Pongsak Mahachoklertwattana
Thammasat University

HOMEWORK SET #7

Note: Please show all the steps leading to the final answer.

1. The z-plane array factor of an array of isotropic elements placed along the z-axis is given by (assume $\beta = 0$)

$$\text{AF}(z) \quad = \quad (z+1)^4. \tag{1}$$

   Determine

   (a) Number of elements of the discrete array to have such an array factor

   (b) Normalized excitation coefficients of each of the elements of the array **the ones at the edges to be unity**

   (c) Angles in theta of all the nulls of the array factor when spacing d between the elemenets is $d = \lambda/4$ and $d = \lambda/2$.

   Also use Matlab (or other software) to plot the array factor.

2. Determine the current distribution and the approximate radiation pattern of a linear array placed along the z-axis whose desired radiation pattern is symmetrical about $\theta = \pi/2$ and it is given by

$$\text{AF}(\theta) \quad = \quad \begin{cases} 1 & 40 \le \theta < 140^\circ \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

   Use 11 and 21 elements with a spacing of $d = \lambda/2$ between them. Also use Matlab (or other software) to plot the array factor.

3. Show that

$$\int_{-l/2}^{l/2} \frac{k}{\pi\sqrt{2}} \frac{\sin\left(\frac{kz'}{\sqrt{2}}\right)}{\frac{kz'}{\sqrt{2}}} e^{j\xi z'} dz' \quad = \quad \frac{1}{\pi}\left\{ S_i\left[\frac{l}{\lambda}\pi\left(\cos\theta + \frac{1}{\sqrt{2}}\right)\right] - S_i\left[\frac{l}{\lambda}\pi\left(\cos\theta - \frac{1}{\sqrt{2}}\right)\right]\right\} \tag{3}$$

   where $\xi = k\cos\theta$ and S_i is the sine integral function given by

$$S_i(x) \quad = \quad \int_0^x \frac{\sin y}{y} dy. \tag{4}$$