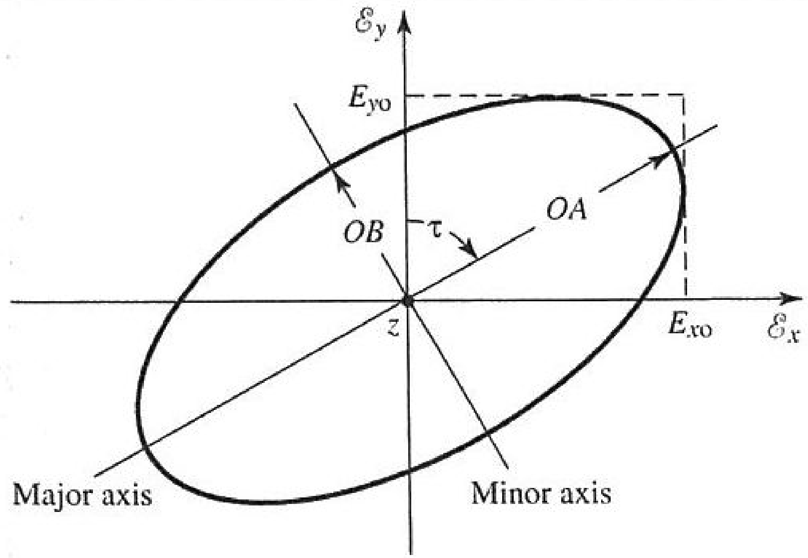
(ii)if $\delta = \phi_y - \phi_x \neq \pm \dfrac{n\pi}{2} \begin{cases} > 0; \text{ CW/REP} \\ < 0; \text{ CCW/LEP} \end{cases}$

where $n = 0,1,2,\ldots$ FOR $\forall E_{xo}, \forall E_{yo}$

Polarization (7)

$$\text{Axial Ratio (AR)} = \frac{\text{Major Axis}}{\text{Minor Axis}};\ 1 \le \text{AR} \le \infty$$

$$\tau = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{2E_{xo}E_{yo}}{E_{xo}^2 - E_{yo}^2}\cos\delta\right)$$



Polarization Loss Factor (PLF)

- **Electric field of incoming wave**

$$\vec{\mathbf{E}}_w = \hat{\rho}_w E_i$$

- **Electric field of receiving antenna**

$$\vec{\mathbf{E}}_a = \hat{\rho}_a E_a$$

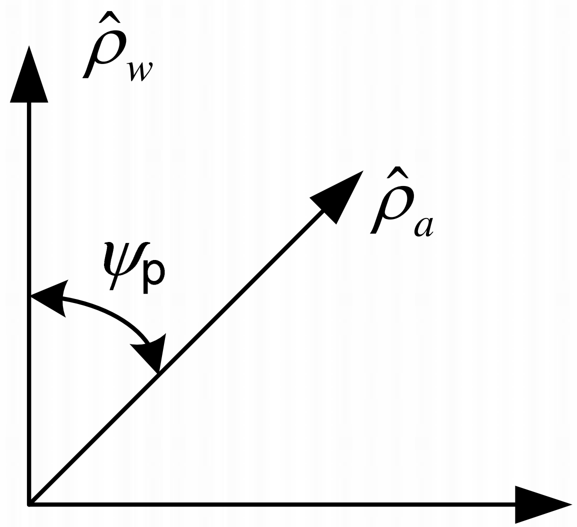
where

$\hat{\rho}_w$: unit vector of the wave

$\hat{\rho}_a$: polarization vector

$$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\cos\psi_p|^2$$

(dimensionless)



PLF example

LCP wave: δ=-π/2, ϕ_x=0 ⇨ ϕ_y=-π/2

$$E_x = E_o e^{j\phi_x} e^{jkz}; E_y = E_o e^{j\phi_y} e^{jkz}$$

$$\vec{\mathbf{E}} = (\hat{x}E_o + \hat{y}E_o e^{-j\pi/2})e^{jkz} = (\hat{x} - j\hat{y})E_o e^{jkz} = \hat{\rho}_w \sqrt{2} E_o e^{jkz}$$

where $\hat{\rho}_w = \dfrac{\hat{x} - j\hat{y}}{\sqrt{2}}$

If the antenna is also LCP, $\hat{\rho}_a = \hat{\rho}_w^*$

$$\text{PLF} = \left| \frac{\hat{x} - j\hat{y}}{\sqrt{2}} \cdot \frac{\hat{x} + j\hat{y}}{\sqrt{2}} \right|^2 = 1 = 0\,\text{dB}$$

