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HOMEWORK SET #3

Note: Please show all the steps leading to the final answer.

1. Define the right and left circularly polarized phasors as follows

$$\hat{R} = \frac{\hat{x} + j\hat{y}}{\sqrt{2}}, \quad (1)$$

$$\hat{L} = \frac{\hat{x} - j\hat{y}}{\sqrt{2}}. \quad (2)$$

   Show that any phasor $\mathbf{E} = \hat{x}E_x + \hat{y}E_y$ can be written as

$$\mathbf{E} = \frac{1}{\sqrt{2}}[\hat{R}E_R + \hat{L}E_L], \quad (3)$$

   where

$$E_{\substack{R \\ L}} = E_x \mp jE_y. \quad (4)$$

   Note that E_R and E_L are the right and left circularly polarized components of $\mathbf{E}$. This implies that any linearly polarized wave can be decomposed to be a sum of two circularly polarized waves.

2. Derive equations (1) and (2) on page 5 of the chap3 slide.

3. Derive the electric and magnetic fields due to a magnetic current source. (Similar to equations (3) and (4) on page 6 of the chap3 slide)

4. Assume that the electric field can be given in terms of a plane wave, i.e.,

$$\mathbf{E} = \mathbf{E}_0 e^{-j\bar{k}\cdot\bar{r}}, \quad (5)$$

   where $\mathbf{E}_0$ denotes the complex constant vector. Also, $\bar{k} = \hat{k}k$ with the unit vector $\hat{k}$ denoting the propagation direction and $k = |\bar{k}| = \omega\sqrt{\mu\epsilon}$ denoting the propagation constant. Likewise, $\bar{r}$ denotes the vector representing the field point. If the field point is far away from the source, i.e., in the "source-free" region, show that
   (a) $\bar{k} \cdot \mathbf{E} = 0$, i.e., the electric field is perpendicular to the propagation direction.
   (b) $\mathbf{H} \cdot \mathbf{E} = 0$, i.e., the electric field is perpendicular to the magnetic field.
   (c) $\bar{k} \cdot \mathbf{H} = 0$, i.e., the magnetic field is also perpendicular to the propagation direction.

5. Determine the radiation resistance, the radiation intensity, and the directivity of the small dipole.