If the antenna is RCP, $\hat{\rho}_a = \dfrac{\hat{x} - j\hat{y}}{\sqrt{2}}$

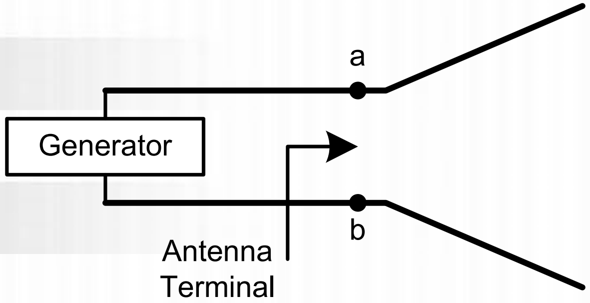
$$\text{PLF} = \left| \frac{\hat{x} - j\hat{y}}{\sqrt{2}} \cdot \frac{\hat{x} - j\hat{y}}{\sqrt{2}} \right|^2 = 0$$

Input Impedance

Impedance presented by the antenna at its terminal

Transmitting case

$$Z_A(\omega) = R_A(\omega) + jX_A(\omega) \qquad [\Omega]$$

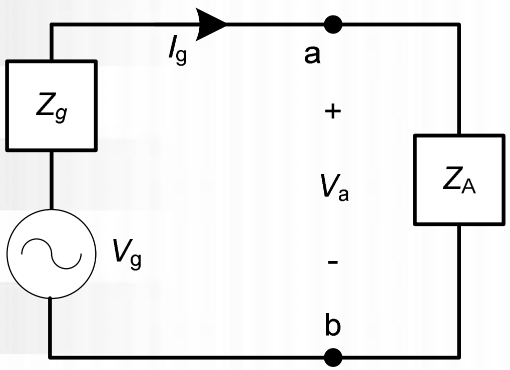


$$R_A = R_r + R_{loss}$$

R_r : Radiation resistance

R_{loss} : Loss resistance (ohmic, dielectric)

$$R_{loss} = R_{ohmic} + R_d$$



$$Z_g = R_g + jX_g, V_g = I_g(Z_g + Z_A)$$

V_g, I_g are peak values

Maximum power delivered to the antenna occurs when conjugate matched:

$$R_A = R_g, X_A = -X_g$$

Input Impedance (2)

When conjugate matched:

$$I_g = \frac{V_g}{2R_A} \Rightarrow P_{rad} = \frac{1}{2}|I_g|^2 R_r = \frac{|V_g|^2}{8}\frac{R_r}{(R_r + R_{loss})^2}$$

Radiated Power

$$P_{loss} = \frac{1}{2}|I_g|^2 R_{loss} = \frac{|V_g|^2}{8}\frac{R_{loss}}{(R_r + R_{loss})^2}$$

Power loss to heat

$$P_g = \frac{1}{2}|I_g|^2 R_g = \frac{|V_g|^2}{8}\frac{1}{R_r + R_{loss}}$$

Power loss in R_g

NOTE: $P_g = P_{rad} + P_{loss}$

Input Impedance (3)

Power supplied by generator when conjugate matched:

$$P_s = \mathrm{Re}[\frac{1}{2} V_g I_g^*] = \frac{1}{2} V_g \frac{V_g^*}{2R_A} = \frac{|V_g|^2}{4R_A} = \frac{|V_g|^2}{4R_g}$$

$$\Rightarrow \quad P_g = \frac{1}{2} P_s$$

$$P_{in} = P_{rad} + P_{loss} = \frac{1}{2} P_s \quad \Rightarrow \quad P_g = P_{in}$$

$$e_{cd} = \frac{P_{rad}}{P_{in}} = \frac{R_r}{R_r + R_{loss}}$$

antenna radiation efficiency

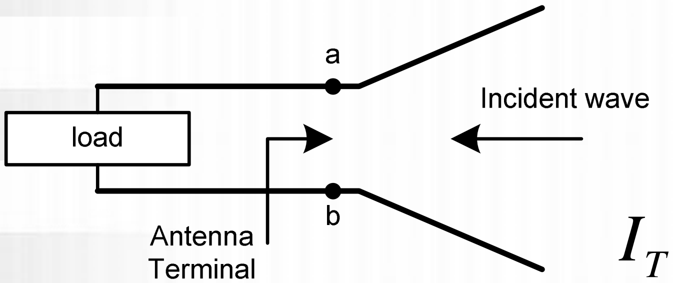
$$\text{If } R_{loss} = 0 \Rightarrow P_{loss} = 0, P_{rad} = P_{in}, e_{cd} = 1$$

Receiving Antenna

Impedance presented by the antenna at its terminal

$$Z_A = R_A + jX_A, Z_T = R_T + jX_T$$

Receiving case



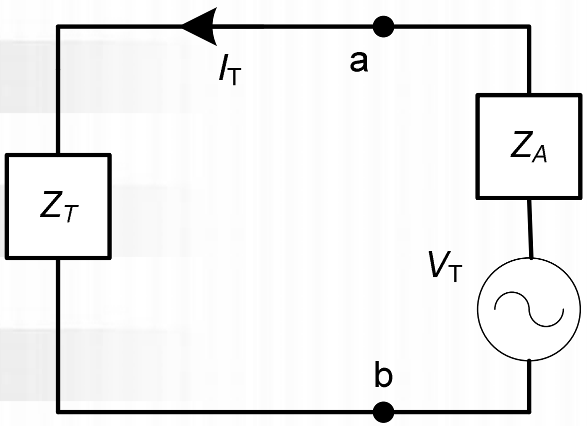
under conjugate matched condition

$$R_T = R_A = R_r + R_{loss}, X_T = -X_A$$

$$I_T = \frac{V_T}{2R_T} \Rightarrow P_T = \frac{1}{2}|I_T|^2 R_T = \frac{|V_T|^2}{8R_T}$$

V_T, I_T are peak values

Power delivered to load



$$P_{scatt} = \frac{1}{2}|I_T|^2 R_r = \frac{|V_T|^2}{8}\frac{R_r}{(R_r + R_{loss})^2}$$

Power scattered or re-radiated

Receiving Antenna (2)

Power supplied by generator when conjugate matched:

$$P_{loss} = \frac{1}{2} |I_T|^2 R_{loss} = \frac{|V_T|^2}{8} \frac{R_{loss}}{(R_r + R_{loss})^2}$$

$$\Rightarrow \quad P_T = P_{scatt} + P_{loss}$$

Power lost to heat

$$P_c = \frac{1}{2} \mathrm{Re}[V_T I_T^*] = \frac{|V_T|^2}{4R_T}$$

captured/collected power

$$\Rightarrow \quad P_T = \frac{1}{2} P_c$$

$$P_{scatt} + P_{loss} = \frac{1}{2} P_c \qquad P_c = P_{scatt} + P_{loss} + P_T$$

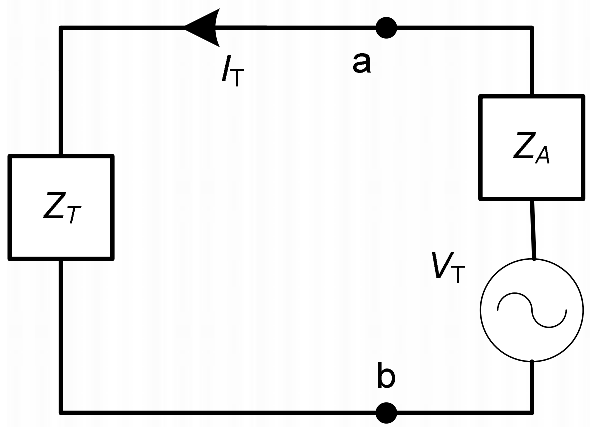
Antenna equivalent Area

- **Used to describe the power capturing characteristics of an antenna when a wave impinges on it**

$$A_e(\theta,\phi) = \text{effective area (aperture) in direction } (\theta,\phi)$$

$$= \frac{\text{Power available at terminals of receiving antenna}}{\text{Incident power flux density from direction } (\theta,\phi)}$$

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2}{2}\frac{R_T}{W_i} \quad [\text{m}^2]$$



P_T : power delivered to load R_T

W_i : power density of incident wave

Antenna Equivalent Area (2)

$$A_e = \frac{|V_T|^2}{2W_i}\left[\frac{R_T}{(R_r + R_{loss} + R_T)^2 + (X_A + X_T)^2}\right]$$

Under conjugate matched condition:

$$A_{em} = \frac{|V_T|^2}{8W_i R_T} = \frac{|V_T|^2}{8W_i}\frac{1}{R_r + R_{loss}}$$

Maximum effective area

$$A_s = \frac{|V_T|^2}{8W_i}\frac{R_r}{(R_r + R_{loss})^2}$$

A_e : effective area, which when multiplied by the incident power density, is equal to power delivered to load R_T

A_s : scattering area, which when multiplied by incident power density, is equal to the scattered or re-radiated power

Antenna Equivalent Area (3)

Under conjugate matched condition:

$$A_{loss} = \frac{|V_T|^2}{8W_i} \frac{R_{loss}}{(R_r + R_{loss})^2} \quad \Rightarrow \quad A_e = A_s + A_{loss}$$

$$A_c = \frac{|V_T|^2}{4W_i R_T} = \frac{|V_T|^2}{8W_i} \frac{R_r + R_{loss} + R_T}{(R_r + R_{loss})^2} \quad \Rightarrow \quad A_c = A_e + A_s + A_{loss}$$

A_{loss} : loss area, which when multiplied by the incident power density, is equal to power delivered to load R_{loss}

A_c : Captured area, which when multiplied by incident power density, is equal to the total power captured by antenna

Antenna Equivalent Area (4)

Example 2.5: a uniform plane wave is incident upon very short dipole, whose radiation resistance is $R_r=80(\pi l/\lambda)^2$. Assume that $R_{loss} = 0$, the maximum effective area reduces to

$$A_{em} = \frac{|V_T|^2}{8W_i R_r}$$

Since the dipole is very short, the induced current can be assumed to be constant and of uniform phase. The induced voltage is

$$V_T = El$$

For a uniform plane wave, the incident power density is given by

$$W_i = \frac{E^2}{2\eta}$$

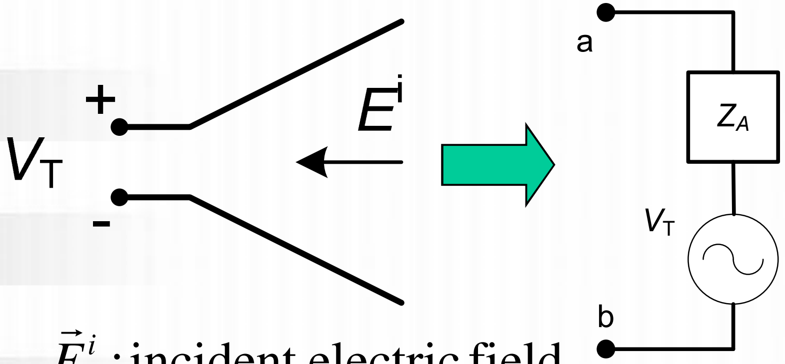
thus

$$A_{em} = \frac{(El)^2}{8(E^2/2\eta)(80\pi^2 l^2/\lambda^2)} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$

Vector Effective Length

- **Vector effective length (or height) is a quantity used to determine the voltage induced on the open-circuit terminals of an antenna when a wave impinges on it. It is a far-field quantity.**

$$\vec{l}_e(\theta,\phi) = \hat{\theta} l_\theta(\theta,\phi) + \hat{\phi} l_\phi(\theta,\phi) \quad [\text{m}]$$



$$\boxed{V_{oc} = \vec{E}^i \cdot \vec{l}_e}$$

$$\boxed{\vec{\mathbf{E}}_a = \hat{\theta} E_\theta + \hat{\phi} E_\phi = -j\eta \frac{kI_{in}}{4\pi r} e^{-jkr} \vec{l}_e}$$

$\vec{E}^i$: incident electric field

$V_T = V_{oc}$: open-circuit voltage

Example 2.6 : The electric field of a short dipole is given by

$$\vec{\mathbf{E}}_a = \hat{\theta} j\eta \frac{kI_{in} l e^{-jkr}}{8\pi r} \sin\theta \quad \Rightarrow \quad \vec{l}_e = -\hat{\theta} \frac{l}{2} \sin\theta$$

Effective area & Directivity

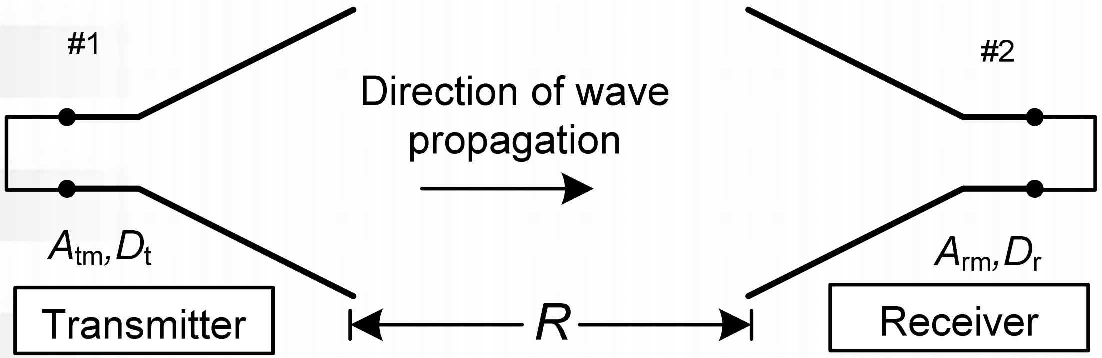
If antenna #1 were isotropic, its radiated power density at a distance R would be

$$W_0 = \frac{P_t}{4\pi R^2}$$

where P_t is the total radiated power. Because of the directivity, the actual power density becomes

$$W_t = W_0 D_t = \frac{P_t D_t}{4\pi R^2}$$

The power collected by the antenna would be



$$P_r = W_t A_r = \frac{P_t D_t A_r}{4\pi R^2}$$

$$\text{or } D_t A_r = \frac{P_r}{P_t} 4\pi R^2 \quad (1)$$

Effective area & Directivity(2)

If antenna #2 is used as a transmitter, 1 as a receiver, and the medium is linear, passive and isotropic, one obtains

$$D_r A_t = \frac{P_r}{P_t} 4\pi R^2 \quad (2)$$

From (1) and (2),

$$\frac{D_t}{A_t} = \frac{D_r}{A_r}$$

Increasing the directivity of an antenna increases its effective area:

$$\frac{D_{0t}}{A_{tm}} = \frac{D_{0r}}{A_{rm}}$$

If antenna #1 is isotropic,

$$A_{tm} = \frac{A_{rm}}{D_{0r}}$$

Effective area & Directivity(3)

For example, if antenna #2 is a short dipole, whose effective area is $3\lambda^2/8\pi$ and directivity is 1.5, one obtains

$$A_{tm} = \frac{A_{rm}}{D_{0r}} = \frac{3\lambda^2}{8\pi}\frac{2}{3} = \frac{\lambda^2}{4\pi}$$

and

$$A_{rm} = D_{0r}A_{tm} = D_{0r}\frac{\lambda^2}{4\pi}$$

In general, maximum effective area of any antenna is related to its maximum directivity by

$$A_{em} = \frac{\lambda^2}{4\pi}D_0$$

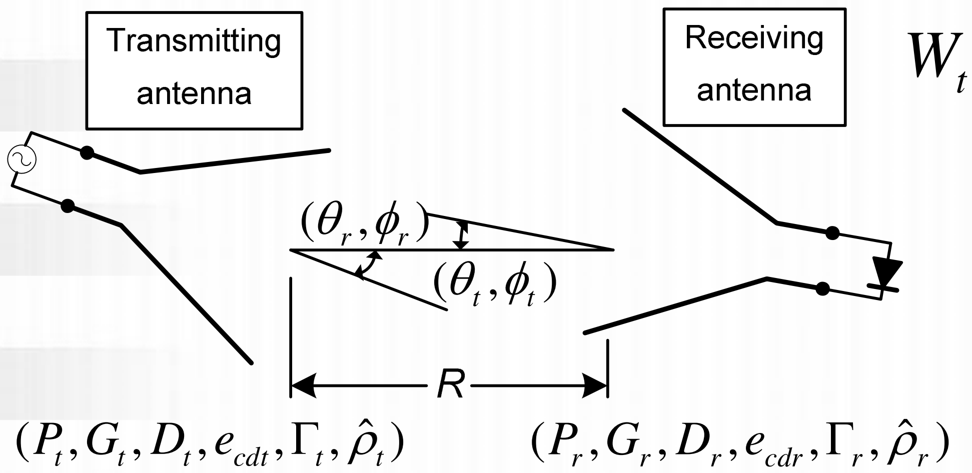
If there're conduction-dielectric loss, polarization loss and mismatch: $\Rightarrow$

$$A_{em} = \frac{\lambda^2}{4\pi}D_0 e_{cd}(1-|\Gamma|^2)|\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

Friis Transmission Equation

- **The Friis transmission equation relates the power received to the power transmitted between two antennas separated by a distance $R > 2D^2/\lambda$.**

Power density at distance R from the transmitting antenna:



$$W_t = \frac{P_t G_t(\theta_t,\phi_t)}{4\pi R^2} = \frac{e_t P_t D_t(\theta_t,\phi_t)}{4\pi R^2}$$

The effective area of the receiving antenna:

$$A_r = e_r D_r(\theta_r,\phi_r)\frac{\lambda^2}{4\pi}$$

The amount of power collected by the receiving antenna:

$$P_r = e_r D_r(\theta_r,\phi_r)\frac{\lambda^2}{4\pi} W_t = e_t e_r \frac{\lambda^2 D_t(\theta_t,\phi_t) D_r(\theta_r,\phi_r) P_t}{(4\pi R)^2} |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

Friis Transmission Equation(2)

The ratio of the received to the input power:

$$\frac{P_r}{P_t} = e_t e_r \frac{\lambda^2 D_t(\theta_t,\phi_t) D_r(\theta_r,\phi_r)}{(4\pi R)^2} |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

or

$$\frac{P_r}{P_t} = e_{cdt} e_{cdr} (1-|\Gamma_t|^2)(1-|\Gamma_r|^2) \frac{\lambda^2 D_t(\theta_t,\phi_t) D_r(\theta_r,\phi_r)}{(4\pi R)^2} |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

For reflection and polarization-matched antennas aligned for maximum directional radiation and reception:

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{0t} G_{0r}$$

Radar Cross Section

- **Radar cross section or echo area (σ) of a target is defined as *the area intercepting that amount of power which, when scattered isotropically, produces at the receiver a density which is equal to that scattered by the actual target*.**

$$\sigma = \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{|\vec{\mathbf{E}}^s|^2}{|\vec{\mathbf{E}}^i|^2}\right] = \lim_{R\to\infty}\left[4\pi R^2 \frac{|\vec{\mathbf{H}}^s|^2}{|\vec{\mathbf{H}}^i|^2}\right]$$

σ : radar cross section [m^2]

R : observation distance from targe [m]

W_i, W_s : incident, scattered power density [W/m^2]

$\vec{\mathbf{E}}^i, \vec{\mathbf{E}}^s$: incident, scattered electric field [V/m]

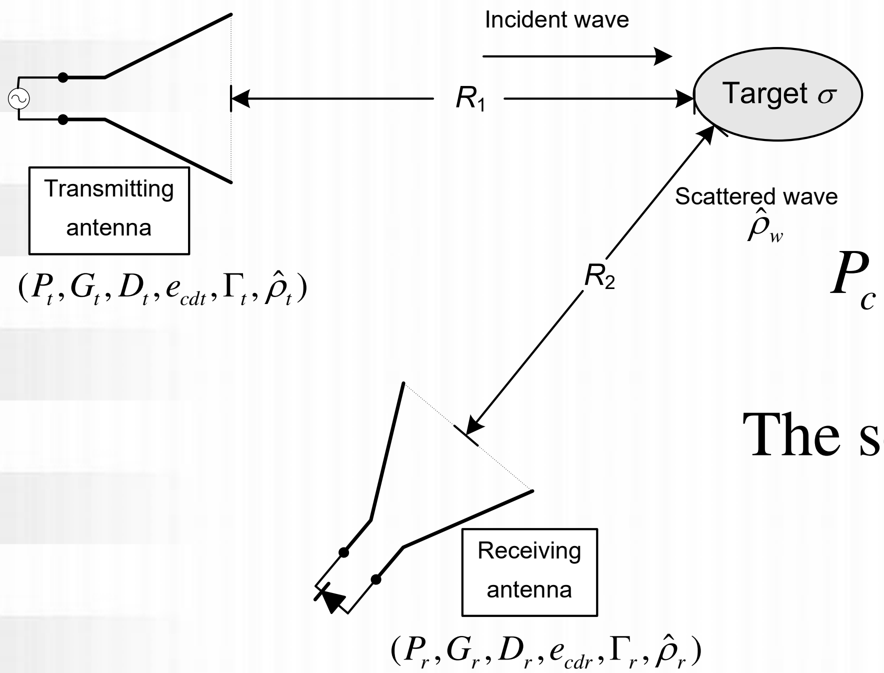
$\vec{\mathbf{H}}^i, \vec{\mathbf{H}}^s$: incident, scattered magnetic field [A/m]

Table 2.2 RCS OF SOME TYPICAL TARGETS

Object	Typical RCSs [22] RCS (m^2)	RCS (dBsm)
Pickup truck	200	23
Automobile	100	20
Jumbo jet airliner	100	20
Large bomber *or* commercial jet	40	16
Cabin cruiser boat	10	10
Large fighter aircraft	6	7.78
Small fighter aircraft *or* four-passenger jet	2	3
Adult male	1	0
Conventional winged missile	0.5	−3
Bird	0.01	−20
Insect	0.00001	−50
Advanced tactical fighter	0.000001	−60

Radar Range Equation

- **The radar range equation relates the power delivered to the receiver load to the power transmitted by an antenna, after it has been scattered by a target with a radar cross section of σ.**



The amount of captured power at the target with the distance R_1 from the transmitting antenna:

$$P_c = \sigma W_t = \sigma \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R_1^2} = \frac{e_t P_t D_t(\theta_t, \phi_t)}{4\pi R_1^2}$$

The scattered power density:

$$W_s = \frac{P_c}{4\pi R_2^2} = e_t \sigma \frac{P_t D_t(\theta_t, \phi_t)}{(4\pi R_1 R_2)^2}$$

Radar Range Equation(2)

The amount of power delivered to the load:

$$P_r = A_r W_s = e_t e_r \sigma \frac{P_t D_t(\theta_t,\phi_t) D_r(\theta_r,\phi_r)}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2$$

Thus,

$$\frac{P_r}{P_t} = A_r W_s = e_t e_r \sigma \frac{D_t(\theta_t,\phi_t) D_r(\theta_r,\phi_r)}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2$$

With polarization loss:

$$\frac{P_r}{P_t} = A_r W_s = e_{cdt} e_{cdr} (1-|\Gamma_t|^2)(1-|\Gamma_r|^2)\sigma \frac{D_t(\theta_t,\phi_t) D_r(\theta_r,\phi_r)}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2 |\hat{\rho}_w \cdot \hat{\rho}_r|^2$$

For reflection and polarization-matched antennas aligned for maximum directional radiation and reception:

$$\frac{P_r}{P_t} = \sigma \frac{G_{0t} G_{0r}}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2$$

Brightness Temperature

- **Energy radiated by an object can be represented by an equivalent temperature known as Brightness Temperature T_B (K).**

$$T_B(\theta,\phi) = \varepsilon(\theta,\phi)T_m = \left(1-|\Gamma|^2\right)T_m$$

where

ε : emissivity (dimensionless) $0 \le \varepsilon \le 1$

T_m : molecular (physical) temperature (K)

$\Gamma(\theta,\phi)$: reflection coefficient of the surface for the polarization of wave

<u>Example</u> Ground 300 K

Antenna Temperature

- **Energy radiated by various sources appears at antenna terminal as antenna temperature, given by**

$$T_A = \frac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta,\phi)G(\theta,\phi)\sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi} G(\theta,\phi)\sin\theta d\theta d\phi}$$

where

T_A : antenna temperature (effective noise temperature of the antenna radiation resistance ; K)

$G(\theta,\phi)$: gain (power) pattern of the antenna

Noise Power

- **Assuming no losses or other contributions between antenna & receiver, noise power transferred to receiver:**

$$P_r = kT_A \Delta f$$

where

P_r **: antenna noise power (W)**

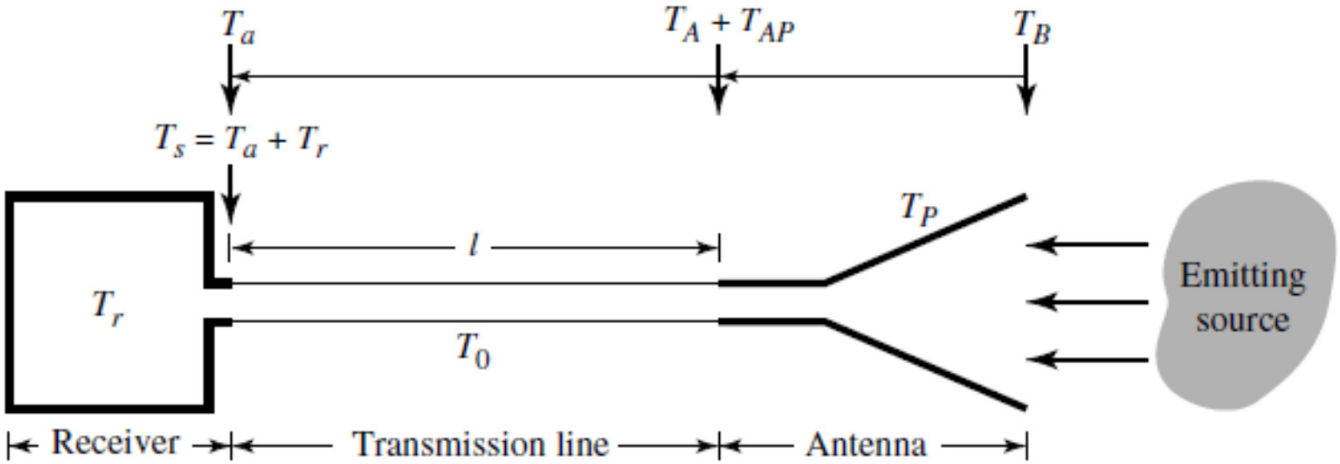
k **: Boltzmann's constant (1.38×10^{-23} J/K)**

T_A **: antenna temperature (K)**

Δf **: bandwidth (Hz)**

System Noise Power Model

- Assume antenna & transmission line are maintained at certain temperature, and transmission line is lossy, then the model below can be used to include all contributions.



Antenna Temperature (2)

- **Effective antenna temperature at receiver terminals:**

$$T_a = T_A e^{-2\alpha l} + T_{AP} e^{-2\alpha l} + T_0 (1 - e^{-2\alpha l})$$

where

T_a : antenna temperature at receiver terminals (K)

T_A : antenna noise temperature at antenna terminals (K)

T_{AP} : antenna temperature at antenna terminals due to physical temperature (K)

$$T_{AP} = (e_A^{-1} - 1) T_P$$

α : attenuation constant of transmission line (Np/m)

e_A : thermal efficiency of antenna (dimensionless)

l : length of transmission line (m)

T_0 : physical temperature of transmission line (K)

System Noise Power

- **noise power transferred to receiver:** $P_r = kT_a\Delta f$
- **If there's thermal noise in receiver:**

$$P_s = k(T_a + T_r)\Delta f = kT_s\Delta f$$

where

P_s **: system noise power (W)**

T_a **: antenna noise temperature at receiver terminals**

T_r **: receiver noise temperature at receiver terminals**

$T_s = T_a + T_r$ **: effective system noise temperature at receiver terminals**

Ex 2.16 **Effective antenna temp = 150 K. Antenna is maintained at 300 K and has thermal efficiency 99%. It is connected to a receiver through 10-m waveguide (loss = 0.13 dB/m, temp = 300 K) Find effective antenna temperature at receiver terminals.**

$$T_{AP} = (e_A^{-1} - 1)T_P = (.99^{-1} - 1)300 = 3.03$$

$$\alpha(\mathrm{Np/m}) = \alpha(\mathrm{dB/m})/8.68 = 0.0149, \alpha l = 0.149$$

$$T_a = T_A e^{-2\alpha l} + T_{AP} e^{-2\alpha l} + T_0(1 - e^{-2\alpha l})$$

$$= 150e^{-2(.149)} + 3.03e^{-2(.149)} + 300(1 - e^{-2(.149)})$$

$$= 190.904\,\mathrm{K}$$